Lecture 6.0:

Detection of Optical Radiation & Laser Ranging:

A Gentle Introduction (or likely a Reminder…)

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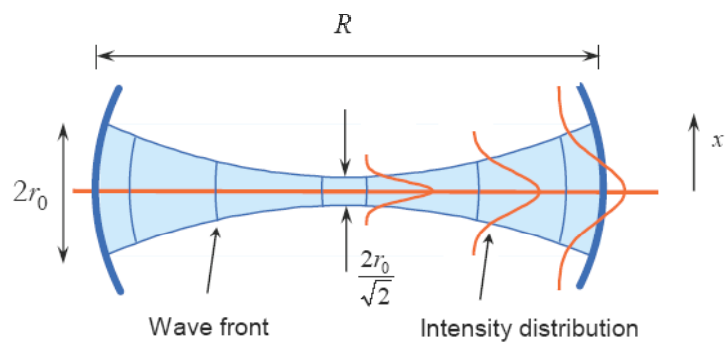
Overview

- Direct Detection Laser Radar Systems
  - Laser Fundamentals
  - Introduction to Laser Radar (LADAR)
  - Application and Advantages
- Detection Regimes and Figures of Merit
  - Laser Radar System Design
  - Laser Radar Theory
- Coherent Detection
  - Heterodyne
  - Homodyne detection
  - Signal-to-Noise Calculation
- Potential Opportunity for Gravitational Experiments

Laser Fundamentals



Wavelength range

- 10 - 15 nm → 100 - 500 μm (100 eV → 0.01 eV)
- tunable lasers: dye laser, diode laser, Ti:Sapphire laser …

Monochromaticity

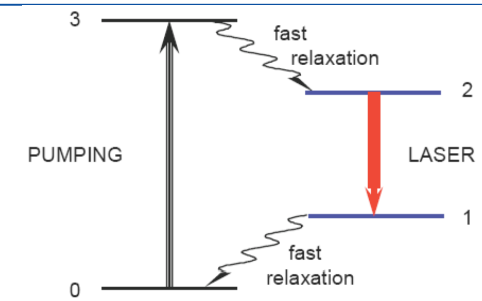
- typically $\Delta\nu$ ~ 1 MHz - 1 GHz
- at best $\frac{\Delta\lambda}{\lambda} = \frac{\Delta\nu}{\nu} \approx \frac{1-100 \text{ Hz}}{5 \cdot 10^{14} \text{ Hz}} \sim 10^{-15} - 10^{-12}$

Directionality

- $\delta\Theta \approx \frac{\lambda}{d}$, (d = beam diameter)
- typically $\delta\Theta$ ~1 mrad, with extra collimation → 1 μrad

Coherence

- coherence time $\Delta\tau = 1/\Delta\nu$
  - e.g. $\Delta\nu$ = 1 MHz → $\Delta\tau$ = 1 μs
- coherence length $\Delta z = c \cdot \Delta\tau$
  - e.g. $\Delta z = c \cdot \Delta\tau = 3 \cdot 10^8$ m/s · 1 μs = 300 m

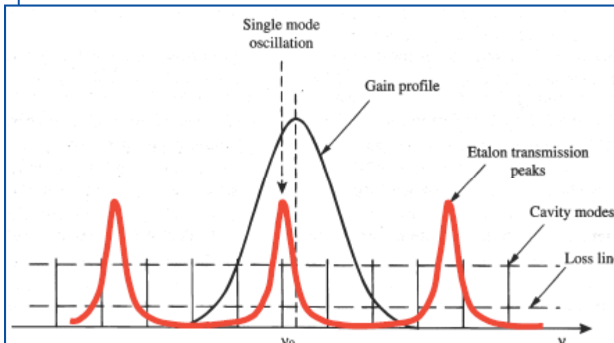


Spectral brightness

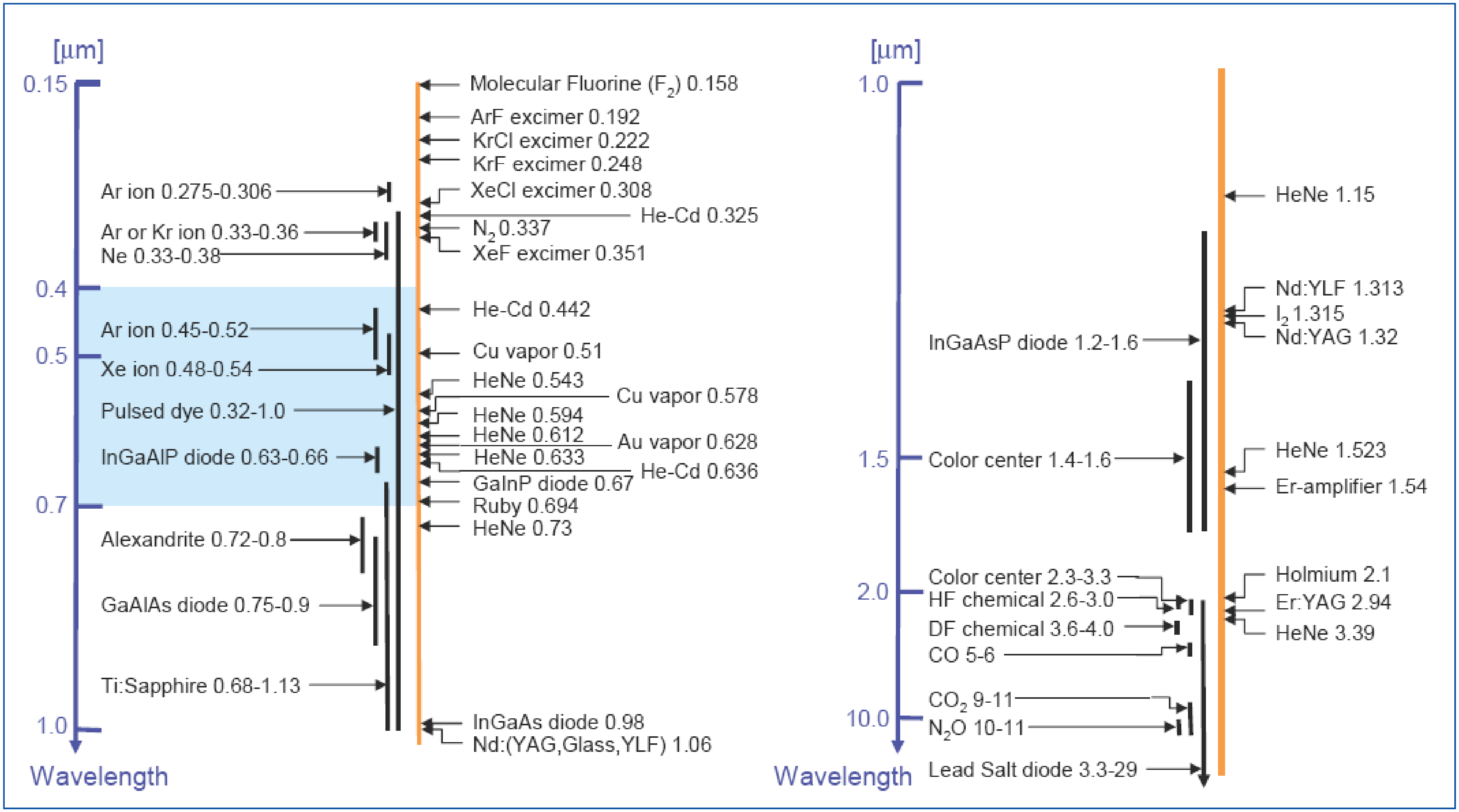
- $\beta_\nu = P_\nu / A\,\Delta\Omega\,\Delta\nu$ [W/cm²-sr-Hz]
- Sun $\beta_\nu \sim 1.5 \cdot 10^{-12}$ W/cm²-sr-Hz
- HeNe-laser (1 mW) β_ν ~ 25 W/cm²-sr-Hz
- Nd:glass-laser (10 GW) $\beta_\nu \sim 2 \cdot 10^8$ W/cm²-sr-Hz

Operation mode

- CW (continuous wave)
- pulsed operation
  - shortest pulses < 10 fs (10^{-14} s)
  - peak power at best tens of TW



Most Common Laser Lines



- Lasers based on Neodymium are the most common solid-state lasers
  - Supports both CW or pulsed operations; YAG – most common host material

Types of Laser

Typical Lasers:	Carrier Wavelength (μm):
CO_2	9.2 – 11.2 (nominally 10.6)
Er:YAG	2
Raman-Shifted Nd:YAG	1.54
Nd:YAG	1.064
GaAlAs	0.8 – 0.904
HeNe	0.63
Frequency-Doubled Nd:YAG	0.532

Detection Technique	Interferometer Type	Modulation Technique
Direct Detection	N/A	Pulsed Amplitude Modulation (AM)
Coherent Detection	Heterodyne Homodyne Offset Homodyne	Pulsed Amplitude Modulation (AM) Frequency Modulation (FM) Hybrid (AM/PM, Pulse Burst) None (CW)

Source: The IR & EO Systems Handbook, Vol. 6, Ch. 1

The Electromagnetic Spectrum

Spectral Band:	Wavelength Range (µm):
Vacuum Ultraviolet (UV)	0.05 to 0.20
Short UV (UV-C)	0.20 to 0.29
Mid-Wave UV (UV-B)	0.29 to 0.32
Long-Wave UV (UV-A)	0.32 to 0.40
Visible	0.40 to 0.70
Violet	0.40 to 0.46
Blue	0.46 to 0.49
Green	0.49 to 0.55
Yellow	0.55 to 0.58
Orange	0.58 to 0.60
Red	0.60 to 0.70
Near Infrared (IR) (NIR)	0.7 to 1.1
Short-Wave IR (SWIR)	1.1 to 2.5
Mid-Wave IR (MWIR)	2.5 to 7.0
First thermal imaging band	3.0 to 5.5
Blue spike plume	4.1 to 4.3
Red spike plume	4.3 to 4.6
Long-Wave IR (LWIR)	7.0 to 15.0
Second thermal imaging band	8.0 to 14.0
Very-Long-Wave IR (VLWIR)	> 15
Extreme IR	15 to 100
Near Millimeter	100 to 1000
Millimeter	1000 to 10,000 (1 to 10 mm)

Satellite & Lunar Laser Ranging at 0.532 µm (→ Green, 0.49 to 0.55)

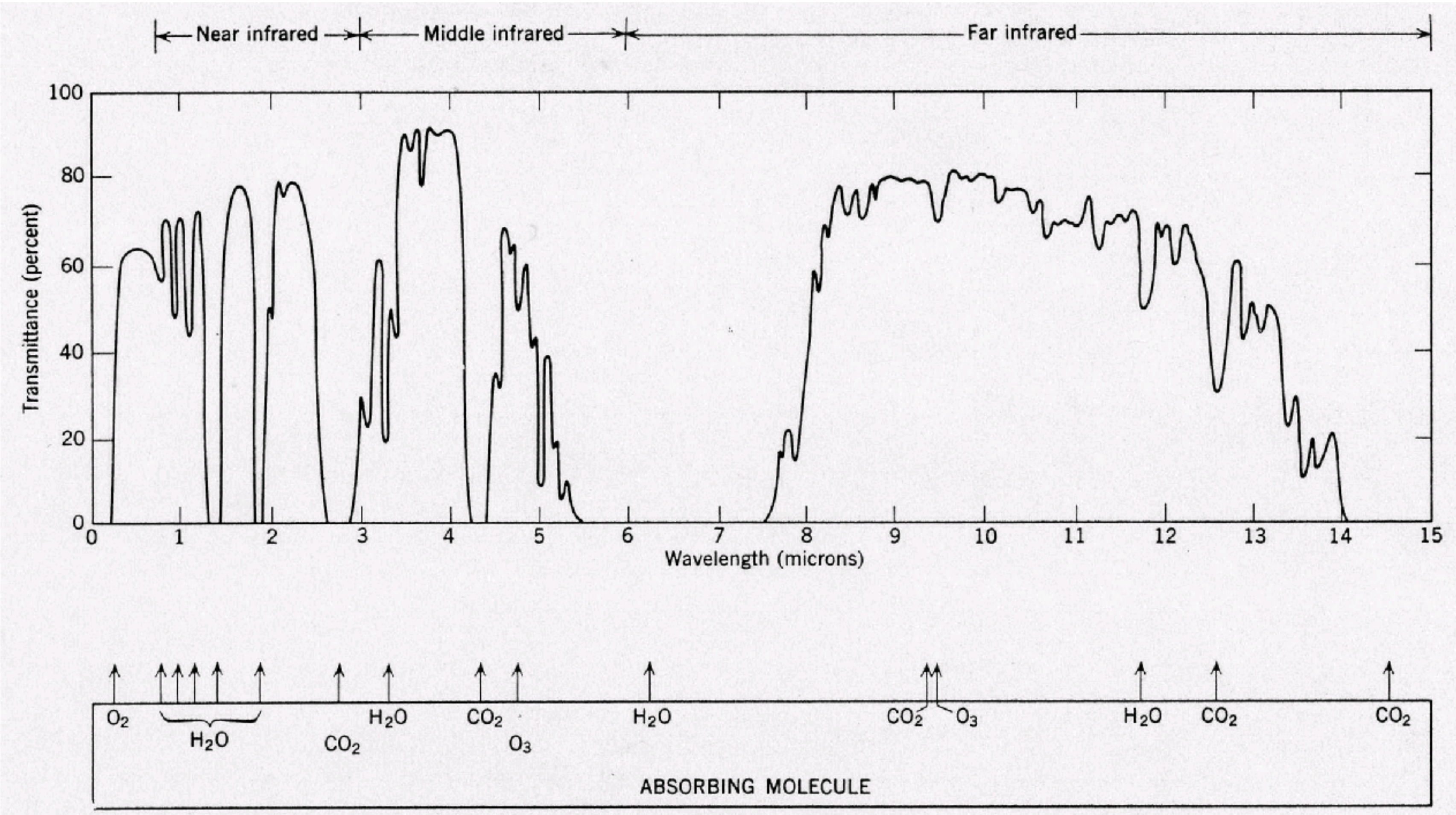
Semi-Active Laser (SAL) Seeker at 1.064 µm (→ 0.60 to 0.70 / 0.7 to 1.1 boundary)

