Lecture 14.0: Advances in Ultra-Cold Atom Physics:
Development of Precision Frequency Standards (i.e., Clocks) and Matter-Wave Interferometers (i.e., Quantum Sensors)

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Summary

• Clocks:
  – \(\mu\)-wave atomic clocks:
    • NIST F-1 Cs fountain \(\mu\)-wave clock: \(5 \times 10^{-16}\) accuracy (1 day)
  – \(\mu\)-wave atomic clocks on a chip:
    • NIST Rb chip-scale \(\mu\)-wave clock: \(6 \times 10^{-12}\) accuracy (1 sec)
  – Optical atomic clocks
    • NIST Hg+ clock now \(< 3 \times 10^{-17}\) (\(10^4\) sec)

• Matter-wave interferometers:
  – Gravity gradiometer:
    • Demonstrated differential acceleration sensitivity: \(3 \times 10^{-9}\) g/Hz\(^{1/2}\)
  – Gyroscope
    • Achieved stability \(2 \times 10^{-6}\) deg/hr\(^{1/2}\) ARW (angle random walk)
  – Accelerometer
    • Demonstrated performance \(2.3 \times 10^{-9}\) g/Hz\(^{1/2}\)
**Optical Clocks in Space**

- Atomic clock ensemble for space applications based on the optical transitions of strontium and ytterbium atoms
- Stability and accuracy of at the $10^{-17}$ - $10^{-18}$ level
- Such performances will impose major efforts to improve existing techniques for time and frequency transfer both space-ground and space-space
• Atom Interferometry Sensors for Space Applications
  – Space-based instrument for the measurement of tiny rotations and acceleration and for the detection of faint forces
  – Quantum and metrological sciences; direct applications in inertial navigation, Earth observation, geodesy, and geology

**Sensitivity to accelerations (10^8 atoms):**
Ground: 10^{-10} g/√Hz (expansion time 0.2 s)
Space: 10^{-12} g/√Hz (expansion time 3 s)

**Sensitivity to rotations (10^8 atoms):**
Ground: 10^{-9} rad/√Hz (expansion time 0.025 s)
Space: 8 \times 10^{-12} rad/√Hz (expansion time 3 s)
Earth rotation rate: 7.2 \times 10^{-5} rad/s

from E. Rasel et al. (2005)
Outline

• Clocks
  – $\mu$-wave atomic clocks
  – *Optical* atomic clocks

• Measurement of optical frequencies
  – Optical frequency chains
    – Frequency comb technique

• Matter-wave interferometers
  – Gravity gradiometer
  – Gyroscope
  – Accelerometer
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<th><strong>Atomic Clocks</strong></th>
<th><strong>Atom Interferometers</strong></th>
<th><strong>Degenerate Quantum Gases</strong></th>
</tr>
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</table>
| **Fundamental Physics:**  
• Standard Model Extension tests  
• Universality of the gravitational red-shift  
• Time variations of fundamental constants  
• Gravitational red-shift  
• Shapiro time delay and $1/c^3$ effects  
• Gravitational waves detection | **Fundamental Physics:**  
• Weak Equivalence Principle tests  
• Measurement of fundamental constants  
• Time variations of fundamental constants  
• Measurement of the gravito-magnetic effect  
• Tests of the Newton’s law at short distances  
• Gravitational waves detection | **Fundamental Physics:**  
• Thermodynamics of the phase transition at ultra-low temperatures  
• Collective excitations in the weak trapping regime  
• BEC coherence properties in microgravity  
• Role of interactions in BEC: dipolar forces and short range interactions  
• Dynamics of Bose mixtures in microgravity |
| **Applications:**  
• Atomic time scales (TAI)  
• Time & Frequency metrology  
• Deep space navigation  
• Doppler tracking  
• Synchronization of DSNA  
• VLBI  
• Time & Frequency transfer  
• Gravity mapping  
• Planetary exploration | **Applications:**  
• Inertial navigation  
• Earth observation and monitoring  
• Geology and vulcanology  
• Gravity and gravity-gradient mapping  
• Planetary exploration | **Applications:**  
• Atomic sources for atom interferometry  
• High-resolution interferometric measurements with dilute coherent matter waves |
Clocks

