



Курс лекций весеннего семестра 2011 года:
«Современные Проблемы Астрономии»
Государственный Астрономический Институт им. П.К. Штернберга



Lecture 10.2:

Basics Elements of a Navigational Program: *Orbit Determination and Parameter Estimation*

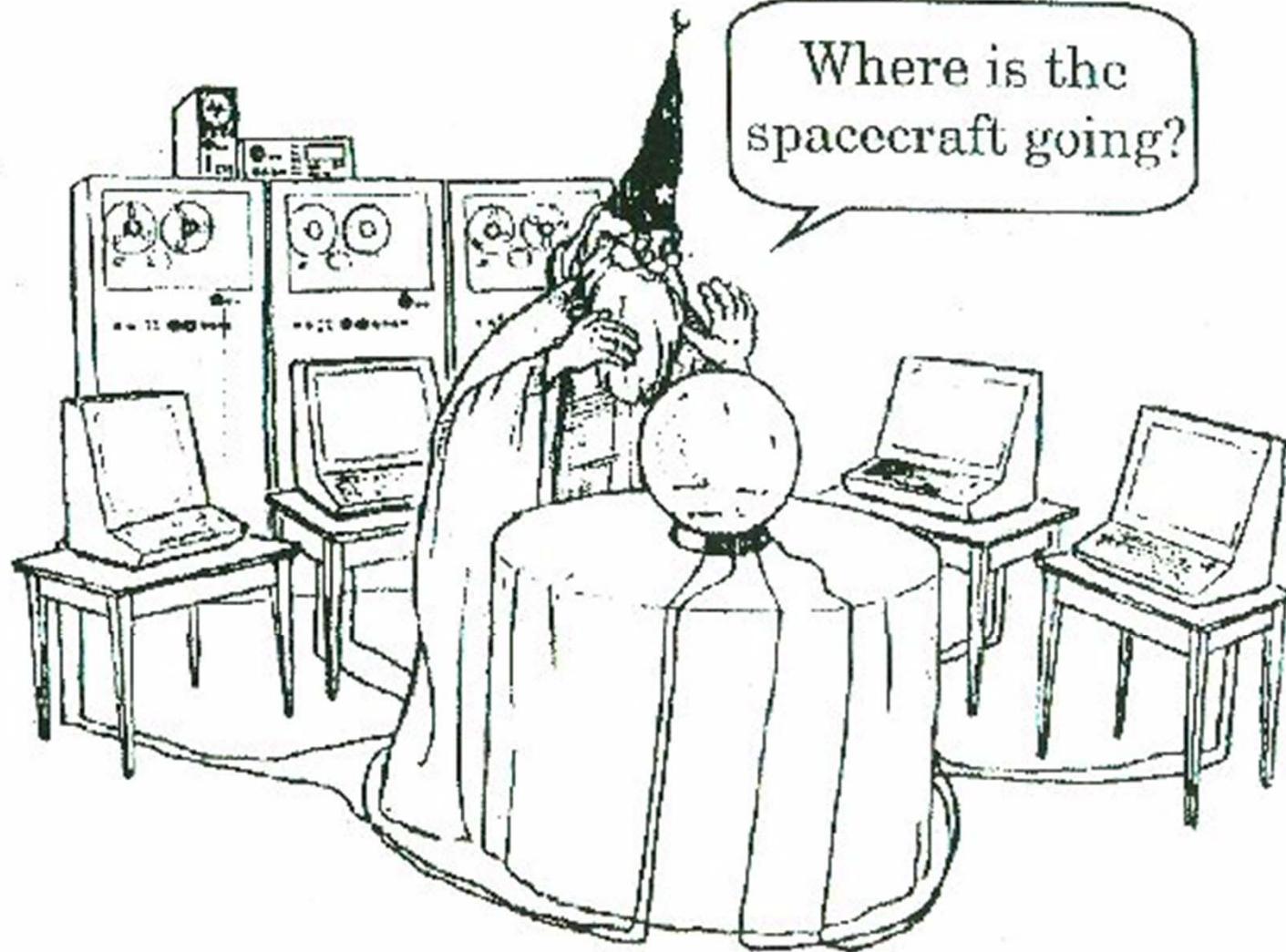
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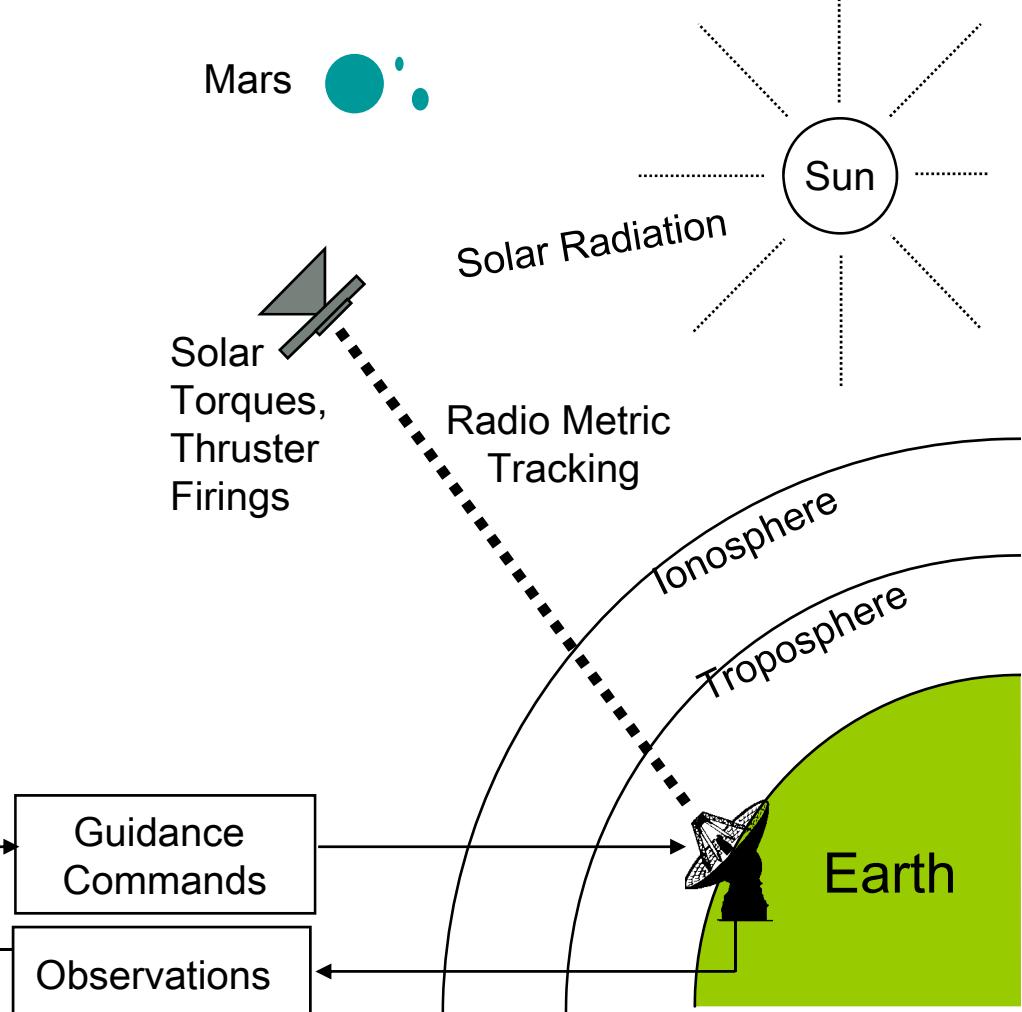
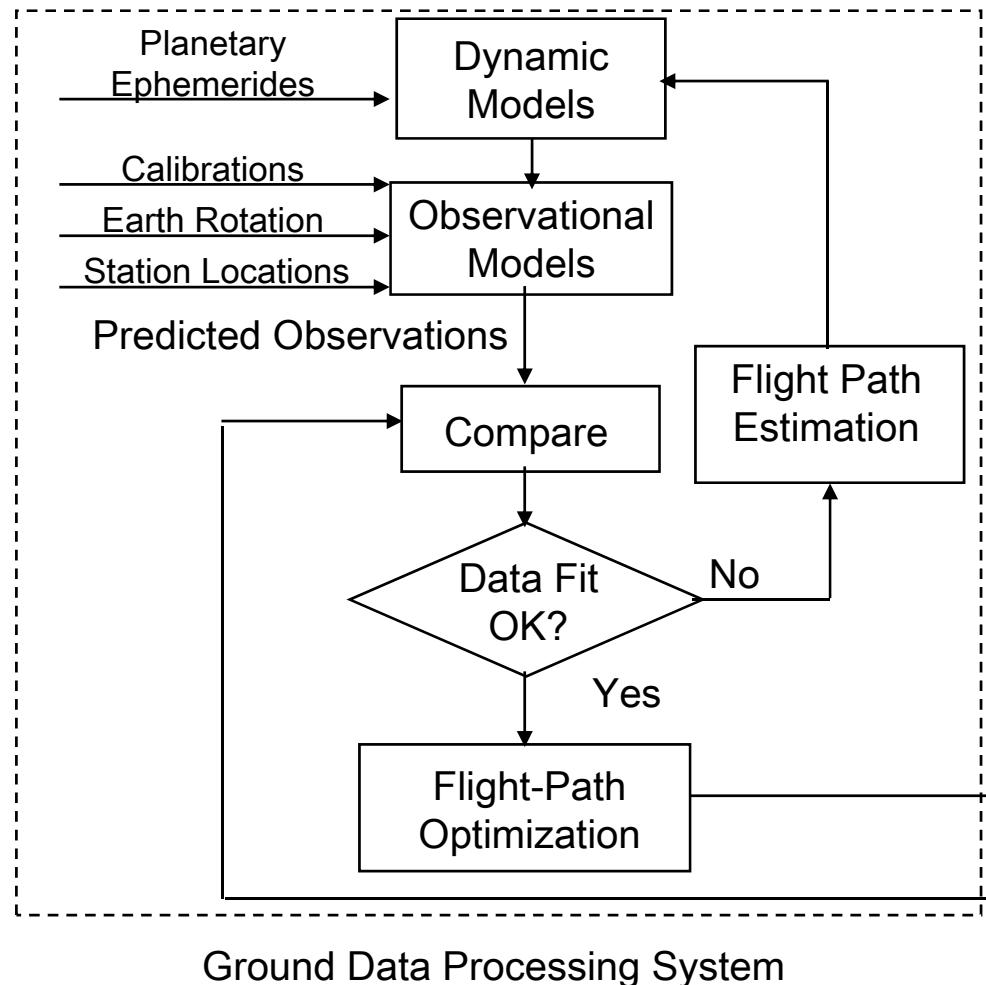
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*Курс Лекций: «Современные Проблемы Астрономии»
для студентов Государственного Астрономического Института им. П.К. Штернберга
7 февраля – 23 мая 2011*