Passive MWIR Imaging Infrared (IIR) Seeker (→ Mid-Wave IR, 2.5 to 7.0; → Long-Wave IR, 7.0 to 15.0)

Active Millimeter Wave (MMW) Radar Seeker at 35 GHz (8.6 mm wavelength) (→ Extreme IR / Near Millimeter rows)

Transmittance of the Atmosphere

(2 km Horizontal Path at Sea Level)

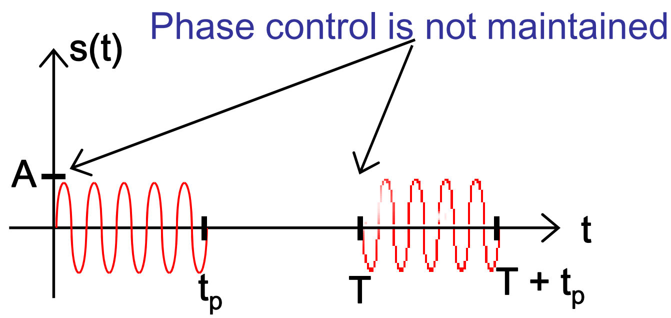


Source: RCA Electro-Optics Handbook

Types of LADAR (or Laser Transponder)

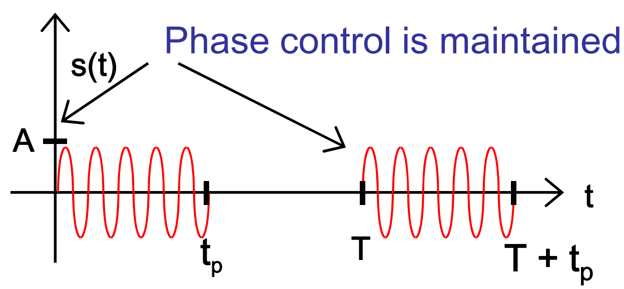
Direct Detection:

Incoherent laser system which measures range and intensity. Uses backscattered signal strength to provide a measurement.



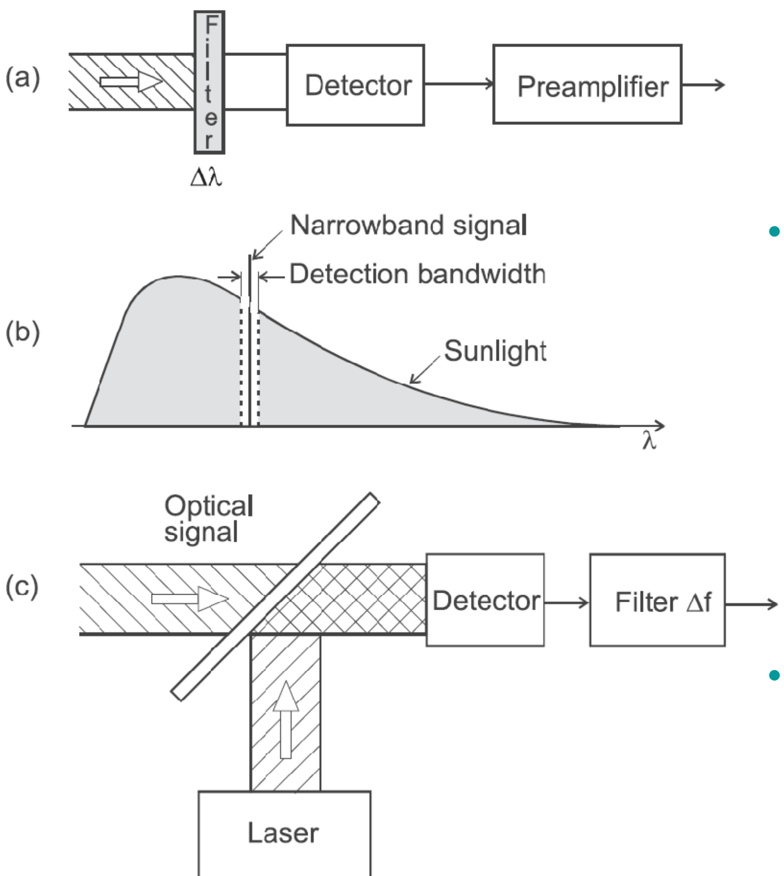
Coherent Detection (Heterodyne or Homodyne):

Coherent laser system which measures range, intensity, and Doppler. Uses optical phase alignments to mix and detect phase shifts (Doppler).



Source: Laser Radar Systems, A. V. Jelalian

Coherent Detection



Comparison of coherent versus incoherent optical detection

- Fig. (a) presents a **direct** detection system. To narrow the received optical bandwidth we have applied a filter to limit the spectral range of radiation reaching the detector — Fig. (b).
- Optical filters based on the Bragg effect can have 5-nm bandwidth for λ=1.56 μm, corresponding to a detection band, Δf, equal to 600 GHz. This wide bandwidth, of what is a reasonably narrow optical filter illustrates that application of an optical filter cannot easily ensure narrow wavelength selection of a photoreceiver.
- A widely-spaced Fabry-Perot filter can achieve Δf of several hundred MHz, but is costly and difficult to stabilize, so impractical for most systems.
- A simple addition of a beam-splitter & an additional coherent light source (local oscillator), provides a coherent detection scheme, that can use either **heterodyne** and **homodyne** detection systems

Basic Concepts of Coherent Detection

- The electric field associated with the received optical signal is given by

$$E_s = A_s \exp[-j(\omega_0 t + \phi_s)]$$

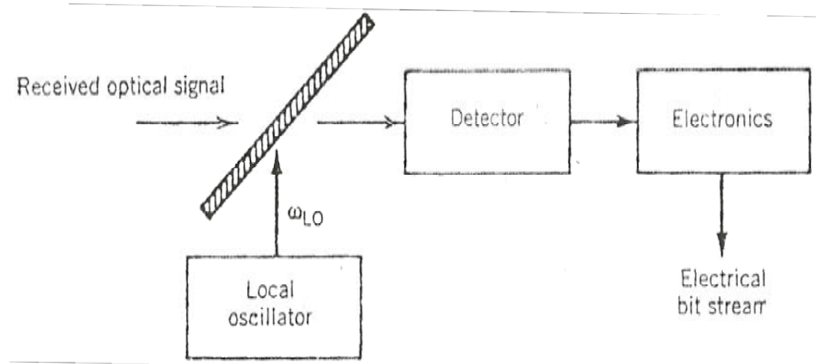
where ω_0 is the carrier frequency, A_s is the amplitude, and ϕ_s is the phase.

- The optical field associated with the local oscillator (LO) is given as

$$E_{LO} = A_{LO} \exp[-j(\omega_{LO} t + \phi_{LO})]$$

where A_{LO}, ω_{LO} , and ϕ_{LO} are the amplitude, the frequency, and the phase of the LO.

Coherent detection scheme



- The photodetector (PD) responds to the intensity $I=|E_s+E_{LO}|^2$. Since the optical power is proportional to I, the received power at the PD is given by

$$P_R(t) = K\,|E_s + E_{LO}|^2 = P_s + P_{LO} + 2\sqrt{P_s P_{LO}}\cos(\omega_{IF} t + \phi_s - \phi_{LO})$$

where $P_s = K A_s^2$, $P_{LO} = K A_{LO}^2$, $\omega_{IF} = \omega_0 - \omega_{LO}$

Homodyne Detection

- Depending on whether or not $\omega_{IF} = 0$, there are two different coherent detection techniques: **homodyne** and **heterodyne** detection.
- In **homodyne** coherent-detection technique the LO frequency ω_{LO} is selected to coincide with the signal-carrier frequency ω_0 so that $\omega_{IF} = 0$.
- The photocurrent ($I = RP$, where R is the detector responsivity) is given by

$$I(t) = R(P_s + P_{LO}) + 2R\sqrt{P_s P_{LO}} \cos(\phi_s - \phi_{LO})$$

Typically, $P_{LO} >> P_s$, and $P_s + P_{LO}$ can be approximated by a constant P_{LO}.

- When the LO phase is locked to the signal phase: $\phi_{LO} = \phi_s$, the homodyne signal

$$I_p(t) = 2R\sqrt{P_s(t) P_{LO}}$$

- In the direct-detection case: $I_{dd}(t) = RP_s(t)$. The average electrical signal power is increased by a factor of $4P_{LO} / \overline{P}_s$ by the use of homodyne detection.
- The photocurrent I_p contains the signal phase explicitly, it is possible to transmit information by modulating the phase or the frequency of the optical carrier.

Heterodyne Detection

- In heterodyne detection, the local-oscillator frequency ω_{LO} is chosen to differ from the signal-carrier frequency ω_0 such that the intermediate frequency ω_{IF} is in the microwave region ($\nu_{\mathrm{IF}} = 1$ GHz)
- Since the optical power is given by

$$P_R(t) = P_s + P_{\mathrm{LO}} + 2\sqrt{P_s P_{\mathrm{LO}}}\cos(\omega_{\mathrm{IF}} t + \phi_s - \phi_{\mathrm{LO}})$$

together with $I = RP$, the detector current is

$$I(t) = R(P_s + P_{\mathrm{LO}}) + 2R\sqrt{P_s P_{\mathrm{LO}}}\cos(\omega_{\mathrm{IF}} t + \phi_s - \phi_{\mathrm{LO}})$$

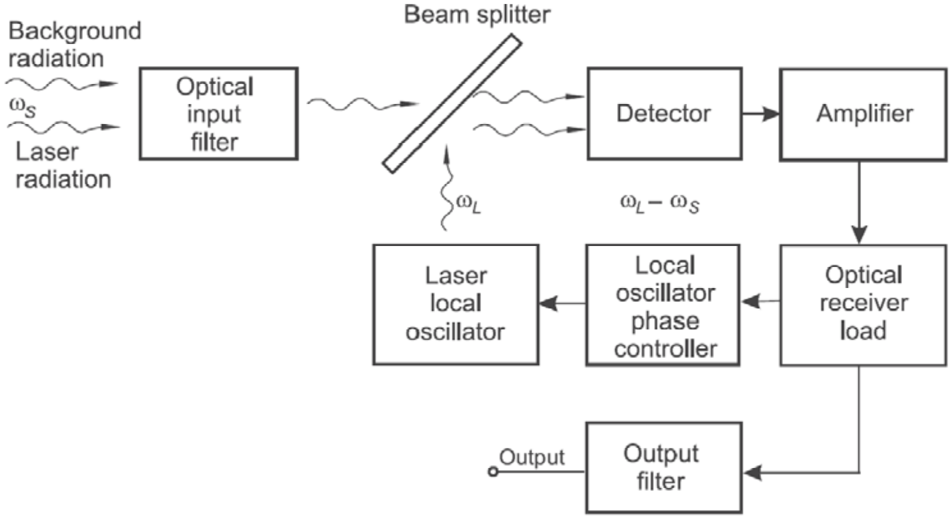
- Since $P_{\mathrm{LO}} >> P_s$ in practice, the nearly constant dc term can be filtered out easily. The heterodyne signal is thus given by

$$I_{ac}(t) = 2R\sqrt{P_s(t) P_{\mathrm{LO}}}\cos(\omega_{\mathrm{IF}} t + \phi_s - \phi_{\mathrm{LO}})$$

- Information can be transmitted through amplitude, phase, or frequency modulation of the optical carrier wave in heterodyne detections.

Coherent Detection Optical Receivers

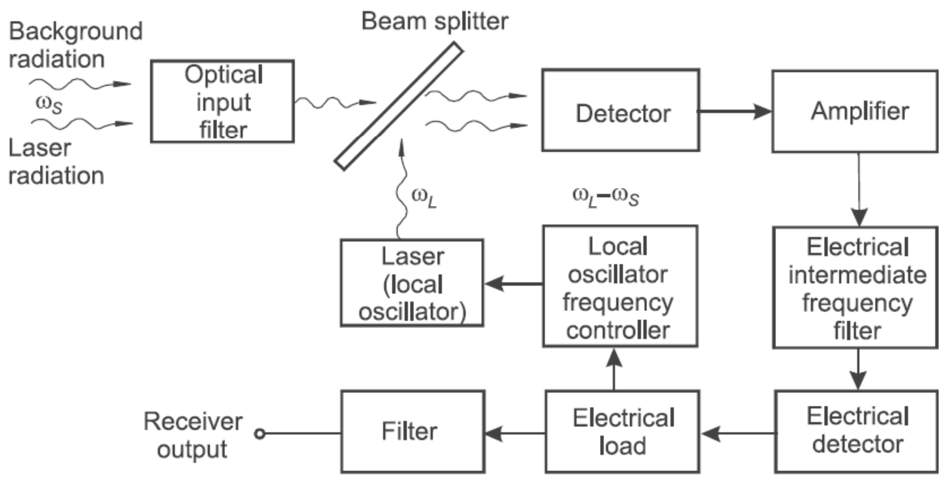
Homodyne detection optical receiver:



$$\frac{S}{N} = \frac{2\eta P_S}{h\nu\Delta f M^x}$$

- Homodyne receivers are used in the most sensitive coherent systems.
- In practice, construction of such receivers is difficult, as LO must:
  - have excellent spectral purity, and
  - no power fluctuations.

Heterodyne detection optical receiver:



$$\frac{S}{N} = \frac{\eta P_S}{h\nu\Delta f M^x}$$

- Heterodyne detection is used for construction of Doppler velocimeters and laser rangefinders and as well as in spectroscopy (particular LIDAR systems).