Sun clock since 3500 B.C.
One period per day

Quartz clock since 1918
32.768 periods per second

Pendulum clock since 1656
One period per second

Cesium atomic clocks since 1955
9.192.631.770 periods per second

Increasing number of oscillations → increasing stability
→ increasing accuracy
ADVANCES IN PRECISION CLOCKS AND QUANTUM SENSORS

“The times they are a-changin’…”

Bob Dylan

Clock Uncertainty (s/day)

10^3

10^1

10^-1

10^-3

10^-5

10^-7

10^-9

10^-11

Year

1000 1200 1400 1600 1800 2000

Huygen’s Pendulum

Chinese water clock

Harrison’s Chronometer

Shortt Clock

Quartz Crystal

Cs Beam Clock

Cs Fountain
Microwave radiation

- Nuclear spin transitions are induced by radiation with a frequency in the GHz region, e.g. the hyperfine transition in $^{133}\text{Cs}$: $\nu_{\text{Cs}} = 9.2$ GHz
- Measure the transition frequency $\nu_{\text{Cs}}$ very precisely
- Define the second as the duration of $\nu_{\text{Cs}}$ periods

Atomic beam magnetic resonance technique (Nobel prize 1944, Rabi)
\[ |e\rangle \]
\[ \hbar \omega_{ef} = h\nu_{ef} = E_e - E_f \]
\[ |f\rangle \]

\[ \omega(t) = \omega_{ef} \times (1 + \epsilon + y(t)) \]

Inaccuracy: \( \epsilon \)

Fractional frequency fluctuations: \( y(t) \)

Fractional frequency instability: \( \sigma_{y(T)} \propto \frac{\sigma_{\delta P}}{Q_{at}} \sqrt{\frac{T_e}{T}} \)

Fluctuations of the transition probability:

Atomic quality factor: \( Q_{at} = \frac{\nu_{ef}}{\Delta\nu} \propto \nu_{ef} T \)
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Cesium Atomic Clocks

Vacuum chamber

Cesium oven

Magnetic shielding

Cesium beam

Microwave cavity (Ramsey cavity)

Hot wire, Cs ionizer

Getter

Magnet (polarizer)

Magnet (analyzer)

One second further after 9 192 631 770 cycles

Microwave radiation at 9 192 631 770 Hz

Tunable microwave generator

Serve control
The Cesium Fountain Clock

Cs Fountain Clock: The Current Standard and the Definition of the Second

Cold Atoms allow long interaction times

- Increasing the resolution by increasing the interaction time
- Uncertainty less then $10^{-16}$ (one second in 60 million years)

$$f_{Cs} = 9,192,631,770 \text{ Hz}$$

NIST: Steve Jefferts, Liz Donley, Tom Heavner, Tom Parker

Cs-Rb fountain clock FO2

- $N_{at} \sim 10^9$
- $\sigma \sim 3 \text{ mm}$
- $T \sim 1 \text{ mK}$
- $v \sim 4 \text{ m/s}$
- $H \sim 1 \text{ m}$
- $100 \text{ ms} \leq T_{load} \leq 500 \text{ ms}$
- $1.1 \text{ s} \leq T_{cycle} \leq 1.5 \text{ s}$
Ramsey Fringes

Linewidth: $0.94 \text{ Hz}$
Quality factor: $Q_{at} = 9.82 \times 10^9$
S/N ratio: $1/\sigma_{\delta P} \sim 5000$
Performances of FO2

Fractional frequency instability

\[ \sigma_y(\tau) = 1.6 \times 10^{-14} \tau^{-1/2} \]

<table>
<thead>
<tr>
<th>Inaccuracy ((\times 10^{-16}))</th>
<th></th>
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</thead>
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<tr>
<td>Second Order Zeeman</td>
<td>3207.0(4.7)</td>
</tr>
<tr>
<td>Blackbody radiation</td>
<td>-127.0(2.1)</td>
</tr>
<tr>
<td>Cold collisions + cavity pulling</td>
<td>0.0(1.0)</td>
</tr>
<tr>
<td>Residual first order Doppler</td>
<td>0.0(2.0)</td>
</tr>
<tr>
<td>Recoil</td>
<td>0.0(1.0)</td>
</tr>
<tr>
<td>Ramsey and Rabi pulling</td>
<td>0.0(1.0)</td>
</tr>
<tr>
<td>Microwave leakage</td>
<td>0.0(2.0)</td>
</tr>
<tr>
<td>Background collisions</td>
<td>0.0(1.0)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>7</td>
</tr>
</tbody>
</table>
ADVANCES IN PRECISION CLOCKS AND QUANTUM SENSORS