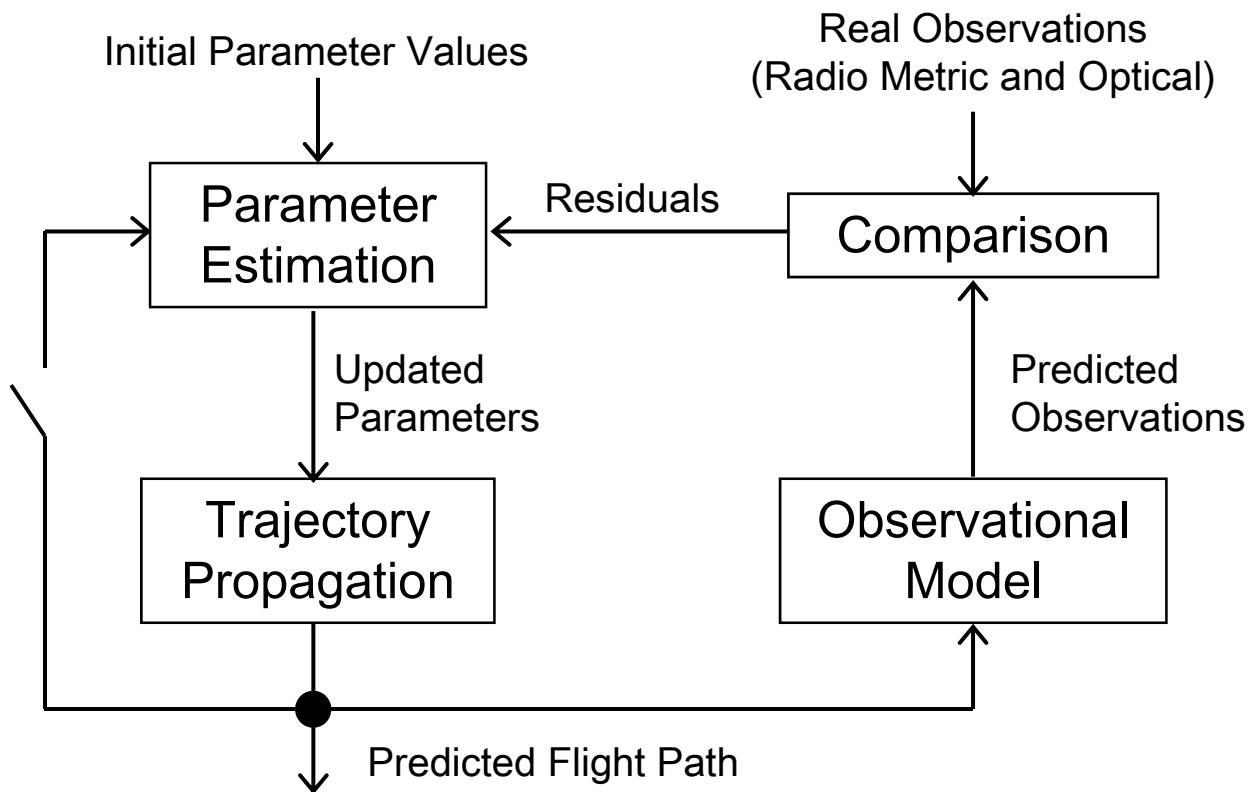
Wizardry of Spacecraft Navigation



Functions: Measurement Acquisition, Flight Path Determination, Maneuver Computation and Command



- Estimation process is performed by adjusting free parameters so as to minimize measurement residuals in some weighted-least-squares sense
 - Measurement accuracies and accuracies with which parameters were known before receiving measurements are taken into account
- Estimated parameters include
 - Spacecraft position and velocity at some reference time
 - Additional optional parameters:
 - Non-gravitational accelerations
 - Discrete velocity changes
 - Planetary and satellite ephemerides
 - Gravity field characteristics
 - Tracking station locations
 - Measurement biases



- **DPTRAJ – Trajectory Propagator**
 - Integrates equations of motion (full set of acceleration models)
 - Allows detailed tuning of models via input
 - Generates S/C ephemeris file (P-file)
 - Passes S/C ephemeris to Orbit Determination Program (ODP)
- **ODP – Orbit Determination Program**
 - Processes radio metric data (range, Doppler, VLBI, etc.) to produce estimates of state, maneuvers, and astrodynamical and geophysical parameters
 - Performs nonlinear estimation for specific modeled parameters via various filtering strategies
 - Produces statistical estimates for estimated parameters and maps solutions to target space
 - Calculates covariance and sensitivity matrices for all parameters estimated



- **MOPS -- Maneuver Operations Program**
 - Determines propulsive maneuvers and associated commandable quantities from S/C trajectory and characteristics
 - Predicts delivery accuracy to target
 - Determines optimized propellant utilization
 - Meets mission target requirements
 - Computes ideal Delta-V and maneuver start time
- **NavUtils -- Navigation Utilities**
 - Includes benchmarks, file comparison, file check, file compression, file creation, file dump, file format conversion, file identification, file merge, file print/plot, file update, file shorten, data scheduling, data simulation, etc.
 - Includes covariance analysis, DSN frequency/pointing predicts, planetary feature models, S/C antenna models, momentum desaturation models, physical constants, time conversion, etc.