Heterodyne Detection and the SNR

- LO amplifies the received signal, thereby improving the SNR. However, the SNR improvement is lower by a factor of 2 (or by 3dB) compared with the homodyne case (referred to as the 3-dB heterodyne detection penalty).
  - The advantage gained at the expense of the 3-dB penalty is that the receiver design is much simplified: there is no need for an optical phase-lock loop.
- Fluctuations in both ϕ_s and ϕ_{LO} still need to be controlled by using narrow-linewidth semi-conductor lasers for both optical sources.
- The receiver current fluctuates due to shot noise & thermal noise. The variance σ^2 of current fluctuations is given by

$$\sigma^2 = \sigma_{\text{shot}}^2 + \sigma_T^2 = 2q(I_{\text{sig}} + I_{\text{d}})\Delta f + \frac{4k_B T}{R_L}\Delta f$$

- In the heterodyne case, SNR is given by

$$\text{SNR} = \frac{\langle I_{\text{ac}}^2 \rangle}{\sigma^2} = \frac{2R^2 \overline{P}_s P_{\text{LO}}}{2q(RP_{\text{LO}} + I_{\text{sig}} + I_{\text{d}})\Delta f + \frac{4k_B T}{R_L}\Delta f}$$

Signal-to-Noise Ratio (SNR)

- The LO power P_{LO} can be made large enough that the receiver noise is dominated by shot noise. Specifically,

$$\sigma_s^2 >> \sigma_T^2 \quad \text{when} \quad P_{LO} \gg \sigma_T^2 / 2qR\Delta f$$

- In the shot-noise limit the SNR is thus given by

$$\text{SNR} = \frac{R\overline{P}_s}{qR\Delta f} = \frac{\eta \overline{P}_s}{h\nu \Delta f} \qquad \text{where} \qquad R = \frac{\eta q}{h\nu}$$

- At the bit rate B, the average signal power is $\overline{P}_s = N_p h\nu B$. Typically, Δf can be assigned to be $B/2$. By using these values of $\overline{P}_s$ and Δf, the SNR for heterodyne detection is given by the simple expression: $\text{SNR} = 2\eta N_p$.
- For the case of homodyne detection, SNR is larger by a factor of 2 and is given by $\text{SNR} = 4\eta N_p$.

Definitions

LADAR

LAser Detection And Ranging

Light Amplification Stimulated Emission of Radiation

Overview of Laser Radar Systems

- Laser Radar (Ladar) is very similar to Radar (Radio Detection and Ranging)
  - Laser pulses are actively transmitted to illuminate the target of interest
  - Range to a target can be determined by measuring the round-trip time delay between a transmitted pulse and the received reflected pulse
  - Doppler shift due to target radial motion can be measured if the Ladar maintains coherency in the receiver
  - Targets can be resolved in angle (Azimuth / Elevation) based on the Ladar beamwidth
  - Monostatic and Bistatic (separate transmitter and receiver) geometries are possible
  - Applications include Sensors, Seekers and Laser Rangefinders (LRFs)
- Ladar differs from Radar Systems in several distinct ways
  - The wavelength of operation is much shorter (microns instead of mm to cm)
  - The beamwidth can therefore be much narrower than for a radar system with the same aperture size
  - The range is often much shorter due to the atmospheric attenuation at optical and IR wavelengths
  - Weather affects Ladar much more than Radar due to the shorter wavelength
- Because of these considerations, Ladar is more often used for tracking and discrimination rather than for wide-area search

Laser Radar Functions and Measurements

Typical Functions	Measurements
Tracking Moving Target Indication (MTI) Range Imaging Velocimetry Autonomous Vehicle Navigation Target Identification Terrain Mapping	Amplitude (Reflectance) Range (Time Delay) Velocity (Doppler Shift or Differential Range) Angular Position

Source: The IR & EO Systems Handbook, Vol. 6, Ch. 1

Range Measurement, Resolution, and Accuracy

- The range to the target, R, is measured using the round-trip time delay, T_d, of the transmitted signal, using distance = (velocity)(time)

$$2R = cT_d \qquad \text{or} \qquad R = {cT_d}/{2}$$

- The range resolution (ability to resolve closely spaced objects in range) is proportional to the compressed pulsewidth, τ_{compress}, and inversely proportional to the waveform bandwidth, B,

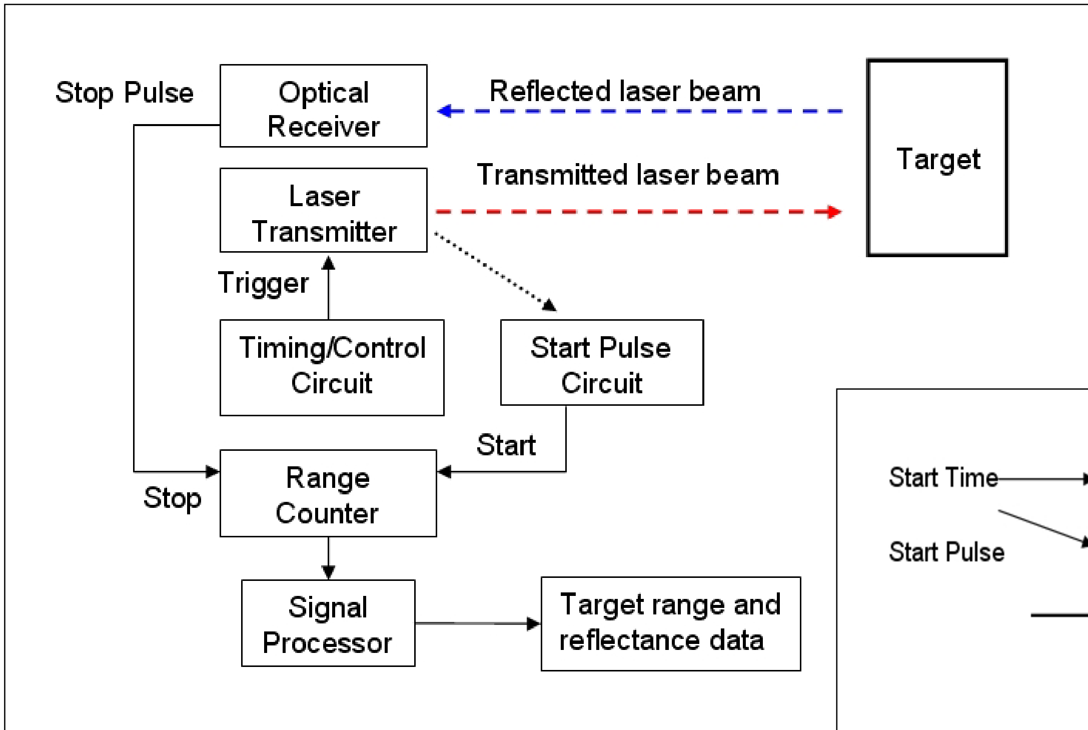
$$\Delta R = \frac{c\tau_{\text{compress}}}{2} = \frac{c}{2B}$$

- The range measurement accuracy is dependent on the quality of the measurement, characterized in terms of signal-to-noise ratio (SNR), as

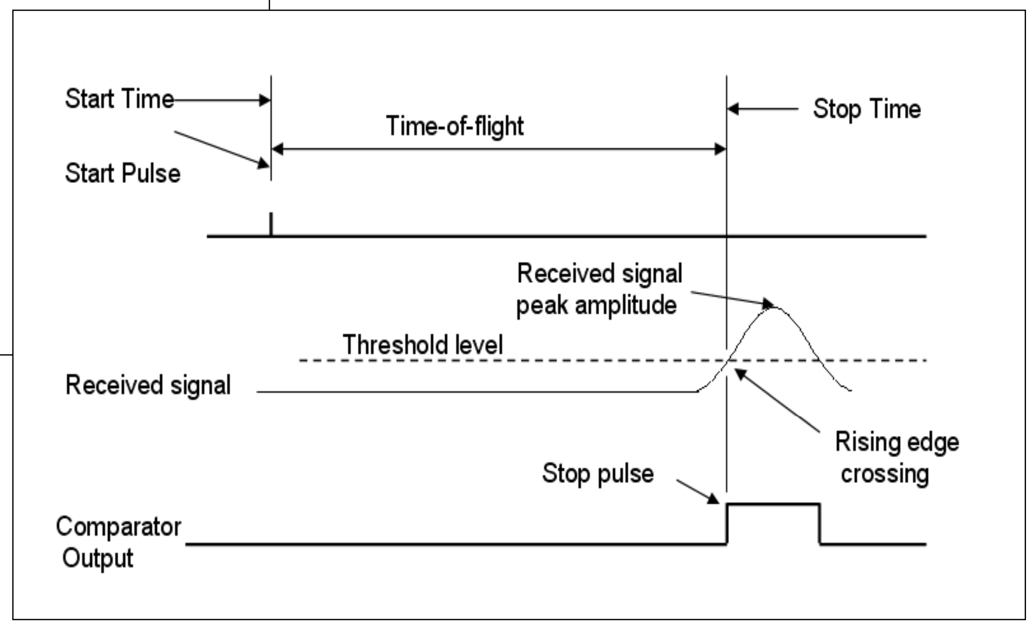
$$\Delta T_R \cong \frac{1/B}{\sqrt{\text{SNR}}}$$

Source: Introduction to Sensor Systems – Havanessian

Block/Timing Diagram



$$\text{Range} = \frac{cT}{2}$$

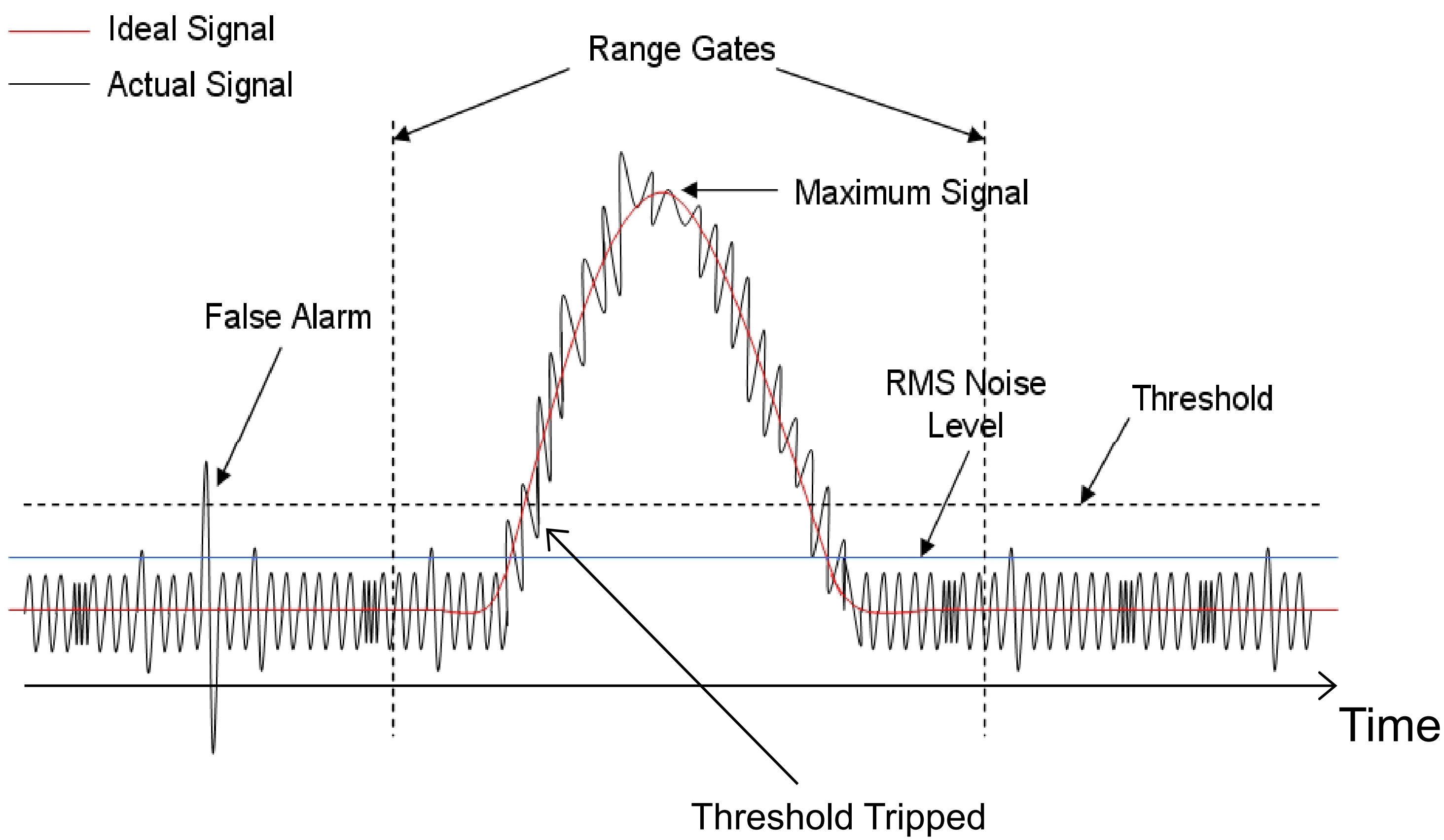


Intensity ~ Material Reflectance

Source: Advanced Design of Direct Detection Laser Radar – H.N. Burns

Range Gating

SNR = Maximum Signal Received / Root Mean Square (rms) of the noise



Source: Advanced Design of Direct Detection Laser Radar – H.N. Burns

Signal-to-Noise Ratio for a Detector

- The common problem of a photon detector:

  i) to terminate the photodetector with a suitable load resistor, and

  ii) to trade off the performance between bandwidth $\Delta f = \frac{1}{2\pi R_L C}$ and signal-to-noise ratio (S/N).