Improvement of Cs μ-wave standards over 50 years
NIST F-1 Cs fountain microwave clock now at 5 x 10^{-16} accuracy
Types of clocks

Higher Precision

- **Primary Standard**
  - Loses 1 sec in: $10^8$ years
  - Size: $10^7$ cm$^3$
  - Power: kW
  - Cost: $1$ M

- **Compact Atomic Clock**
  - Loses 1 sec in: 1000 years
  - Size: 100 cm$^3$
  - Power: 5 W
  - Cost: $1,000

- **Precision Quartz Crystal**
  - Losses 1 sec in: 1000 years
  - Size: 1 cm$^3$
  - Power: 30 mW
  - Cost: $1000

- **Wristwatch Quartz Crystal**
  - Losses 1 sec in: 1 day
  - Size: $10^{-3}$ cm$^3$
  - Power: 10 mW
  - Cost: $1$
Cell performance (table top experiment):

- Cell dimensions 1x1x1mm
- Buffer gas Ar-Ne mixture
- CPT linewidth 1.2kHz @ 3% contrast
  (ratio CPT amplitude to optical absorption)

S. Knappe et al., submitted to Optics Letters (2006)
NIST is currently building structures such as those shown above, that are the size of a grain of rice (V < 10 mm³) and could run on a AA battery (dissipate < 75 mW).
Chip-Scale Atomic Clock from NIST

- **Project Goals:**
  - To develop a compact, low-power micromachined frequency reference physics package capable of providing atomically precise timing for portable, battery-operated applications.

- **The NIST Approach:**
  - … is based on all-optical coherent-population-trapping (CPT) excitation of microwave resonances in a thermal vapor of alkali atoms. The atoms are confined in a miniature cell fabricated using MEMS techniques and the Q-factor is enhanced using a buffer gas.
  - Light from a vertical-cavity surface-emitting laser, modulated at the atomic resonance frequency by an external local oscillator (LO), creates the necessary excitation. The LO is locked to the atomic resonance using a servo.

- **Potential Applications:**
  - Commercial: compact, low-power frequency references with exceptional long-term frequency stability
  - Military GPS receivers: direct P(Y) code acquisition and resistance to jamming
  - Wireless communications systems: enhanced security of transmitted data
  - Application of cell fabrication technology to advanced devices and applications such as magnetometers, gyroscopes and fiber telecommunications

- **Major Accomplishments by May 2006:**
  - Constructed functioning chip-scale physics package based on $^{87}\text{Rb}$ with $V = 9.5\ \text{mm}^3$, $P = 75\ \text{mW}$ and with improved short- and long-term frequency stability: $\sigma_y(1\ \text{sec.}) = 4 \times 10^{-12}$ and $\sigma_y(1\ \text{hr.}) = 2 \times 10^{-11}$
  - Demonstrated chip-scale atomic magnetometer: $\text{dBmin} \sim 200\ \text{pT/Hz}$

- **Long-Term Milestones:**
  - $V = < 1\ \text{cm}^3$, $P < 30\ \text{mW}$, and drift $= < \pm 1\ \mu\text{s/day}$
• The Goal of the Program:
  – To create ultra-miniaturized, low-power, atomic time and frequency reference units that will achieve, relative to present approaches:
    >200X reduction in size (from 230 cm^3 to <1 cm^3), >300X reduction in power (from 10 W to <30 mW), performance (±1×10^{-11} accuracy, drift of <1 µs/day).

• The chip-scale atomic clock will enable:
  – A ultra-miniature wristwatch-size ultra low power time & frequency references for high-security UHF communication and jam-resistant GPS receivers.
  – These time reference units will improve mobility & robustness of any systems & platforms w/ sophisticated UHF communication and/or nav. requirements.
  – In military GPS receivers – will improve the jamming margin in a high-jamming environment, reacquisition capability, and position identification accuracy.
  – In surveillance, these clocks can be used to improve resolution in Doppler radars, to enhance accuracy of location identification of radio emitters.