- **NavLibs – Navigation Libraries**
 - ~ 70 libraries
 - ~ 4000 subroutines
 - ~ 750,000 LOC (FORTRAN 77)
 - Portable
 - All Use NAVSYS
 - Reusable
 - Shared throughout system and among users
- **NAVSYS Library -- System Dependent Functions**
 - Colocates all computing system dependent features
 - Isolates system dependent features to single library on which other modules depend
 - Uses common IEEE definition for errors/exceptions
- **Numerous other programs and modules not listed above**

Navigation Software Migrations



- 1960s: IBM 704/7094 → Univac 1108
- 1970s: Univac 1108 → Unisys 1100
- 1980s: Unisys 1100 → DEC Vax
- 1980s: DEC Vax → SUN Sparc
- 1990s: SUN Sparc → HP 9000, DEC Alpha, SGI (General Unix)

- Equations of motion

$$\ddot{\vec{r}} = \vec{A}_{grav} + \vec{A}_{non-grav}$$

- Gravitational accelerations

- Static & time varying gravitational field of the PCB
- Third body perturbations
- One-body relativistic perturbations

- Non-gravitational accelerations

- Atmospheric drag, solar radiation, PCB radiation
- Spacecraft thruster firing, gas leaks, maneuvers

- Observed & computed 2-way range observables

$$\rho_o^{up+down} = c \Delta t + \rho_{tropo}^{up+down} + \rho_{iono}^{up+down} + \epsilon$$

$$\rho_c = |\vec{r}_{sc} - \vec{r}_{sta}|_{phase}^{up} + |\vec{r}_{sc} - \vec{r}_{sta}|_{phase}^{down} + \rho_{rel}^{up+down} + \rho_{bias}$$

- Observed & computed 2-way Doppler observables

$$\dot{\rho}_o = \frac{c}{f_t} \left[\frac{(N_{12} - N_{trop} - N_{iono})}{(t_2 - t_1)} - \right] + \epsilon$$

$$\dot{\rho}_c = \frac{1}{t_2 - t_1} [\Delta \rho_c + \Delta \rho_{rel}] + \frac{c}{f_t} \Delta f$$

- Location of the DSN station in Earth-fixed coordinates

$$\vec{r}_{sta} = \vec{r}_{marker} + \vec{S}_{plate} + \vec{S}_{phase} + \vec{S}_{dtide} + \vec{S}_{pole} + \vec{S}_{load}$$

- State equations

$$\dot{X} = F(X, t); \quad X(t_0) = X_0^*$$

- Liberalized state equations

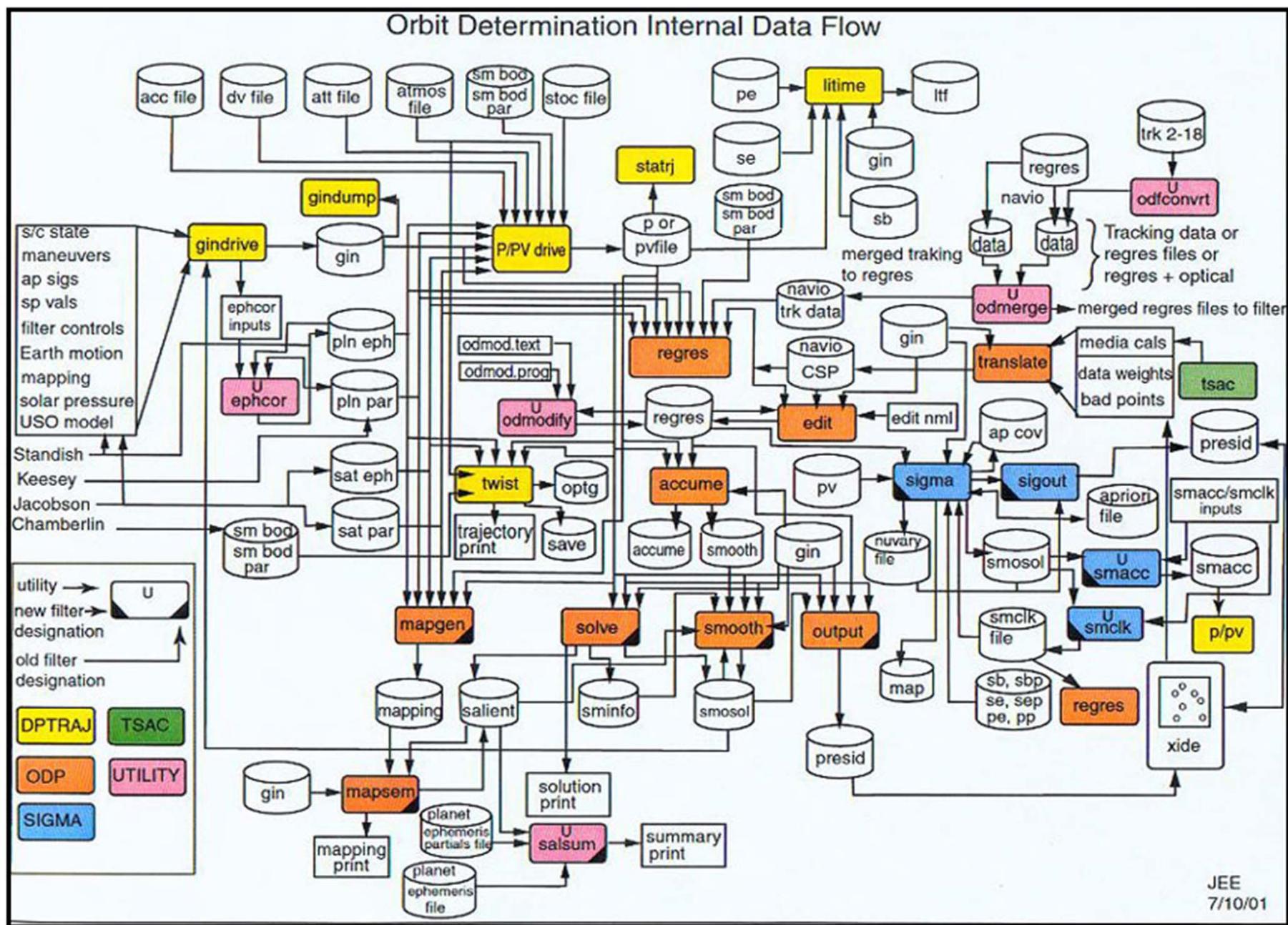
$$\dot{x} \cong \left[\frac{\partial F(X, t)}{\partial X} \right]_{X^*} \cdot x \quad x(t_0) = x_0; \quad x(t) \square X(t) - X^*(t)$$