- The total noise is a sum of shot noise and thermal (Johnson) noise:

$$I_n^2 = 2q(I_{ph} + I_d)\Delta f + \frac{4kT\Delta f}{R_L} \quad \Rightarrow \quad N = \left[2q(I_{ph} + I_d)\Delta f G^2 F + \frac{4kT\Delta f}{R_L}\right]^{1/2}$$

G – internal gain; F – excess noise factor

- The signal-to-noise ratio is given as:

$$\frac{S}{N} = \frac{I_{ph}}{[2q(I_{ph} + I_d)\Delta f F + (4kT\Delta f / R_L G^2)]^{1/2}}$$

- Introduce the threshold of quantum regime: $I_{ph0} = I_d + \frac{2kT/q}{R_L F G^2}$

- The signal-to-noise ratio becomes: $\frac{S}{N} = \frac{I_{ph}}{[2q(I_{ph} + I_{ph0})\Delta f F]^{1/2}}$

Detection Regimes

- For signals: $I_{ph} > I_{ph0}$ and $F = 1$: $\Rightarrow$ $\frac{S}{N} = \left(\frac{I_{ph}}{2q\Delta f}\right)^{1/2}$

This S/N is the **quantum noise limit of detection**. This limitation cannot be overcome by any detection system, whether operating on coherent or incoherent radiation. This is a direct consequence of the quantitative nature of light and the Poisson photon arrival statistics.

- For signals: $I_{ph} < I_{ph0}$ and $F = 1$:

$$\Rightarrow \quad \frac{S}{N} = \frac{I_{ph}}{(2qI_{ph0}\Delta f)^{1/2}}$$

That is, the S/N ratio is proportional to the signal, and the noise has a constant value, primarily given by the load resistance.
This is **the thermal regime of detection**.

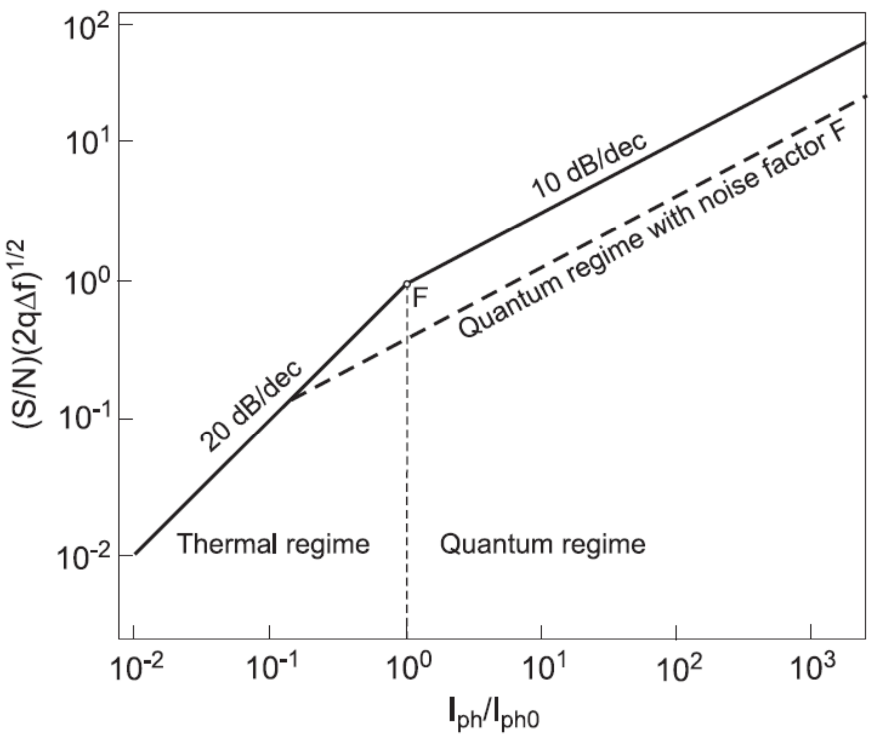


Figure: The S/N ratio of a photodetector, as a function of the input signal, in the thermal and quantum regimes of detection.

Figure of Merit: Detectivity

- Detectivity: $$D^* = \frac{(A\Delta f)^{1/2}}{P}\frac{S}{N}$$ used to compare different detectors
- The ultimate performance of detectors is reached when the detector and amplifier noise are low compared to the photon noise.
- The photon noise is fundamental, as it arises from the detection process itself, as a result of the discrete nature of the radiation field.

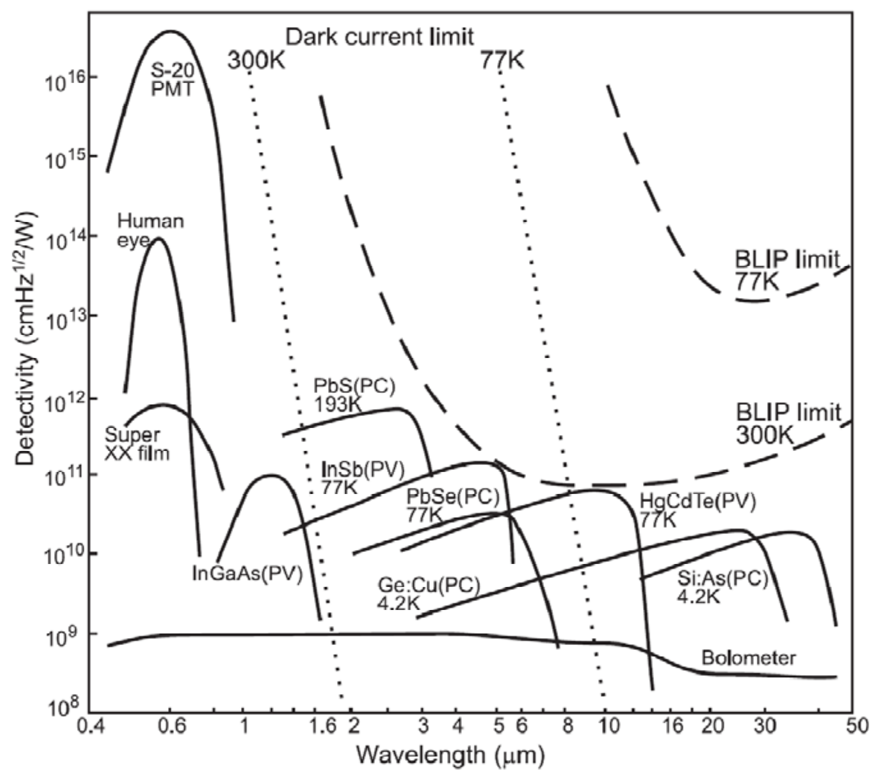
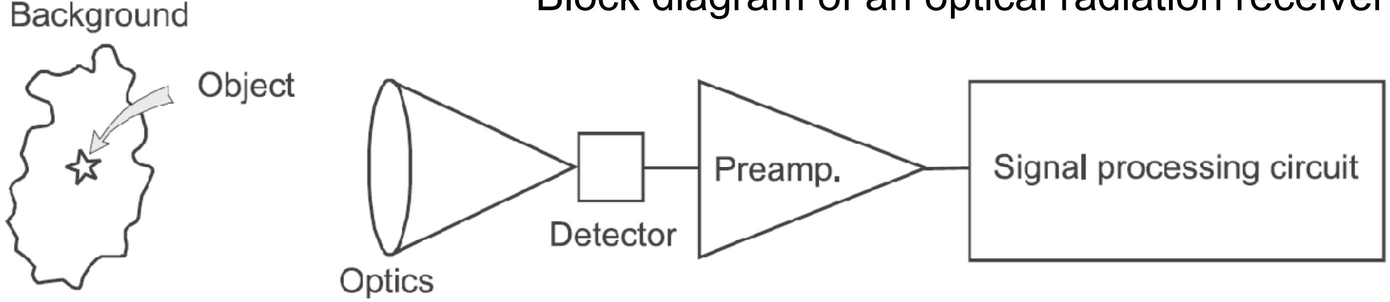


Figure: Detectivity as a function of wavelength for a number of different photodetectors.

The background limited infrared photodetector (BLIP) and the dark current limits are indicated. PC — photoconductive detector, PV — photovoltaic detector, and PMT — photomultiplier tube

Direct Detection Systems

Block diagram of an optical radiation receiver



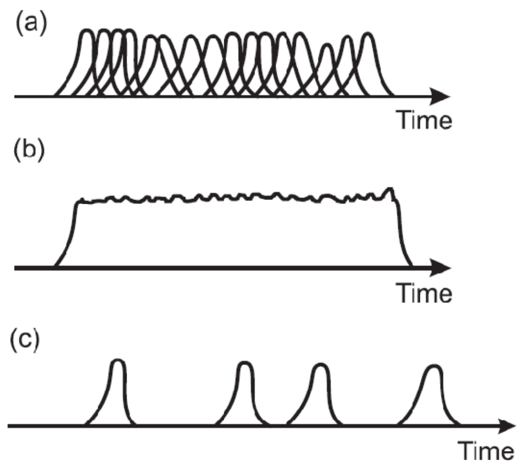
- In direct detection systems the detector converts the incident radiation into a photo-signal that is then processed electronically.
- A preamplifier should have low noise and a sufficiently wide bandwidth to ensure faithful reproduction of the temporal shape of an input signal.
- One needs to minimize noises, i.e., background noise, photodetector noise, biasing resistors noise, and any additional noises of signal processing.
- If further noise minimization of the first photoreceiver stages is not possible, advanced methods of optical detection can sometimes be used to recover information carried by optical radiation signals of extremely low power.
- Heterodyne and homodyne detection can be used to reduce the effects of amplifier noise.

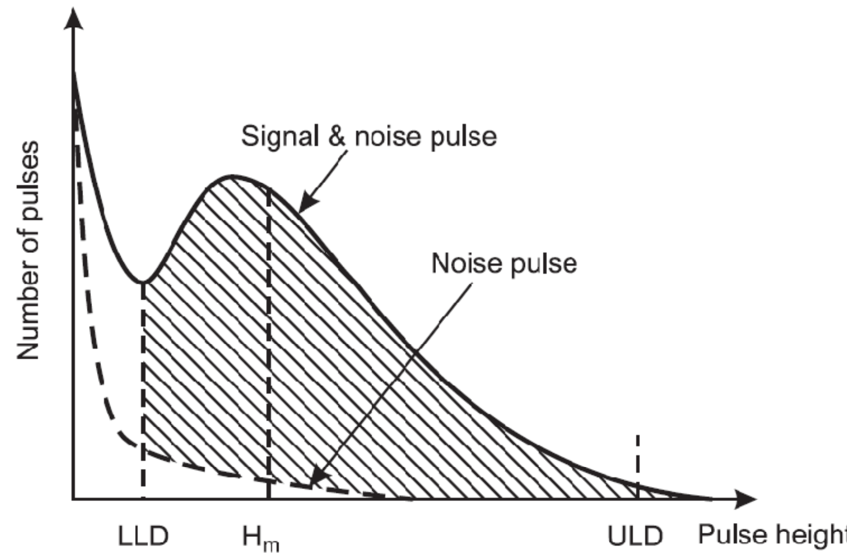
Photon Counting Techniques

- Photon counting is one effective way to use a photomultiplier tube for measuring very low light (e.g. astronomical photometry and etc.)

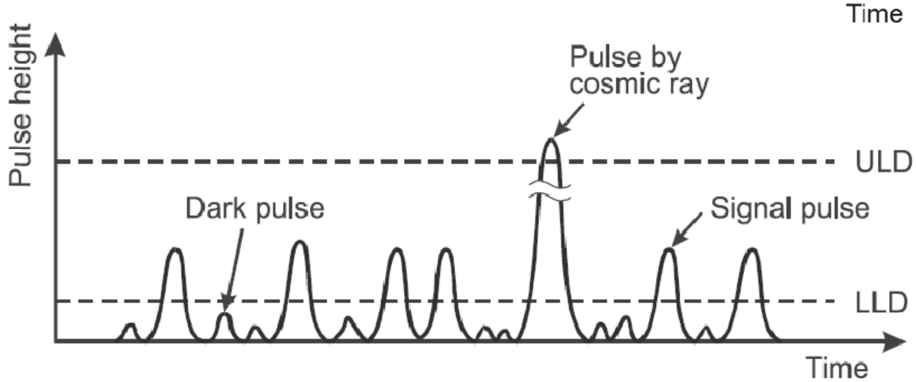
A pulse signals at the output of a photomultiplier tube

Single photon events





Typical pulse height distribution.



Output pulse and discriminator level

LLD: the low level discrimination

ULD: is the upper level discrimination

Avalanche photodiodes (APDs) can get an output pulses for each detected photons; in Geiger mode 1 photon can trigger 10^8 carriers

APD is well suited for application which requires high sensitivity and fast response time: laser rangefinders, fast receiver modules, lidar, ultrasensitive spectroscopy.

SNR for Photodiodes

$$\frac{S}{N} = \frac{I_{ph}}{\left[2q\left(I_{ph} + I_d + I_b\right)\Delta f + \frac{4kT\Delta f}{R_L} + I_a^2\right]^{1/2}}$$

The first term represents the shot noise component of the photocurrent, the dark current and the background, whereas the second term is the thermal noise of load resistance of a photodetector, and the third term is the preamplifier noise.

$$\frac{S}{N} = \left(\frac{I_{ph}}{2q\Delta f}\right)^{1/2} = \left(\frac{\eta\Phi_e\lambda}{2hc\Delta f}\right)^{1/2} = \left(\frac{\eta A E_e\lambda}{2hc\Delta f}\right)^{1/2}$$

λ is the wavelength of incident radiation, Φ_e is the incident radiant flux [W] and E_e is the detector's irradiance. This noise is also called quantum limited noise.