• The Key Focus:
  – Innovative solutions in micro or nano-fabrication, materials processing, design, transduction mechanism, interconnects, to improve the time referencing.
  – Research elements include confinement & stabilization of Ce, Rb, or other species, excitation and detection of the hyperfine-transition resonance of the chosen species, and phase locking or direct coupling with MEMS resonators:
    • temperature stability, magnetic shielding, hermetic encapsulation, maintenance of atomic ground-state coherence within the confinement cell;
    • integration with vertical cavity surface emitting laser (VSCEL) or other photon and/or microwave sources and photo detector with the confinement cell;
    • integration, phase locking, and/or direct coupling with MEMS resonators.
Benefits of Space

- **Weightlessness**
  - Long interrogation times
  - Narrow clock transitions
    - Linewidth: $100 \text{ mHz}$
    - Instability: $7 \cdot 10^{-14}$ at 1 s
      $3 \cdot 10^{-16}$ at 1 day
    - Accuracy: $\sim 10^{-16}$
- Low mechanical vibrations
- Possibility of worldwide access
ADVANCES IN PRECISION CLOCKS AND QUANTUM SENSORS

PHARAO: A Cold-Atom Clock in \( \mu \)-gravity
Microwave clocks
These clocks get even better in space: longer observation times, lower velocity.

NIST – U. Colorado – JPL
Primary Atomic Reference Clock in Space (PARCS)
Short term servo loop:

Locks PHARAO local oscillator to SHM ensuring a better short and mid-term stability

Long term servo loop:

Corrects for SHM drifts providing the ACES clock signal with the long-term stability and accuracy PHARAO

Stability of the ACES clock signal:
- $3 \times 10^{-15}$ at 300s (ISS pass)
- $3 \times 10^{-16}$ at 1 day
- $1 \times 10^{-16}$ at 10 days

Accuracy: $\sim 1 \times 10^{-16}$
• Technology demonstrator for cold atom based missions
• First $\mu g$ experiments with cold atoms
• Validation in space of complex laser systems
• Validation of a new generation of atomic clocks
• Precursor of optical clocks: towards the $10^{-18}$ stability and accuracy regime
• Demonstration of stable and accurate time and frequency transfer
• Long-distance clock-to-clock comparisons
• Contribution to high performance global time scale

These results will arrive in time to prepare the next generation of atomic quantum sensors for space
## ACES Mission Objectives I

### Test of a new generation of space clocks

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<th>ACES Mission Objectives</th>
<th>ACES performances</th>
<th>Scientific background and recent results</th>
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<tr>
<td>Cold atoms in a micro-gravity environment</td>
<td>Study of cold atom physics in microgravity.</td>
<td>Such studies will be essential for the development of atomic quantum sensors for space applications (optical clocks, atom interferometers, atom lasers).</td>
</tr>
<tr>
<td>Test of the space cold atom clock PHARAO</td>
<td>PHARAO performances: frequency instability lower than $3 \cdot 10^{-16}$ at one day and inaccuracy at the $10^{-16}$ level. The short term frequency instability will be evaluated by direct comparison to SHM. The long term instability and the systematic frequency shifts will be measured by comparison to ultra-stable ground clocks.</td>
<td>Frequency instability: optical clocks show better performances; their frequency instability can be one or more orders of magnitude better than PHARAO, but their accuracy is still around the $10^{-15}$ level. Inaccuracy: at present, cesium fountain clocks are the most accurate frequency standards.</td>
</tr>
<tr>
<td>Test of the space hydrogen maser SHM</td>
<td>SHM performances: frequency instability lower than $2.1 \cdot 10^{-15}$ at 1000 s and $1.5 \cdot 10^{-15}$ at 10000 s. The medium term frequency instability will be evaluated by direct comparison to ultra-stable ground clocks. The long term instability will be determined by the on-board comparison to PHARAO in FCDP.</td>
<td>SHM performances are extremely competitive compared to state-of-the-art as the passive H-maser developed for GALILEO or the ground H-maser EFOS C developed by the Neuchâtel Observatory:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Maser</td>
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<tr>
<td></td>
<td>GALILEO</td>
<td>$3.2 \cdot 10^{-14}$</td>
</tr>
<tr>
<td></td>
<td>EFOS C</td>
<td>$2.0 \cdot 10^{-15}$</td>
</tr>
</tbody>
</table>