- Observation equations

$$Y_j = G(X_j, t_j) + \varepsilon_j; \quad E[\varepsilon_j] = 0; \quad E[\varepsilon_i \varepsilon_j^T] = \delta_{ij} f_j^{-1} R_j$$

- Linearized observation equations

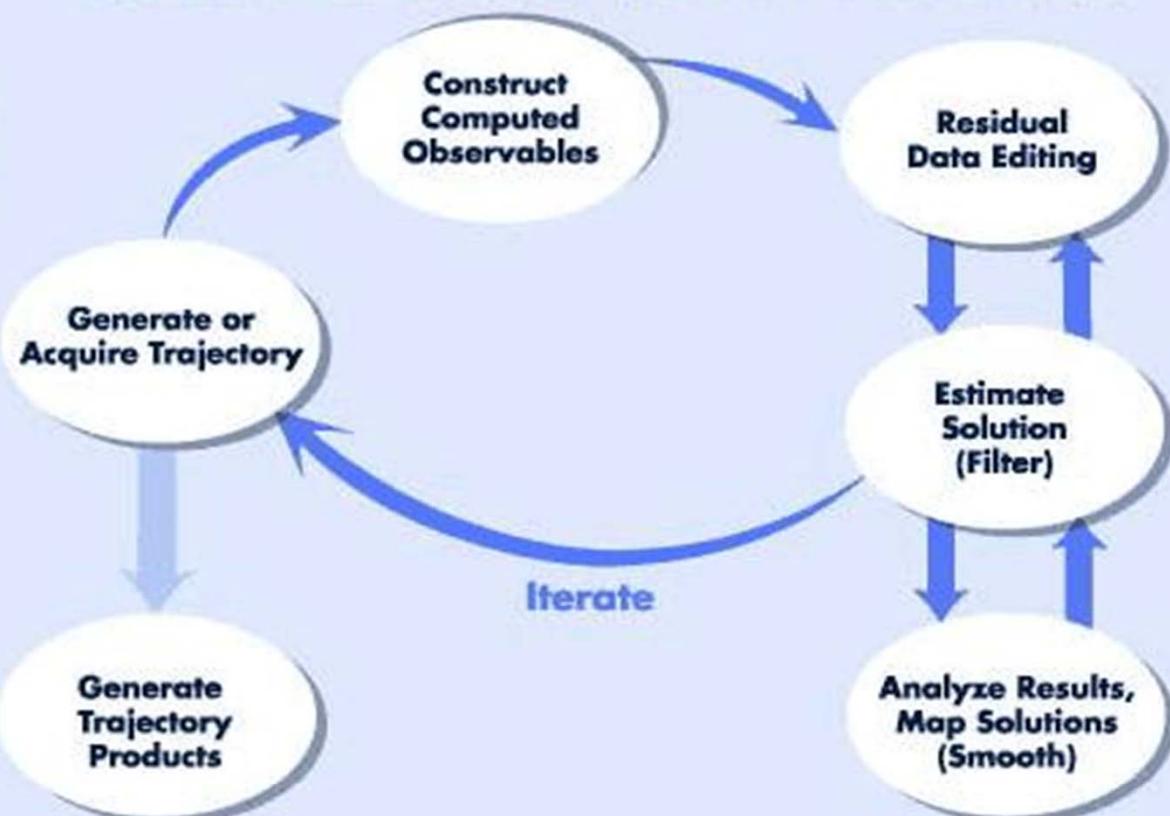
$$y_j \square Y_j - G(X_j^*, t_j) \cong \left[\frac{\partial G(X_j^*, t_j)}{\partial X} \right]_{X^*} \cdot x_j + \varepsilon_j = H_j x_0 + \varepsilon_j$$



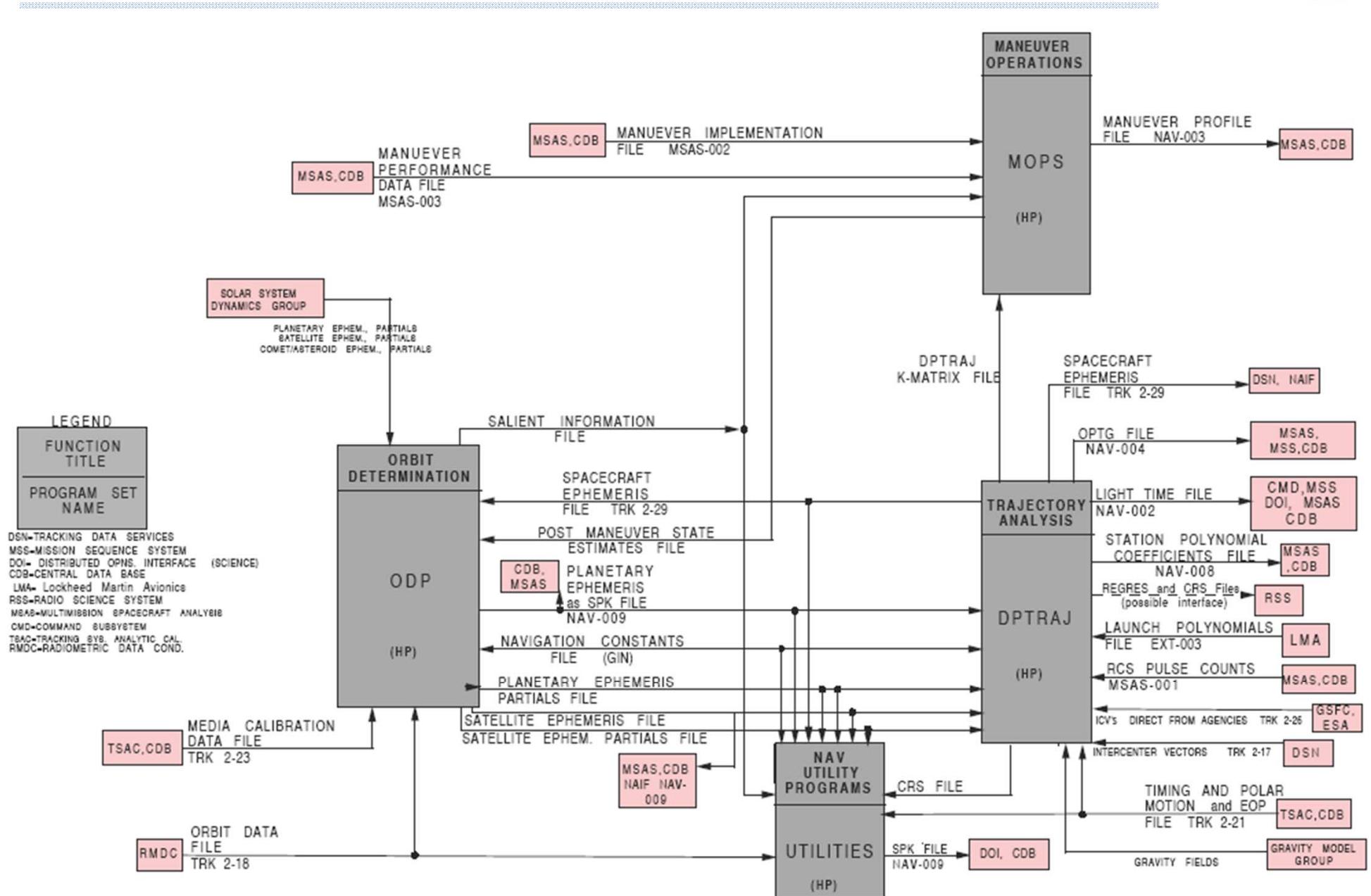
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PROGRAM SETUP