If the power of an optical signal is low, the shot noise is negligible in relation to the thermal noise, then

$$\frac{S}{N} = \frac{\eta q\Phi_e}{h\nu}\left(\frac{R_L}{4kT\Delta f}\right)^{1/2}$$

It is evident that when a photoreceiver is limited by the thermal noise, it is thermally dependent

Laser Transmit Power

Transmit Power (P_T):

$$E_{LP} = \frac{P_{avg}}{n_B \cdot \mathrm{PRF}} \quad \text{[Joules]}$$

$$P_T = \frac{E_{LP}}{\tau} \quad \text{[Watts]}$$

E_{LP} = Laser Pulse Energy [Joules]
PRF = Pulse Repetition Frequency [Hz]
τ = pulse width [s]

P_{avg} = Average Power of Laser Beam [W]
P_T = Transmitted Pulse Power [W]
n_B = number of beams

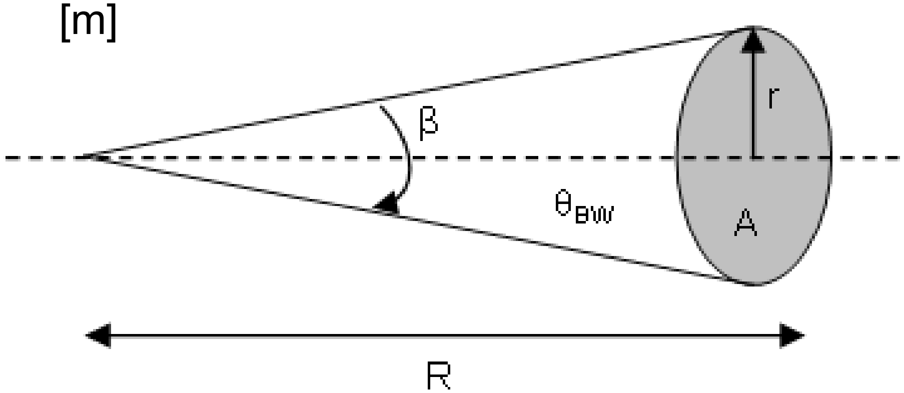
Source: Laser Radar Systems, A. V. Jelalian

Target Radiant Intensity

Beam Divergence:

$$\theta_{BW} = \frac{A}{R^2} = \frac{\pi \cdot r^2}{R^2} \quad \text{[sr]} \qquad r = \tan\left(\frac{\beta}{2}\right) \cdot R \quad \text{[m]}$$

$$\therefore \theta_{BW} = \pi \tan\left(\frac{\beta}{2}\right)^2 \quad \text{[sr]}$$



Radiant Intensity (Power Intensity):

$$P_T \cdot \frac{T_T}{\theta_{BW}} \quad \text{[W/sr]}$$

θ_{BW} = Transmitted Beamwidth [sr]
A = Area subtended by θ_{BW} [m^2]
R = Range to target [m]

r = Radius of Beam Footprint [m]
β = Beam Divergence [rad]
T_T = Transmit Optical Efficiency

Source: Laser Radar Systems, A. V. Jelalian

Atmospheric Attenuation

Atmosphere Attenuation:

$$T_A = e^{-\sigma_{1.06} R[km]}$$

Environment	Visibility (km)	Extinction Coefficient (Visible)	Extinction Coefficient (1.06 μm)
Exceptionally Clear	60	0.0652	0.0303
Very Clear	40	0.0978	0.0485
Standard Clear	23.5	0.1665	0.0900
Clear	15	0.2608	0.1514
Light Haze	8	0.4890	0.3140
Medium Haze	5	0.7824	0.5416
Haze	3	1.3040	0.9796
Thin Fog	1.5	2.6080	2.1890
Light Fog	0.75	5.2160	4.8915
Moderate Fog	0.3	13.0400	14.1597

Target Radiant Intensity at Target (Power Intensity at Target):

$$\frac{P_T \cdot T_T}{\theta_{BW}} \cdot T_A \qquad \text{[W/sr]}$$

Source: Laser Radar Systems, A. V. Jelalian

Laser Power at Target

Target Irradiance (I_T) (Density of Power at Target):

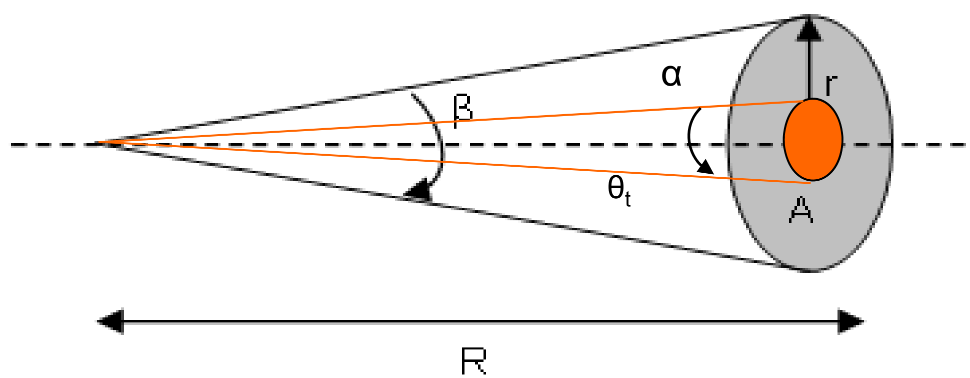
$$I_T = \frac{P_T \cdot T_T \cdot T_A}{\theta_{BW}} \cdot \frac{1}{R^2} \quad \text{[W/m}^2\text{]}$$

Small illuminated Area:

$$dA = \theta_t \cdot R^2 \quad \text{[m}^2\text{]}$$

$$\theta_t = \pi \tan\left(\frac{\alpha}{2}\right)^2 \quad \text{[sr]}$$

"Unresolved Target"



Total Power on Target:

$$P_{tgt} = \frac{P_T \cdot T_T \cdot T_A}{\theta_{BW} \cdot R^2} \cdot dA \quad \text{[W]}$$

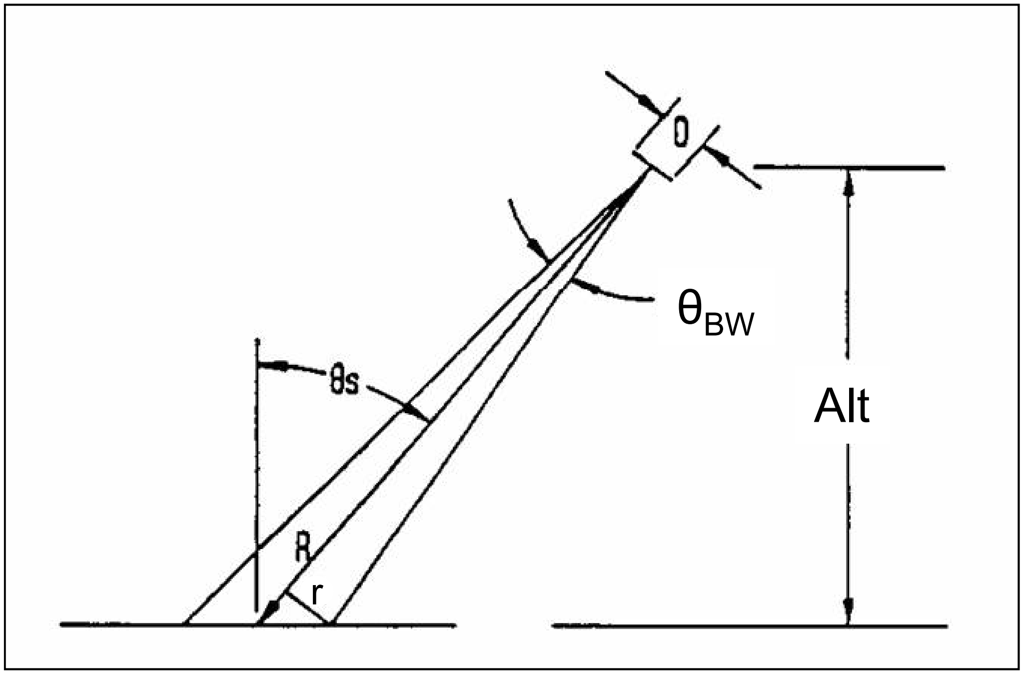
(Unresolved Target)

$$P_{tgt} = P_T \cdot T_T \cdot T_A \quad \text{[W]}$$

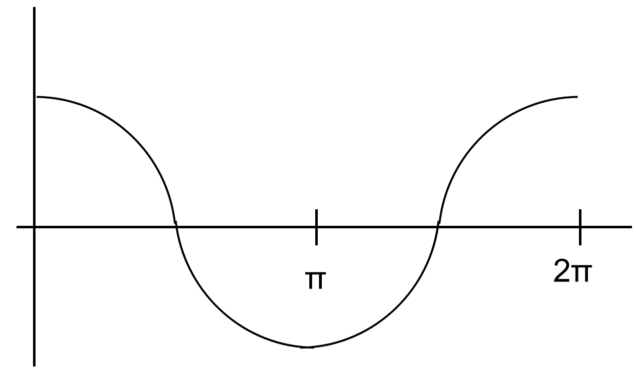
(Resolved Target $\alpha = \beta$, $\theta_t = \theta_{BW}$)

Source: Laser Radar Systems, A. V. Jelalian

Transmit Geometry



Cosine Incident Angle Effect



$$P_{tgt} = \frac{P_T \cdot T_T \cdot T_A \cdot dA}{\theta_{BW} \cdot R^2} \cdot \cos\theta_S \qquad \text{[W]}$$

θ_{BW} = Transmitted Beamwidth [sr]

θ_S = Incident (slant) Angle [rad]

r = Radius of Beam Footprint [m]

Alt = Transmitter Altitude

Source: Laser Radar Systems, A. V. Jelalian

Receiver Signal Power

Receiver Radiant Intensity (Power Intensity at Receiver):

$$\frac{P_T \cdot T_T \cdot T_A \cdot dA \cdot \cos\theta_S}{\theta_{BW} \cdot R^2} \cdot \frac{T_A \rho_T}{\theta_R} \qquad \text{[W/sr]}$$

θ_R radiating source could be an isotropic radiator (emits everywhere - 4π sr), a diffuse radiator (hemispherical radiation - 2π sr), or a Lambertian radiator (uniform radiation - π sr)

Receiver Irradiance (Power Density at Receiver):

$$I_R = \frac{P_T \cdot T_T \cdot T_A^2 \cdot \rho_T \cdot dA \cdot \cos\theta_S}{\theta_{BW} \cdot \theta_R \cdot R^2} \cdot \frac{1}{R^2} \qquad \text{[W/m}^2\text{]}$$

T_A = Atmospheric attenuation

ρ_T = Target Reflectivity

θ_R = Radiating Source

R = Range to Target

Source: Laser Radar Systems, A. V. Jelalian

Detector Signal Power

Receiver Optics Efficiency:

$$A_e = \frac{\rho_A \cdot \pi \cdot D_A^2}{4} \qquad [\mathrm{m}^2]$$

Detector Received Power:

$$P_R = \frac{P_T \cdot T_T \cdot T_A^2 \cdot \rho_T \cdot dA \cdot \cos\theta_S}{\theta_{BW} \cdot \theta_R \cdot R^4} \cdot A_e \qquad [\mathrm{W}]$$

A_e = Receiver Optics Efficiency (m^2)

ρ_A = Optical Efficiency

D_A = Aperture Diameter (m)

Source: Laser Radar Systems, A. V. Jelalian

Laser Range Equation

$$P_R = \frac{P_T \cdot T_T \cdot T_A^2 \cdot \rho_T \cdot dA \cdot \cos\theta_S \cdot A_e}{\theta_{BW} \cdot \theta_R \cdot R^4} \quad \text{[W]}$$

$$P_R = \frac{\pi \cdot P_{avg} \cdot T_T \cdot e^{-2\sigma R} \cdot \rho_T \cdot \rho_A \cdot \theta_t \cdot D_A^2 \cdot \cos\theta_S}{4 \cdot n_B \cdot \mathrm{PRF} \cdot \tau \cdot \theta_{BW} \cdot \theta_R \cdot R^2} \quad \text{[W]}$$

P_R = Received Power

PRF = Pulse Repetition Frequency [Hz]

n_B = number of beams

T_T = Transmit Optical Efficiency

θ_R = Radiating Source

ρ_A = Optical Efficiency

σ = Atmospheric Attenuation Coef.