### Precise and accurate time and frequency transfer

| Test of the time and frequency link MWL | Time transfer stability will be better than 0.3 ps over one ISS pass, 7 ps over 1 day, and 23 ps over 10 days. | At present, no time and frequency transfer link has performances comparable with MWL. |
| Time and frequency comparisons between ground clocks | Common view comparisons will reach an uncertainty level below 1 ps per ISS pass. Non common view comparisons will be possible at an uncertainty level of: 2 ps for $\tau=1000 \text{ s}$ 5 ps for $\tau=10000 \text{ s}$ 20 ps for $\tau=1 \text{ day}$ | |
| | | Existing T&F links | Time stability (1 day) | Time accuracy (1 day) | Frequency accuracy (1 day) |
| | | GPS-DB | 2 ns | 3-10 ns | $4 \cdot 10^{-14}$ |
| | | GPS-CV | 1 ns | 1-5 ns | $2 \cdot 10^{-14}$ |
| | | GPS-CP | 0.1 ns | 1-3 ns | $2 \cdot 10^{-15}$ |
| | | TWSTFT | 0.1-0.2 ns | 1 ns | $2 \cdot 4 \cdot 10^{-15}$ |
## ACES Mission Objectives II

<table>
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<th>ACES performances</th>
<th>Scientific background and recent results</th>
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<tr>
<td><strong>Precise and accurate time and frequency transfer</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Absolute synchronization of ground clocks</td>
<td>Absolute synchronization of ground clock time scales with an uncertainty of 100 ps.</td>
<td>These performances will allow time and frequency transfer at an unprecedented level of stability and accuracy. The development of such links is mandatory for space experiments based on high accuracy frequency standards.</td>
</tr>
<tr>
<td>Contribution to atomic time scales</td>
<td>Comparison of primary frequency standards with accuracy at the $10^{-16}$ level.</td>
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<tr>
<td><strong>Fundamental physics tests</strong></td>
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<tr>
<td>Measurement of the gravitational red shift</td>
<td>The uncertainty on the gravitational red-shift measurement will be below $50 \times 10^{-6}$ for an integration time corresponding to one ISS pass ($\sim 300$ s). With PHARAO full accuracy, uncertainty will reach the $2 \times 10^{-6}$ level.</td>
<td>The ACES measurement of the gravitational red shift will improve existing results (Gravity Probe A experiment and measurements based on the Mössbauer effect). Space-to-ground clock comparisons at the $10^{-16}$ level, will yield a factor 25 improvement on previous measurements.</td>
</tr>
<tr>
<td>Search for a drift of the fine structure constant</td>
<td>Time variations of the fine structure constant $\alpha$ can be measured at the level of precision $\alpha^{-1} \cdot \frac{d \alpha}{dt} &lt; 1 \times 10^{-16}$ year$^{-1}$. The measurement requires comparisons of ground clocks operating with different atoms.</td>
<td>Crossed comparisons of clocks based on different atomic elements will impose strong constraints on the time drifts of fundamental constants improving existing results.</td>
</tr>
<tr>
<td>Search for Lorentz transformation violations and test of the SME</td>
<td>Measurements can reach a precision level of $\delta c / c \sim 10^{-16}$ in the search for anisotropies of the speed of light. These measurements rely on the time stability of SHM, PHARAO, MWL, and ground clocks over one ISS pass.</td>
<td>ACES results will improve previous measurements (GPS-based measurements, Gravity Probe A experiment, measurements based on the Mössbauer effect) by a factor 10 or more.</td>
</tr>
</tbody>
</table>
• Fractional frequency instability at the quantum projection noise
  
  \[ \sigma_y(\tau) = \frac{1}{\pi} \frac{\Delta \nu}{\nu_0} \frac{1}{\sqrt{N_{at}}} \sqrt{\frac{T_c}{\tau}} \]

  – \( \Delta \nu \sim 1 \text{Hz} \), limited by the interaction time (effect of gravity)
  – \( N_{at} \sim 10^6 \), limited by cooling and trapping techniques, collisional shift, etc.