ORBIT DETERMINATION PROGRAM

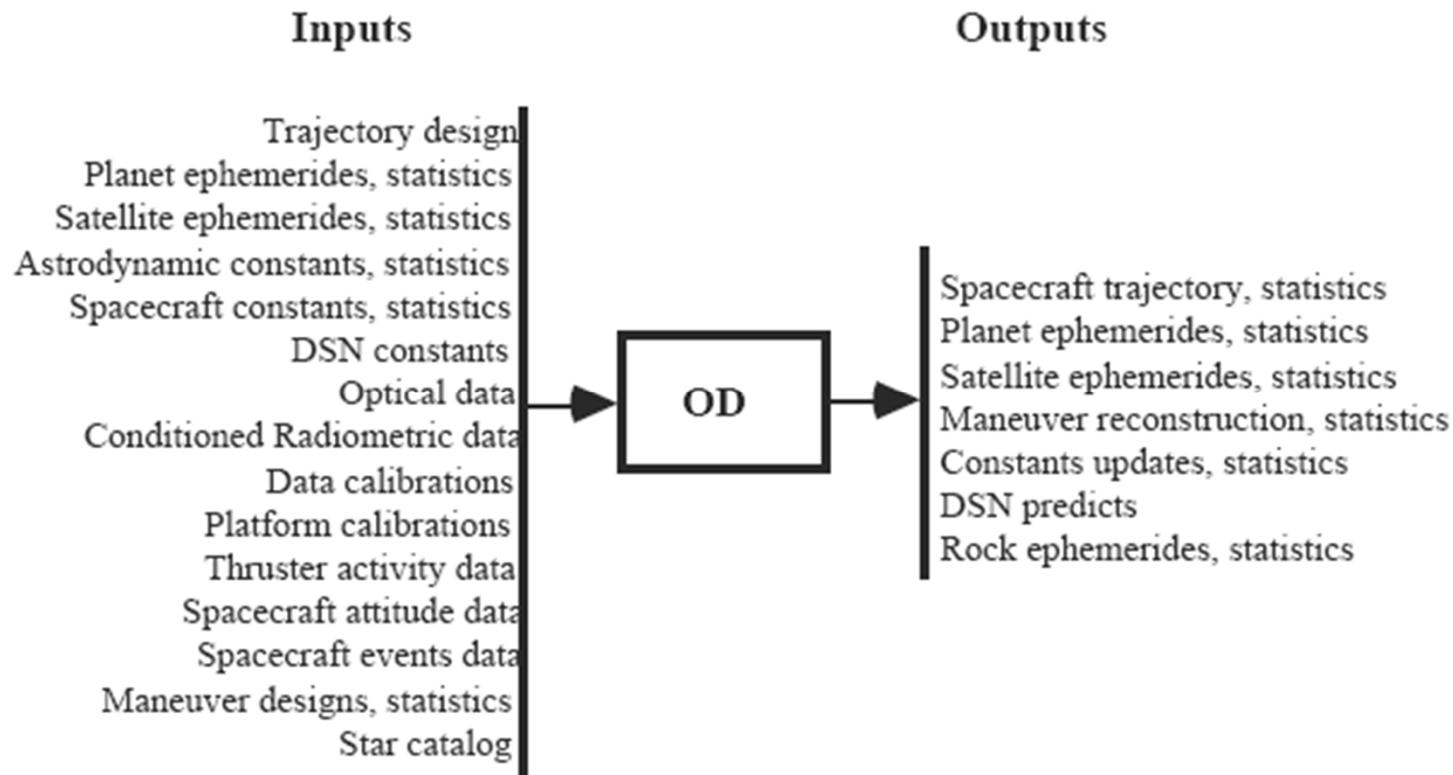


Interface Flow for Cassini Nav Software



Context: The Goal of OD?

- Take an enormous amount of input data and reduce it to manageable, usable, and comprehensible form, i.e., plots, readable summaries, Navio files, etc.

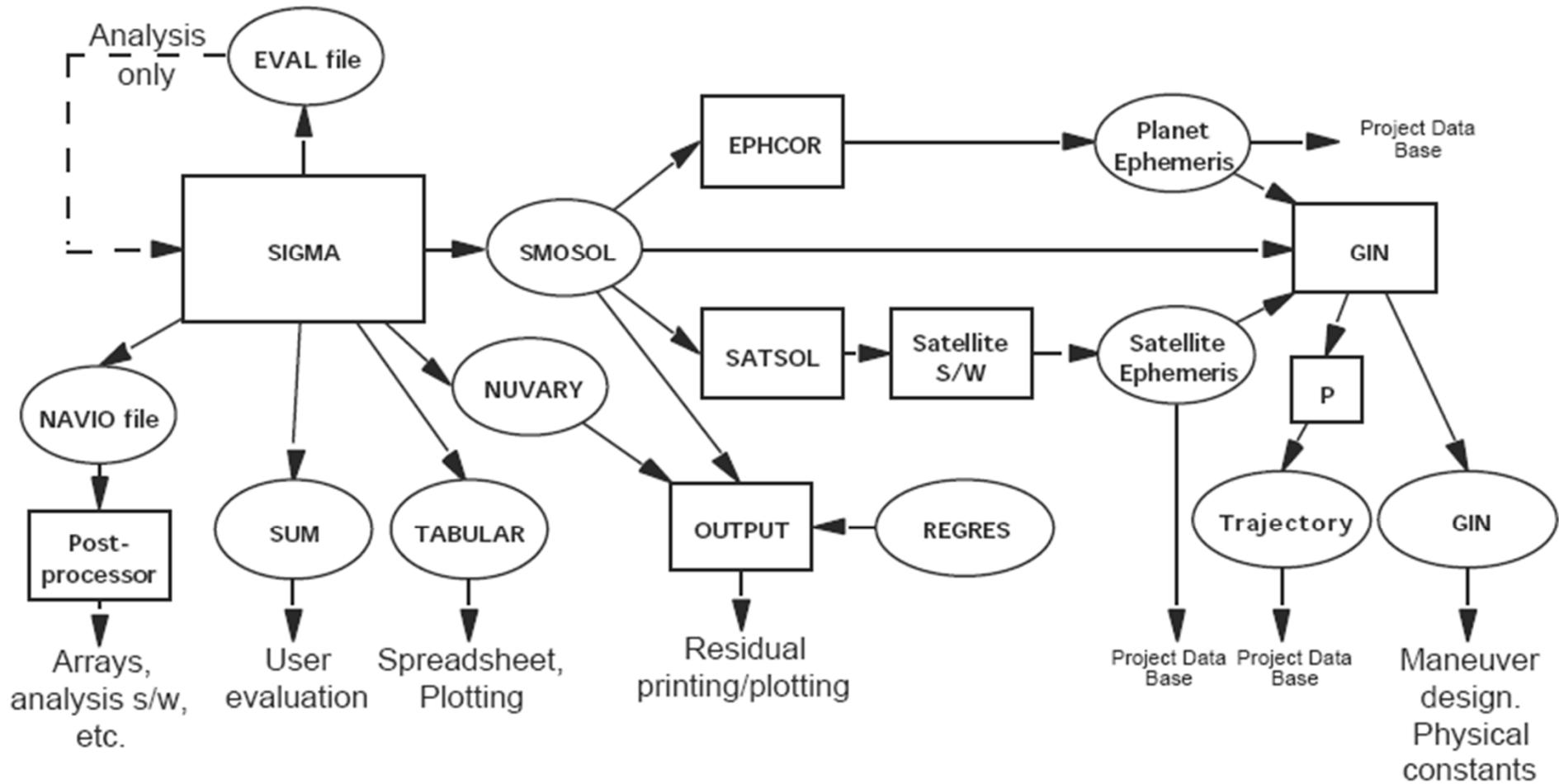


The finest solutions in the world are only random numbers, if you cannot comprehend them.