θ_S = Incident (slant) Angle [rad]

P_{avg} = Average Power of Laser Beam [W]

τ = pulse width [s]

R = Range to target [m]

θ_{BW} = Transmitted Beamwidth [sr]

ρ_T = Target Reflectivity

θ_t = Target Solid Angle [sr]

D_A = Receiver Optics Diameter

Source: Laser Radar Systems, A. V. Jelalian

Noise Calculation

[For an Avalanche Photo Diode (APD)]

Detector:

- Generation and Recombination Current
- Thermal variation of detector

Amplifier:

- Johnson Noise
- Thermal agitation of electrons

$$(I_{DN})^2 = 2q\Delta f_n \cdot \left[\left(P_R + P_{sb} + P_{bks} \right) R_D + i_{DB} + \frac{i_{DS}}{G^2} \right] \cdot \left[G^2 F \right] \quad [\text{m}^2]$$

$$F = k_{eff} G + \left(2 - \frac{1}{G} \right) \left(1 - k_{eff} \right)$$

q = Electron charge (1.6 x 10-19 C)

P_R = Received signal power [W]

P_{bks} = Atmospheric backscatter power [W]

i_{DB} = Bulk dark current [A]

G = APD gain

k_{eff} = Weighted ratio of holes and electrons

Δf_n = Receiver noise bandwidth [Hz]

P_{sb} = Solar background power [W]

R_D = Detector responsively [A/W]

i_{DS} = Surface dark current [A]

F = APD excess noise factor

Source: Laser Radar Systems, A. V. Jelalian

Noise Power

Solar Background Power:

$$P_{sb} = A_e \cdot \delta_\lambda \cdot E_\lambda \cdot T_A \cdot \rho_T \left(\frac{\mathrm{FOV}}{2} \right)^2 \qquad \text{[W]}$$

Atmospheric Backscatter Power:

$$P_{bks} = A_e \cdot T_T \cdot c \cdot E_{LP} \cdot k_{bks} \cdot \frac{T_A^2}{R^2} \qquad \text{[W]}$$

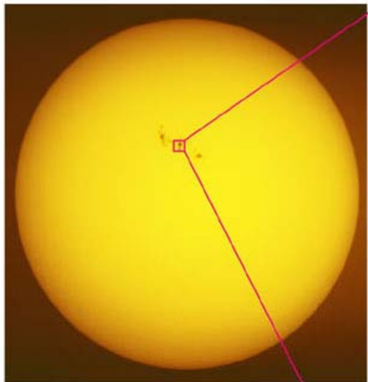
δ_λ = Optical passband [m]

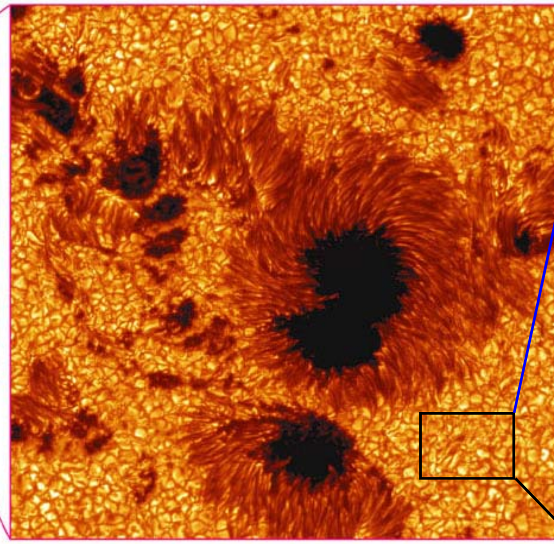
FOV = Detector field of view [rad]

E_λ = Solar spectral irradiance [W/m^2 μm]

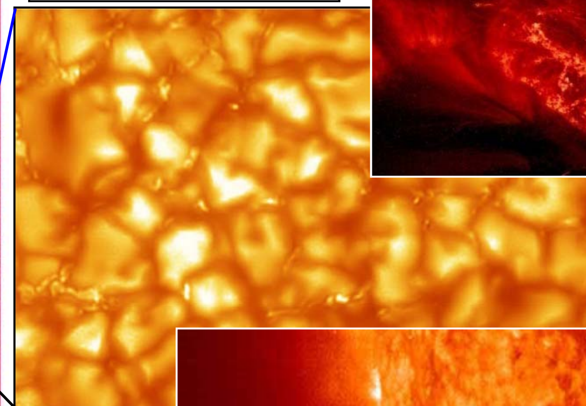
k_{bks} = Backscatter coefficient [~α/8π]

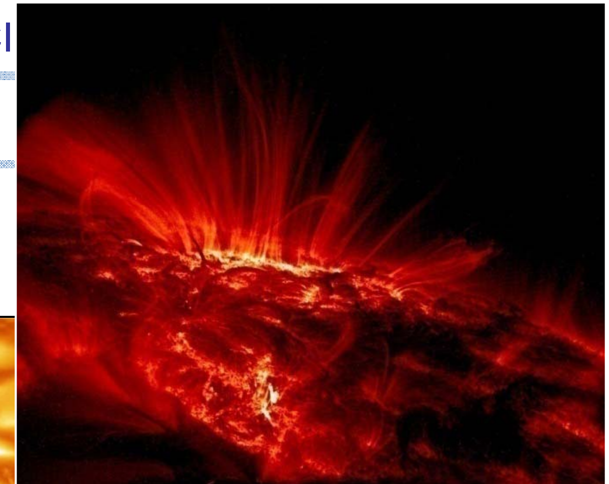
The Solar Corona





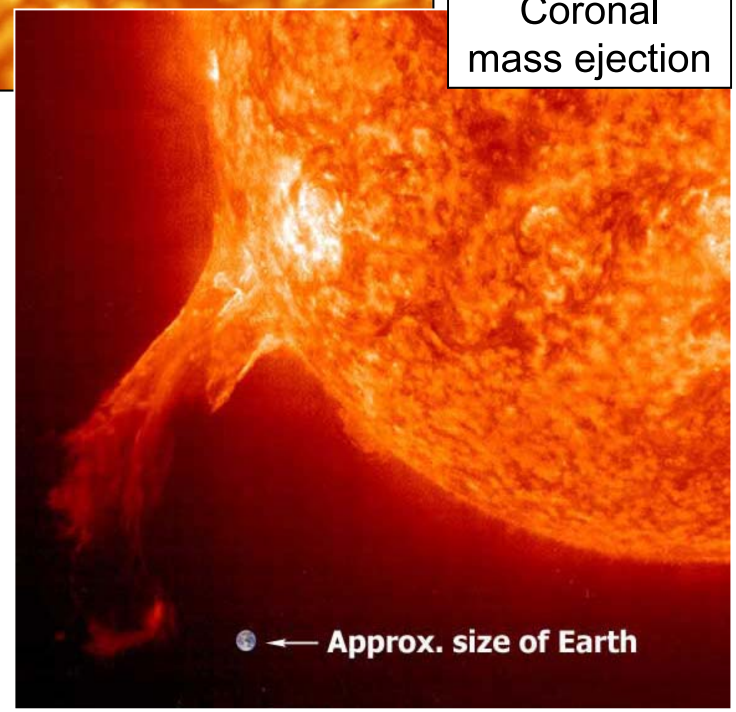
Granulation of solar surface

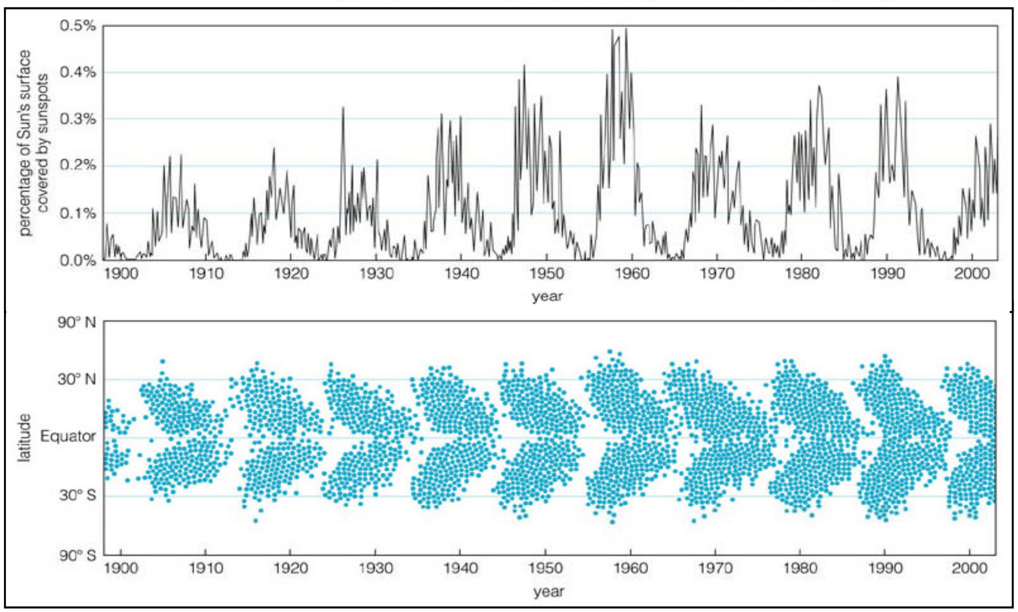




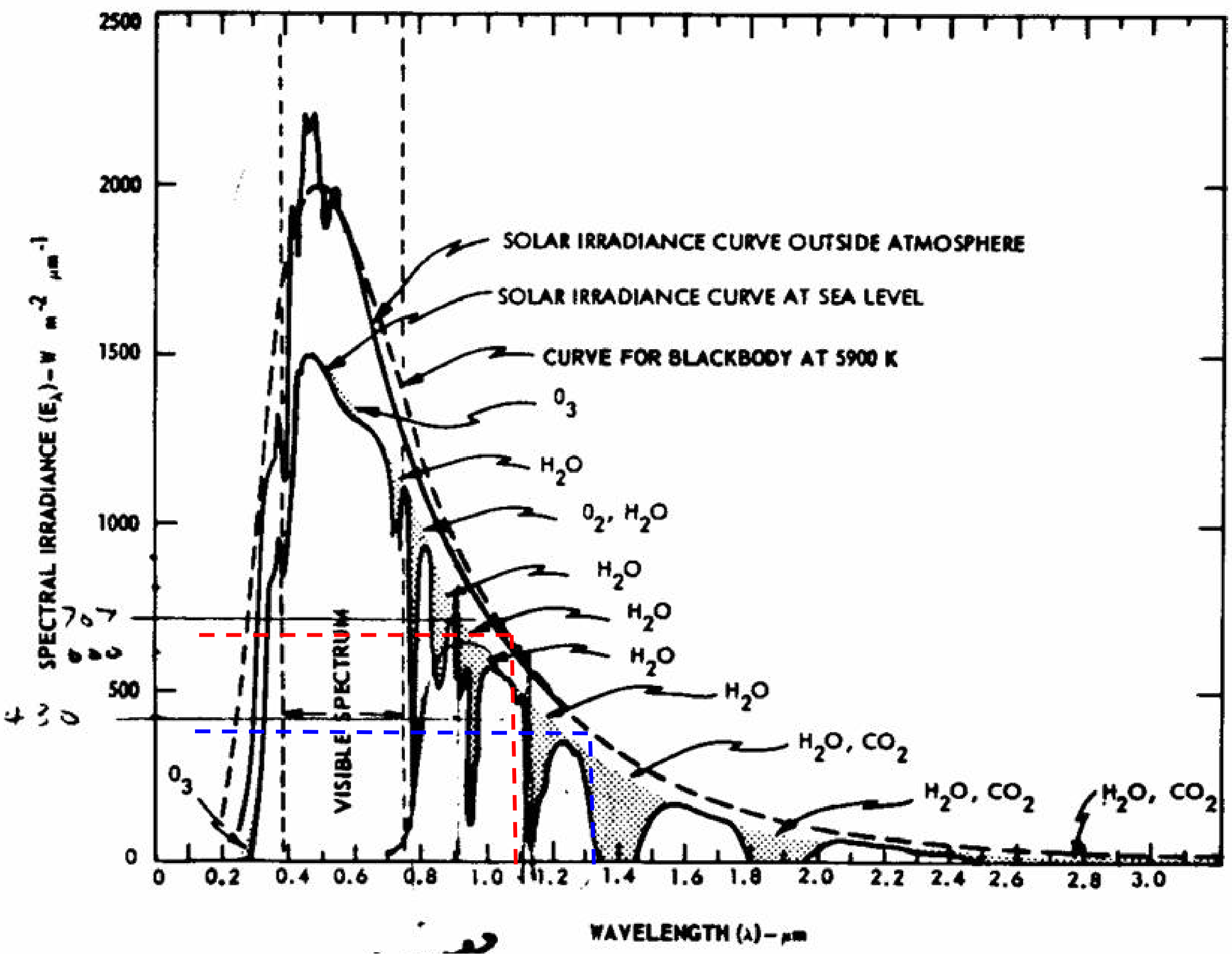
A solar flare

Coronal mass ejection





The Solar Spectrum



For λ = 1.06 µm

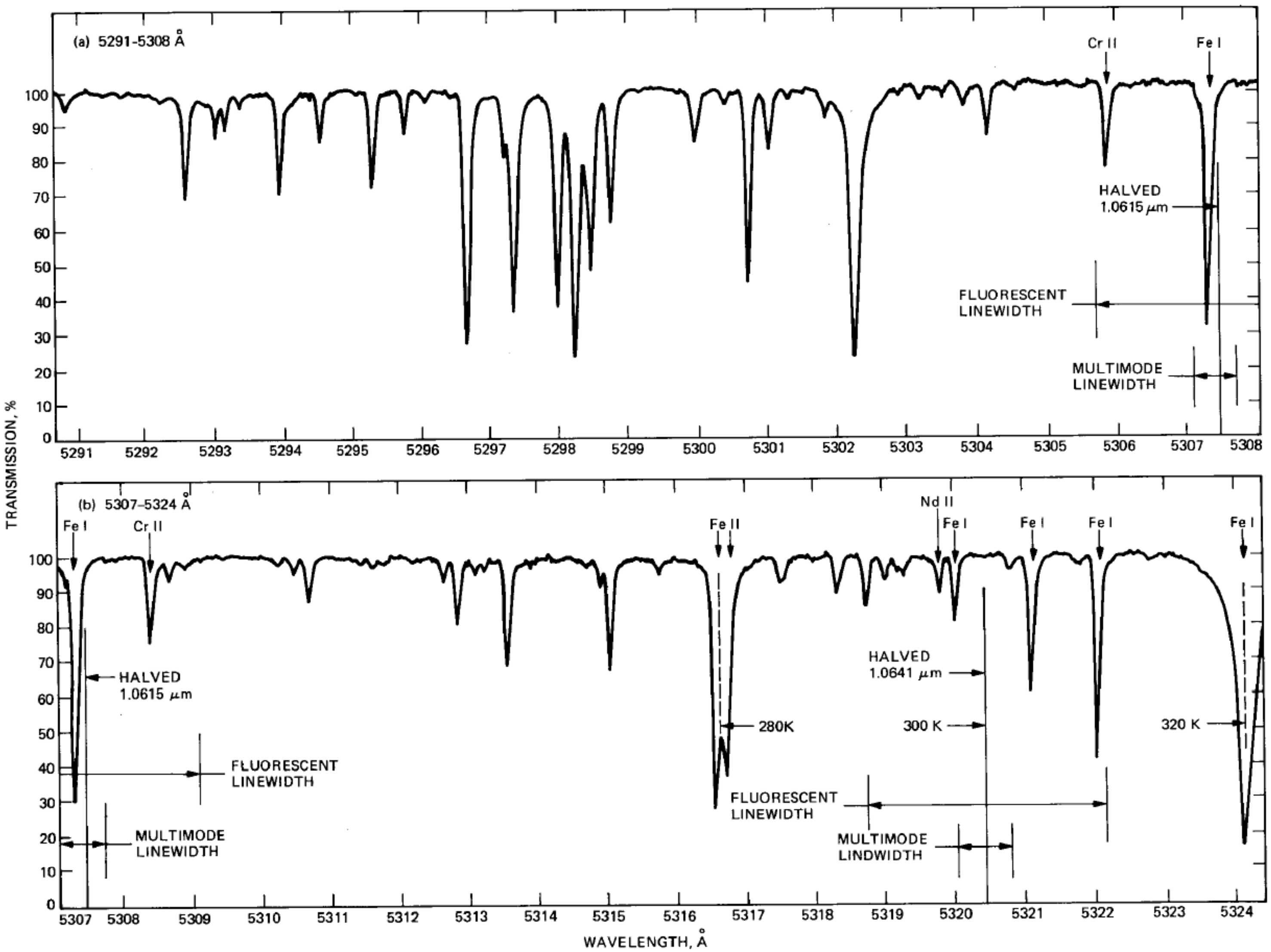
$E_\lambda \approx 700$ W/m^2-µm

For λ = 1.55 µm

$E_\lambda \approx 200$ W/m^2-µm

Source: The Infrared & Electro-Optical Systems Handbook

Solar Spectrum from 5291 to 5324 Angstroms



Laser Range Equation and SNR

$$P_R = \frac{P_T G_T}{4\pi R^2} \cdot \frac{A_{tar}}{4\pi R^2} \cdot \frac{\pi D_A^2}{4} \cdot \eta_A \eta_{sys} = \frac{P_T A_{tar} D_A^4}{16 R^4 \lambda^2 K_a^2} \cdot \eta_A \eta_{sys} \quad \text{[W]}$$