• Solution: increase \( \nu_0 \) → optical transition show a potential increase of 5 orders of magnitude
  
  – \( \mu \)-wave fountain clocks:
    \[ \sigma_y(\tau) \sim 10^{-14} \tau^{-1/2} \]
  – Optical clocks:
    \[ \sigma_y(\tau) \sim 10^{-18} \tau^{-1/2} \]

• Accuracy:
  – theoretical studies foresee the possibility of reaching the \( 10^{-18} \) regime

• Major difficulties:
  – Measurements of optical frequencies (frequency-comb generator)
  – Recoil and first order Doppler effects
  – Downconversion noise of the interrogation oscillator (Dick effect)
Electronic quantum transitions are induced by visible light with a frequency in the THz region!
Optical Atomic Clocks: Science and Metrology on the Femtosecond Time Scale
Advances in Precision Clocks and Quantum Sensors

Principle of Operation of Optical Clocks

from S.A. Diddams et al., Science 293, 825 (2001)
Components of an Optical Clock

- Atom(s)
- Detector & Slow Laser Control
- Laser
- Isolated Optical Cavity
- Fast Laser Control
- Counter & Read Out

- Isolated cavity narrows laser linewidth and provides good short-term stability
- Atoms provide long-term stability and accuracy
- Counter accumulates cycles to generate 1 sec
Atomic Clock Stability

High stability requires: large signal and narrow linewidth

- signal size $\propto N$ number of atoms
- Narrow linewidth and high frequency gives high Q

Fractional frequency instability $\approx \frac{\Delta \nu}{\nu \cdot S/N} = \frac{1}{Q \cdot S/N} \propto \frac{1}{Q \cdot \sqrt{N_{\text{atom}} \tau}}$

Eg. Cold Ca, 400 Hz linewidth, $10^6$ atoms $\approx 10^{-16} @ 1s$

?? atom, 1 Hz linewidth, $10^6$ atoms $\approx 10^{-19} @ 1s$

Trapped single ions vs. laser-cooled neutral atoms ($10^6$)

- Hg$^+$, Yb$^+$, Sr$^+$, In$^+$, Al$^+$…
- Ca, Sr, Mg, Yb, Ag …

Small systematics, narrow lines

High S/N, Doppler sensitivity
Oscillator Stability

Quantum Limited Instability

\[ \sigma(\tau) \sim \frac{\Delta f}{f_0 \sqrt{N}} \frac{1}{\sqrt{\tau}} \]
Phase Noise of Microwave and Optical Oscillators

1 GHz Carrier

- High Quality Quartz
- Commercial µ-wave synthesizer
- Ca Optical
- Sapphire µ-wave Resonator + Divider
- Hg\(^{+}\) Optical Cavity
Ion clock example: Single 199Hg+

Single Hg\(^+\) ion

\[ Q \equiv \frac{\text{clock frequency}}{\text{linewidth}} \]

\[ = 1.6 \times 10^{14} \]
Al\(^{+}/\text{Hg}^{+}\) Optical Clock Stability

Systematic uncertainty of Hg\(^{+}\) clock now < 3\times10^{-17} ! (April 2006)
Cool Alkaline Earth – Strontium

$^{88}\text{Sr} \ ^1S_0 - ^3P_1$ 7.6 kHz 7x10^{-16}

$^{87}\text{Sr} \ ^1S_0 - ^3P_0$  < 10 mHz  ~ 10^{-18}

$\Gamma / 2\pi \delta \nu / \nu$ at 1s

$689 \text{ nm}$

$698 \text{ nm}$

$T \sim 0.5\text{ photon recoil}$

$\sim 220 \text{ nK}$
**Advances in Precision Clocks and Quantum Sensors**

Absolute Frequency of $^{87}\text{Sr} \ ^1S_0 - ^3P_0$

$Q \sim 1 \times 10^{14}$

$\text{Ludlow et al., Phys. Rev. Lett. 96, 033003 (2006).}$

Accuracy soon reaching $1 \times 10^{-15}$

**Projected stability**

$< 1 \times 10^{-15}$ at 1 s