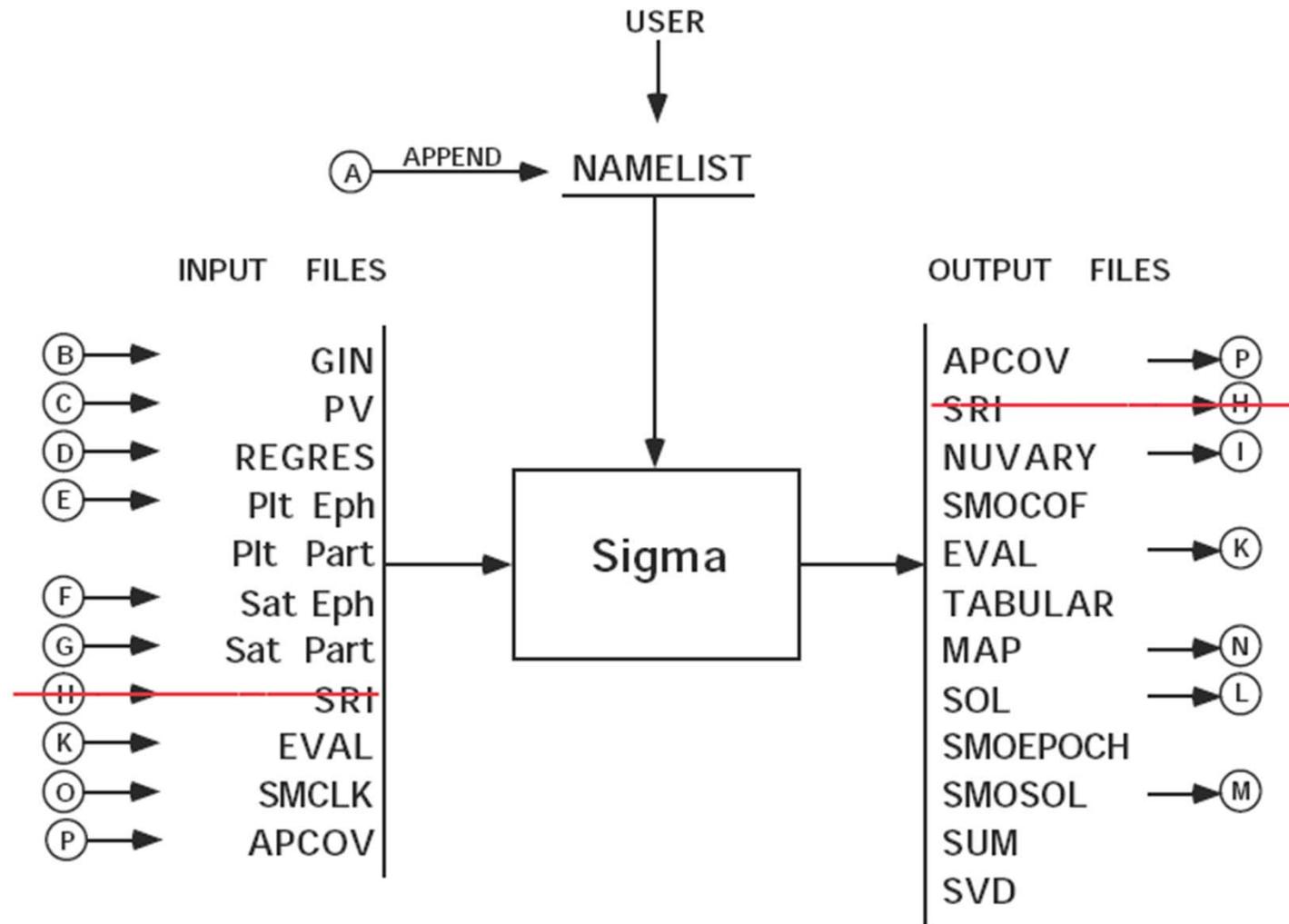
— Anatole France

What is Sigma?

- Reads measurement data in the form of a REGRES file
 - Generates solutions and covariances
 - Maps solutions and covariances
 - Outputs information in convenient formats
 - Spreadsheet data for easy plotting
 - Summary
 - Navio files
- Features:
 - Choice of UD factor or SRI (Square Root Information) filtering
 - SVD (Singular Value Decomposition)
 - Symbolic times
 - Multiple spacecraft
 - Parameter aliases (for covariance studies, not recommended for iterating)
 - Runtime control of filter (NIBs)
 - And more ...



A rather complex maze of relationships – all for one goal to make it simple!



Logic was developed over 20 years of continuous operations in deep space

ODP

Orbit Determination Program

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EDIT1	<input type="checkbox"/>
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GINUPDATE	<input type="checkbox"/>
PDRIVE	<input type="checkbox"/>
PVDRIVE	<input type="checkbox"/>
REGRES	<input checked="" type="checkbox"/>
SEPV	<input type="checkbox"/>
SIGMA	<input type="checkbox"/>
STATRJ	<input type="checkbox"/>
TRANSLATE	<input type="checkbox"/>
TWIST	<input type="checkbox"/>

ORBIT DETERMINATION PROGRAM

**Construct
Computed
Observables**

**Residual
Data Editing**

**Estimate
Solution
(Filter)**

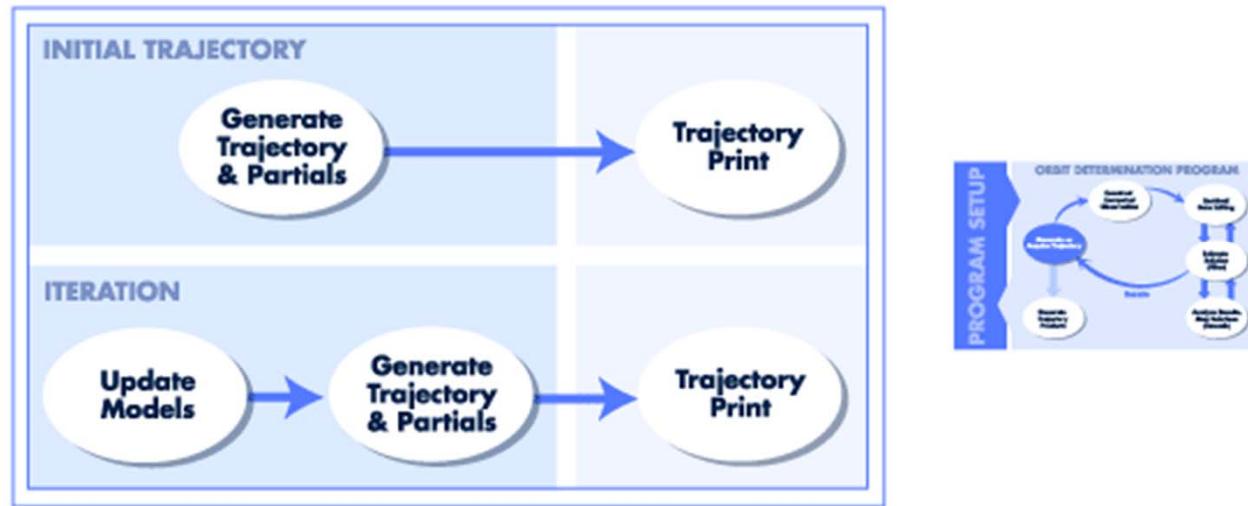
**Analyze Results,
Map Solutions
(Smooth)**