$$\text{SNR} = \frac{P_R \cdot R_D}{\sqrt{(I_{DN})^2}} \quad \text{[A/A]}$$

- P_R = received power [W]
- P_T = transmitter power [W]
- G_T = transmitter antenna gain = $4\pi/\theta_{BW}^2$
- θ_{BW} = transmitter beam-width = $K_A \lambda/D_A$
- A_{tar} = effective target cross-section [m²]
- K_A = aperture illumination constant
- R_D = detector responsivity [A/W]
- R = range to the target [m]
- λ = wavelength [m]
- D_A = aperture diameter [m]
- η_{Atm} = atmospheric Transmission factor
- η_{sys} = system transmission factor
- P_R = received signal power [W]
- I_{DN} = APD Detector Noise [A]

$$\text{SNR}_{\text{incoherent}} = \frac{\eta_D P_{SIG}^{\;2}}{2hfB[P_{BKG} + P_{SIG}] + (\eta_D / \rho^2_i)[P_{DK} + P_{TH}]}$$

Select best noise figure: cool amplifier larger load resistance

Decreased by narrow b-p filter, narrow FOV, lower BKG emission

Smaller detector – smaller DK; higher responsivity – lower NEP

$$P_{TH} = \frac{4kT \cdot B \cdot NF}{R_L}$$

$$\text{SNR}_{\text{coherent}} = \frac{\eta_D P_{LO} P_{SIG}}{hfB[P_{LO} + P_{BKG} + P_{SIG}] + (hf / 2q\rho_i)[P_{DK} + P_{TH}]}$$

$$P_{DK} = \frac{A_d B}{(D^*)^2}$$

$$\text{SNR}_{dB} = 10\log_{10}(\text{SNR})$$

Source: Laser Radar Systems, A. V. Jelalian

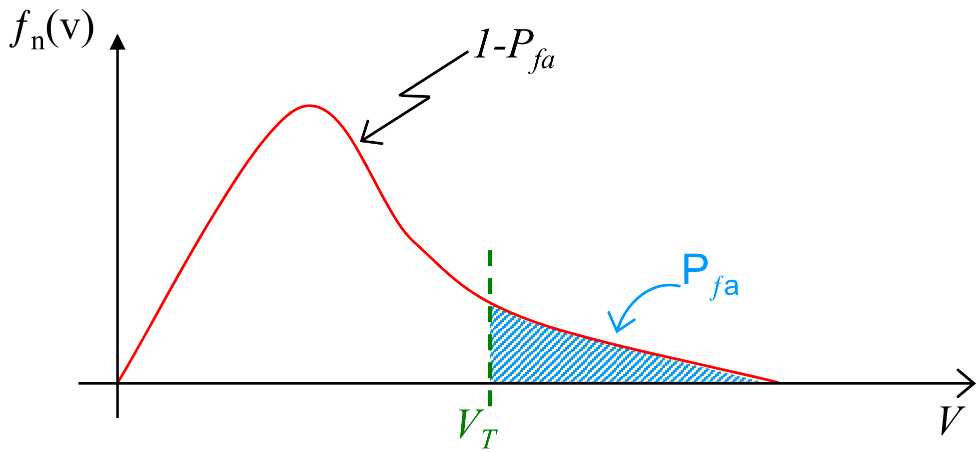
Probability of False Alarm (Pfa)

False alarms occur when noise exceeds a threshold:

$$P_{fa}(V_T) = \int_{V_T}^{\infty} f_N(v)\, dv = \int_{V_T}^{\infty} \frac{v}{\sigma^2} \exp\left[-\frac{v^2}{2\sigma^2}\right] dv$$

Solving this integral yields:

$$P_{fa}(V_T) = \exp\left[-\frac{V_T^{\,2}}{2\sigma^2}\right]$$



V_T/σ	P_{fa}
1	0.607
2	0.135
3	1.11×10^{-2}
4	3.35×10^{-4}
5	3.73×10^{-6}

P_{fa} = Probability of False Alarm
f_N = Probability Density Function

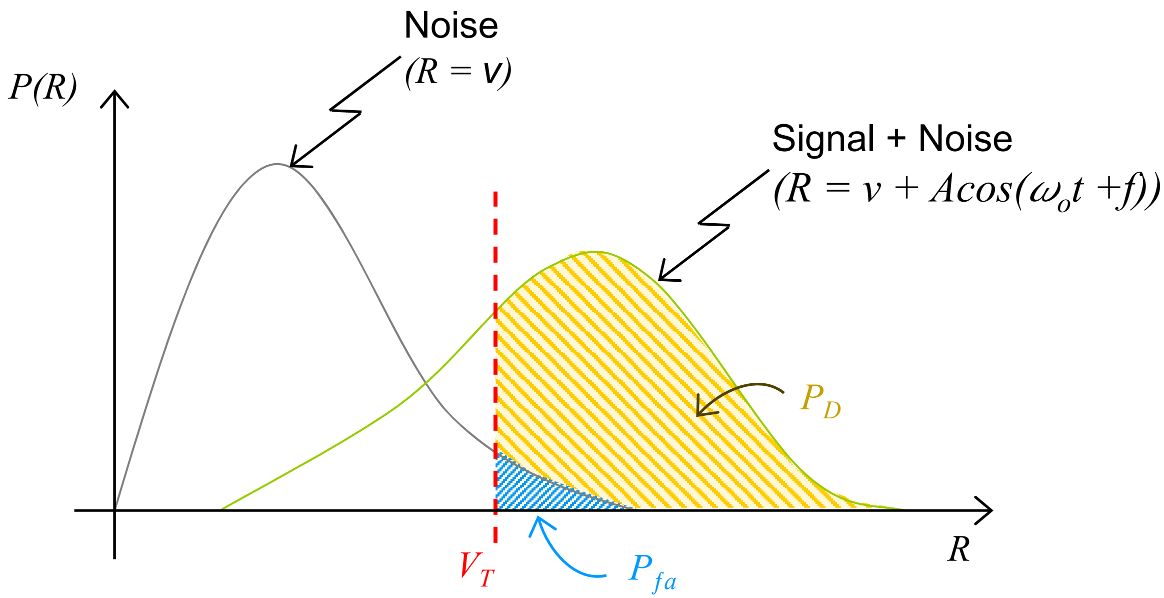
V_T = Threshold Voltage
σ = Variance

Source: Laser Radar Systems, A. V. Jelalian

P_D vs P_{fa} Summary

$$P_D = \int_{\frac{V_T}{\sigma}}^{\infty} z \exp\left[-\left(\frac{z^2}{2} + \frac{S}{N}\right)\right] I_0\left(z\sqrt{\frac{2S}{N}}\right) dz, \qquad P_{fa} = \exp\left[-\frac{V_T^{\,2}}{2\sigma^2}\right]$$

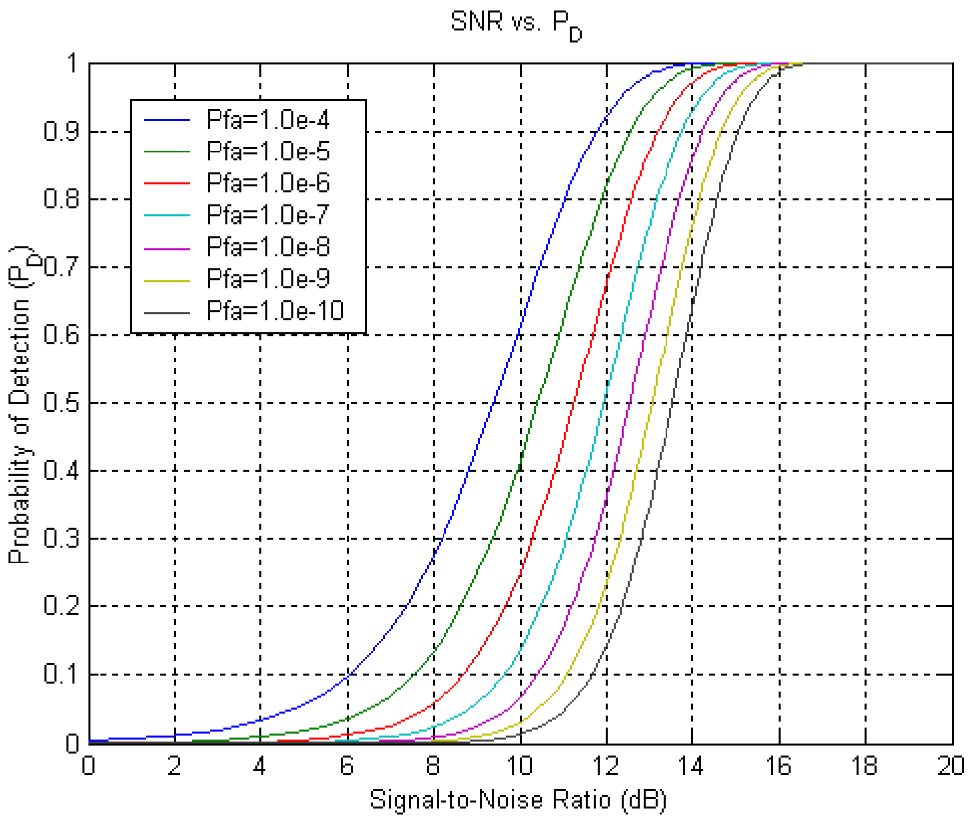
We recognize that $P_D = P_D\left(P_{fa}, \frac{S}{N}\right)$



Raising threshold decreases P_{fa}, but <u>also</u> decreases P_D. Lowering threshold makes radar more sensitive at the expense of increased false alarms

Source: Laser Radar Systems, A. V. Jelalian

Signal-to-Noise Ratio vs. Probability of Detection



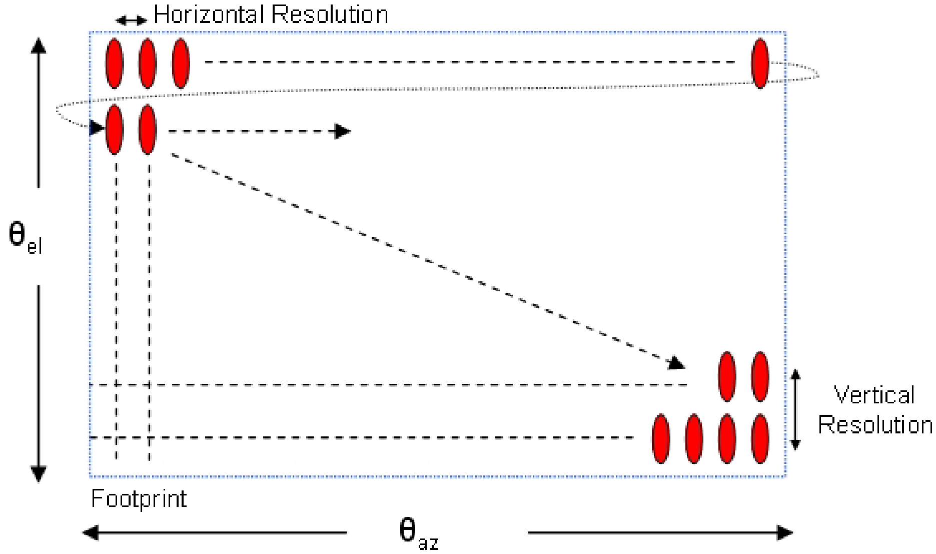
To determine a level of merit of the return signal strength needed to track a target, one should consider the relationship between SNR, probability of detection (P_D), and the probability of false alarm (P_{fa}).

The P_D for a non-fluctuating signal in Gaussian white noise can be approximated by the following equation:

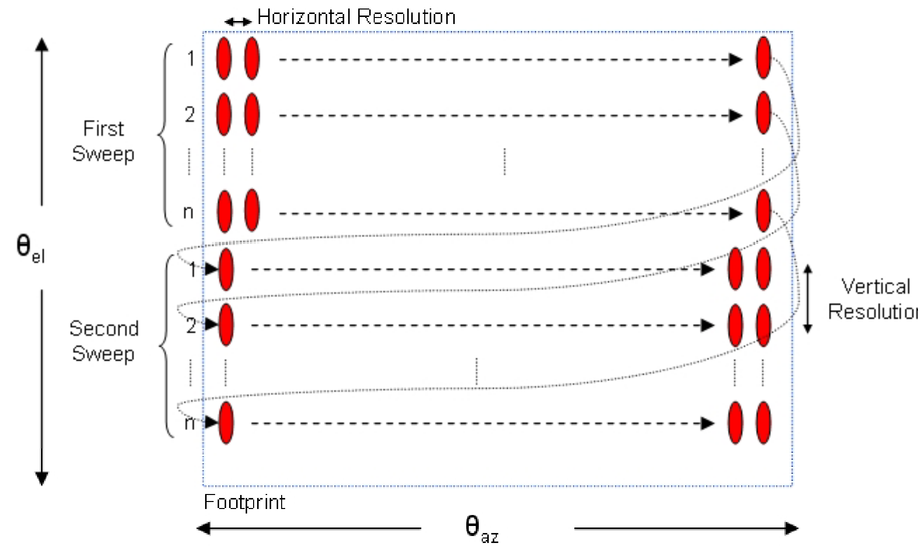
$$P_D = \frac{1}{2}\left(1 + erf\left\{\left(\frac{1}{2} + SNR\right)^{1/2} - \left[\ln\left(\frac{1}{P_{fa}}\right)\right]^{1/2}\right\}\right)$$

Source: Laser Radar Systems, A. V. Jelalian

Scanning Techniques



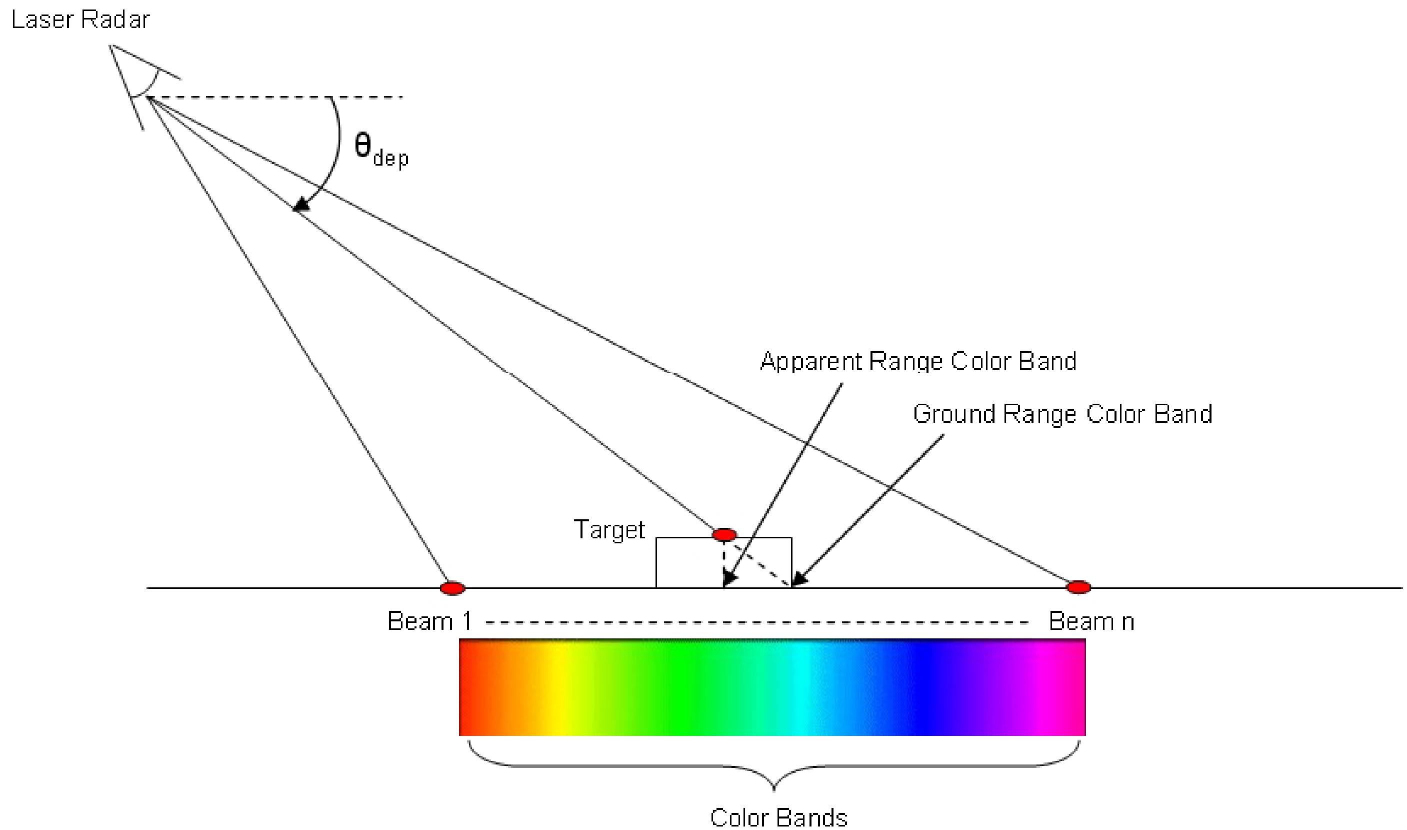
"Raster Scan"



"Broom Scan"

High Resolution Range and Amplitude Information Collected for each Pulse

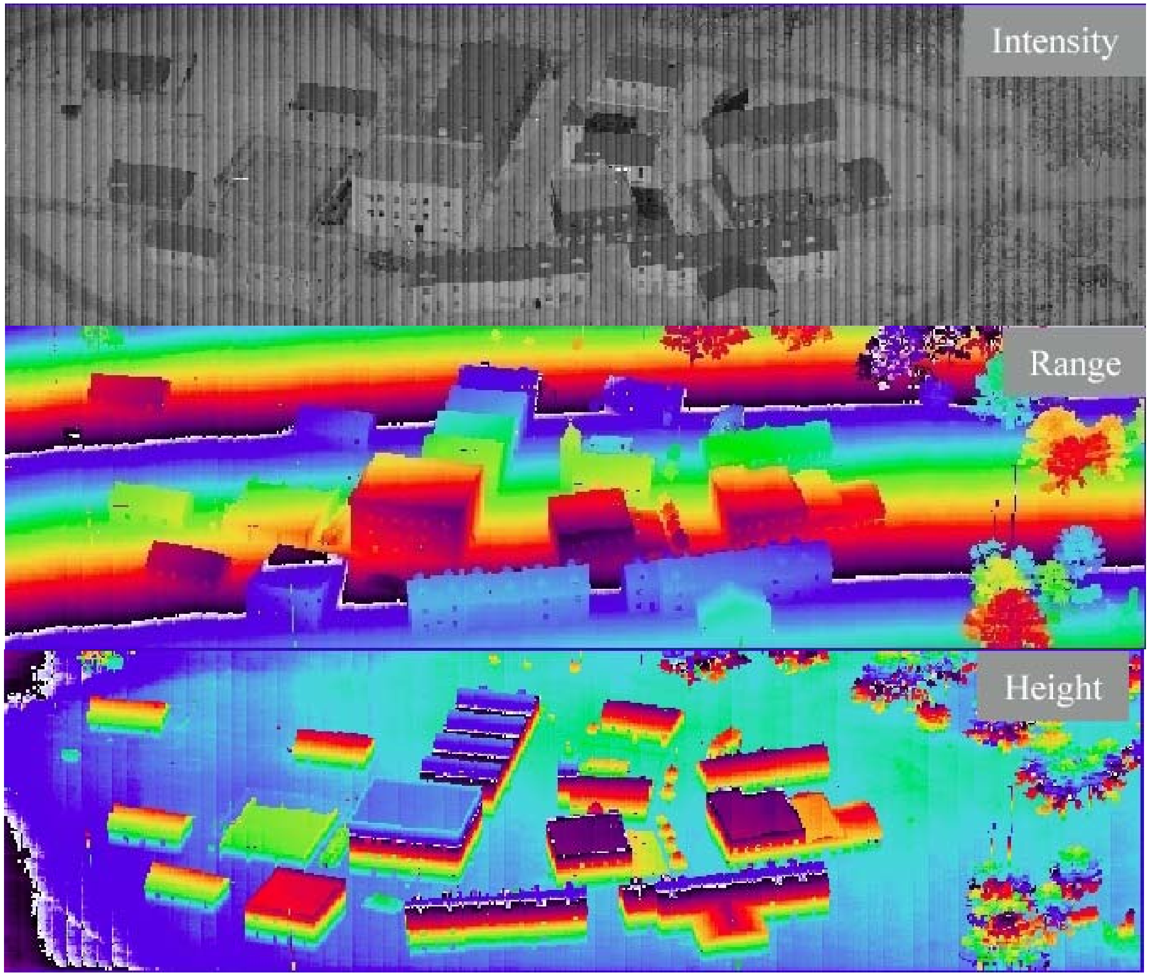
Range "Binning"



Color Bands are encoded to identify objects that vary in range in reference to a ground plane

Source: Introduction to Sensor Systems – Havanessian

LADAR Imagery



Conclusions

- Laser Radar System Performance
  - Ladar is similar in many ways to conventional Radar in that they actively transmit to illuminate the target of interest
  - Range to a target and intensity can be determined by measuring the round-trip time delay between a transmitted pulse and the received reflected pulse
  - Targets can be resolved in angle (Azimuth and Elevation)
  - Laser wavelengths of operation are much shorter (microns instead of millimeters to centimeters)
  - Therefore beamwidth can be much narrower than for a radar system with the same aperture size
  - The range is often much shorter due to the atmospheric attenuation
  - Weather affects Ladar much more than Radar due to the shorter wavelength
- Synthetically Generated Imagery
  - Simulated imagery can be achieved using hardware and software techniques
  - Imagery is used to optimize the performance of high fidelity IFS models and allows a weapon system to be thoroughly tested in various locations and weather conditions that normal test flight imagery might not be available.

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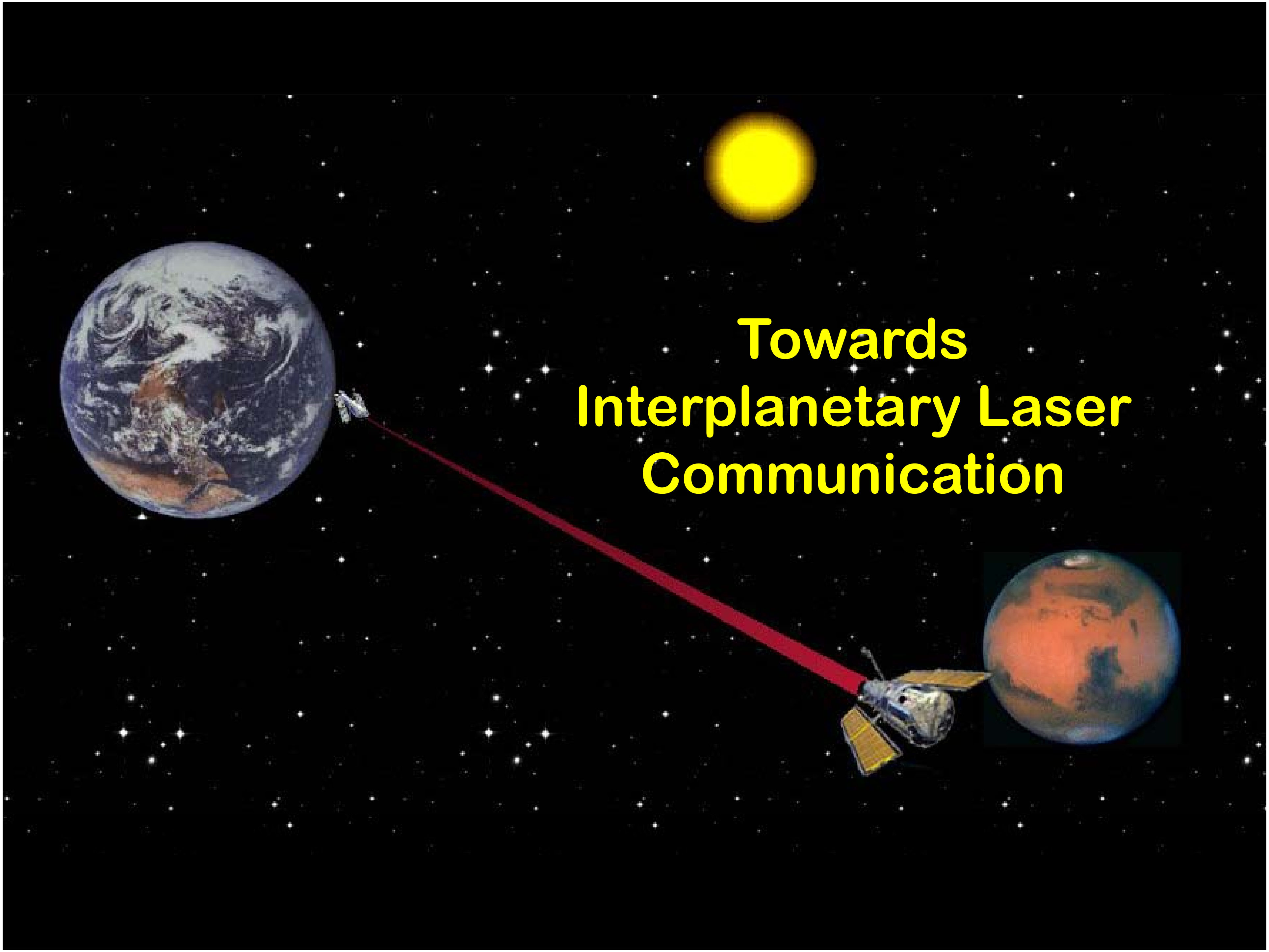
Towards
Interplanetary Laser
Communication