**Generate
Trajectory
Products**

Iterate

- Process
- Tips
- Theory
- Parameters
- Inputs / Setup
- Inputs / Format
- Outputs / Evaluations
- Case Studies

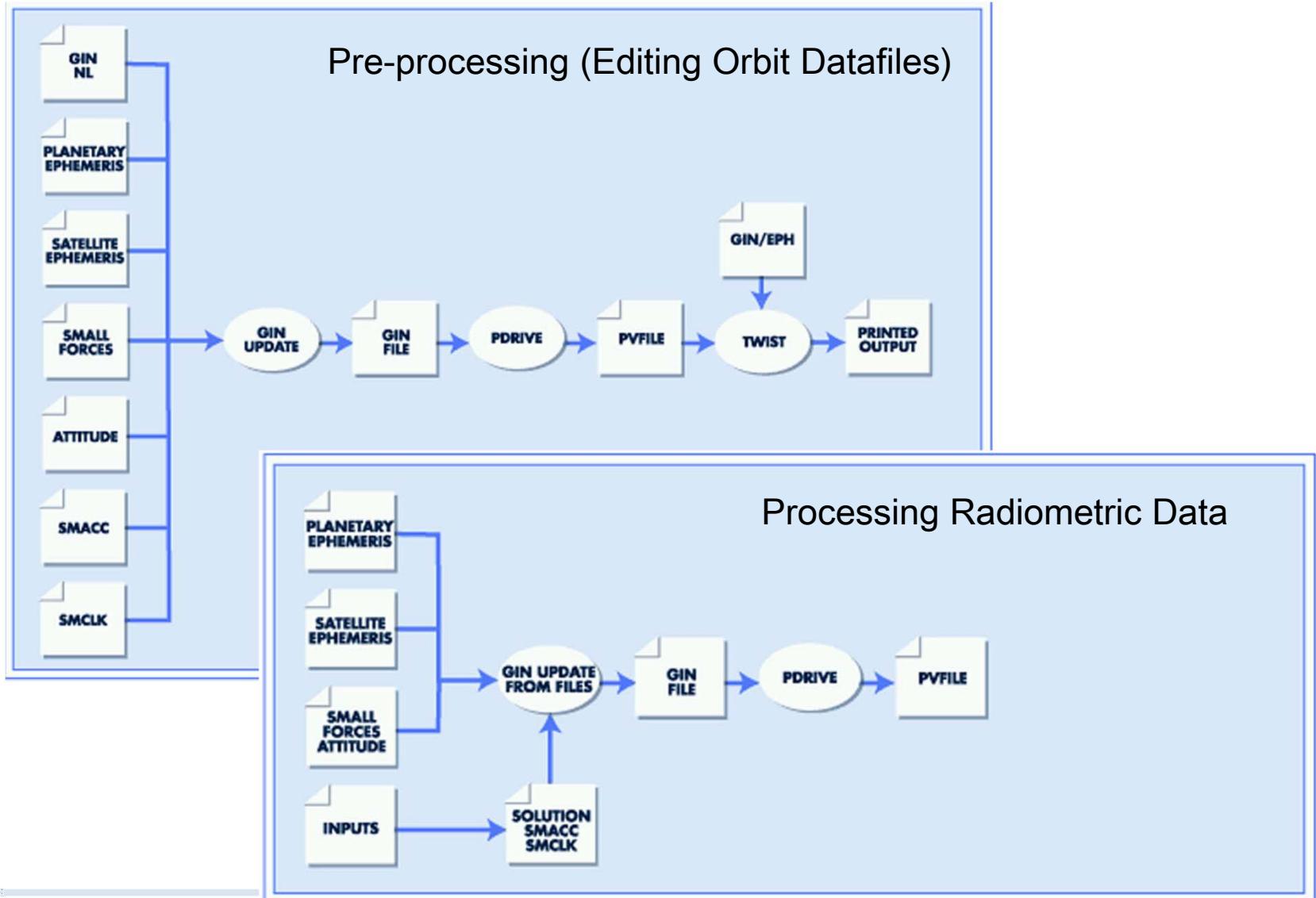
GENERATE OR ACQUIRE TRAJECTORY



- **Generate or Acquire Trajectory**

- Generating the satellite's trajectory involves calculating an initial solution based on a priori estimates of the forces acting on the satellite.
- From the initial solution, an iteration of the trajectory based on updated partials and observables creates an ephemeris within the required tolerances.

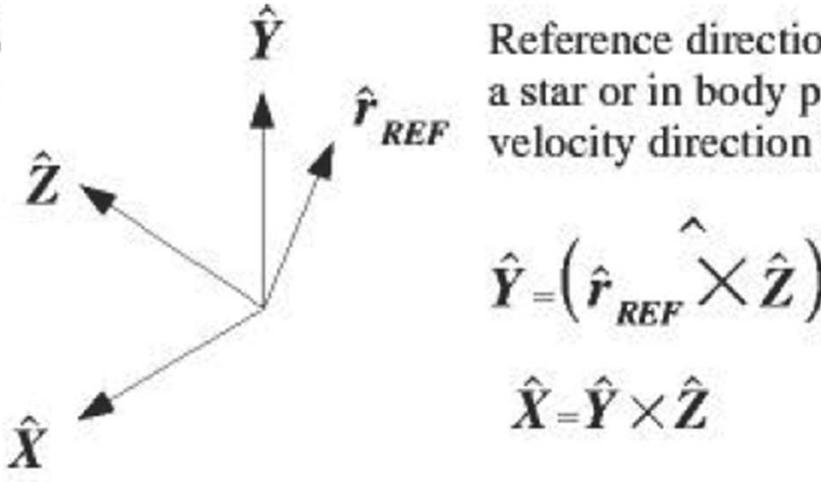
Generate or Acquire Trajectory



Z-axis direction: towards a body, inertial direction, or SRP system

Note that \hat{X} and \hat{r}_{REF} are in opposite directions

Rotations about Z, X, Z applied to get to s/c coordinate system (X^*, Y^*, Z^*)



Reference direction: towards a star or in body position or velocity direction

- External Attitude file

- contains quaternions and rates vs. time, uses Hermite interpolation
 - optional

- GIN inputs:

ACANO : contains the inertial reference direction in reference coordinates

TCANO : interval start time for the reference direction given in ACANO

TUPRS : spacecraft orientation change epochs (format depends on TTYPE)

Per TUPRS time:

UPRS : describes the spacecraft Z direction ('BODY' or 'VECTOR')

DUPRS : defines the spacecraft Z direction at epoch (<unit vector >or <2.D0, RA, DEC>)

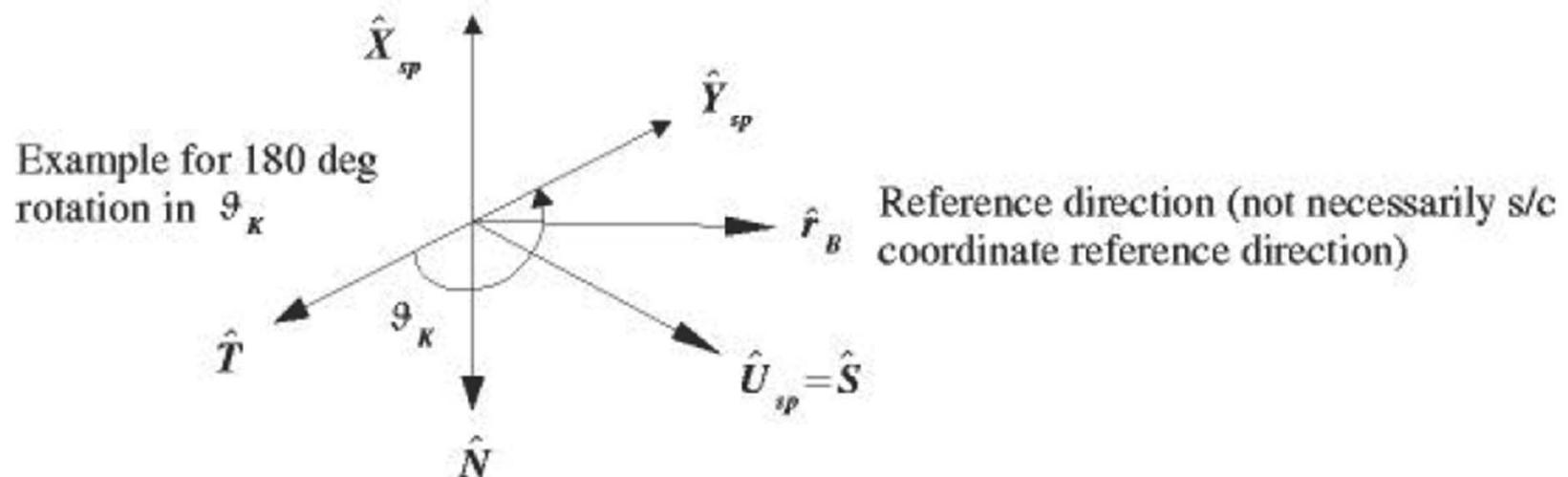
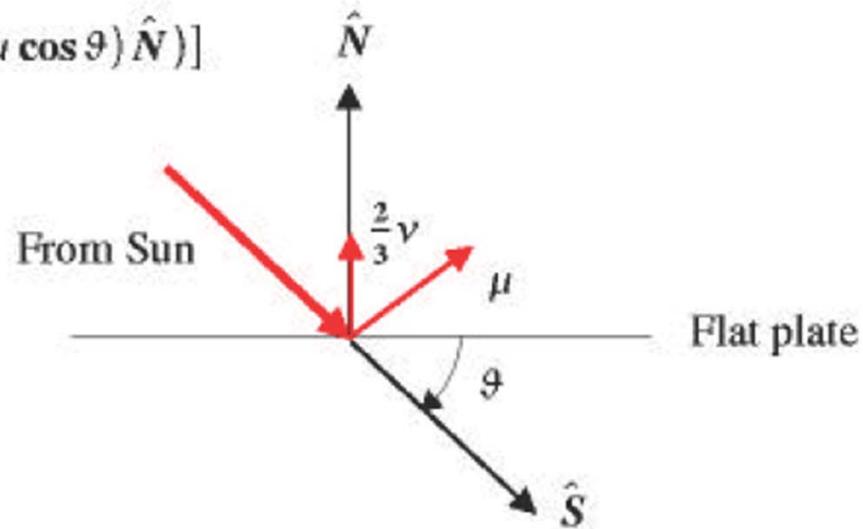
ANGLS : rotations angles used to define the spacecraft coordinate system

REFBS : defines reference direction for spacecraft X ,Y ('STAR' or '<'BODY','POS'> or '<'BODY','VEL'>')

$$\mathbf{F} = \frac{W/m}{c} [\cos \theta A ((1-\mu) \hat{\mathbf{S}} + (\frac{2}{3}\nu + 2\mu \cos \theta) \hat{\mathbf{N}})]$$

$$\mu_{ODP} = \frac{1}{2} \mu_{REAL}$$

$$\nu_{ODP} = \frac{1}{3} \nu_{REAL}$$



\$ Solar Radiation Coordinate System

\$ ANGL: Angle about Sun-S/C axis from Ysp to Sun normal.

REFB = 'STAR',

\$ Reference Direction:

ANGL = 0.0,

\$

\$

UPRC(1 ,1) = 'SC ANG',

DUPRC(1, 1 ,1) = 1.0, 0.0, 0.0,

\$ normal to the surface

\$

\$ Spacecraft Component Model (from LMA)

\$ Cruise : area=7.26 m^2, Diffusive=0.18/3, Specular=0.23/2

\$ Open : area=8.73 m^2, Diffusive=0.18/3, Specular=0.26/2

\$ Science: area=9.03 m^2, Diffusive=0.13/3, Specular=0.31/2

\$

COMP(1) = 'FLAT PLATE',

\$ Science Configuration

CSIZE(1,1) = 9.03,

\$ Area of summed average (sq. m)

SCOFC(1,1) = 0.155,

\$ Specular (summed and /2 of LMA value)

0.0433

\$ Diffusive (summed and /3 of LMA value)

ICMPTM(1) ='26-NOV-2001 18:15', \$ Science Configuration

USECMP(1,1) =1,0,0,0,0

SOLSCL(1) =1.0D0,



- **Sample gin.par file**

```
PARTLS( 1)= 'X','Y','Z','DX','DY','DZ',
$ SRP
      'SPEC01', 'DIFF01',
$ Accelerations
      'ATAR','ATAX','ATAY',  $ X is +X on s/c
$          'GLAR01','GLAX01','GLAY01','GLBT01',
$Wet and dry troposphere models, Night and day ionosphere models for the DSN sites
      'TROPD1','TROPW1','IONOD1','IONON1', $ for ODP
      'TROPD2','TROPW2','IONOD2','IONON2',
      ....
$ DSN station locations in cylindrical coordinates
      'CU24','LO24','CV24', $ 34 m BWG
      'CU34','LO34','CV34',
      ....
$ UT1 and polar motion (of Earth)
      'XPOLE','YPOLE','UT1',
$ Impulsive burns,
'IDLX01','IDLY01','IDLZ01',
'IDLX02','IDLY02','IDLZ02',
$ Finite burns
'FO03','ALPO03','DLTO03','STB03','DRB03',
```

This is only an example, complete list of partials is much larger...



- **Gas Leak**
 - Exponential model (AX, AY, AR, BB, ISTREX, ISTPEX)
 - Quadratic model (SAAP, SAAT)
- **Small Forces**
 - SMFTYP, SMFCRD, ISMFTM, SMFDR, SMFDV
- **Impulsive burn**
 - MB1T, MB1V, MB1P, MB1D, BRNCRD,
- **Finite burn**
 - ITPEQ, BURN, MA1T, MA1D, MA1A, MA1F, MA1M, MA1K,



Dynamic Input Example



```
# Impulsive burn model with duration for SPIN_CONTROL
MB1T(17)      = '25-SEP-2002 16:56:22 UTC', $ end of burn if MB1D, otherwise middle
MB1D(17)      = 68.0,                 $duration
MB1P(17)      = 0.00e+00,             $mass decrement
MB1V(1,17)    = -9.749e-05, 1.385e-05, 3.367e-05, $delta-V in km/sec
```

```
# Finite burn for MAIN_DV
```

```
LPLANE(3)     = '      ', $thrust vector direction
                  $ '      ' (using COORS) or 'SC' or 'VELOC' or 'HORIZ' or 'VIEW1' or 'VIEW2'
LDYN(3)        = 0,       $reference plane: 0 for static, non-zero for dynamic
BURN(3)        = 1,       $termination flag, 1 for duration, 2 for Delta-V, 3 for C3
COORS(1,3)     = '      ','SPACE','EARTH','MEAN ','EQUATO',
MA1T(3)        = '25-SEP-2002 17:01:40 UTC',
MA1D(3)        = 342.0,   $duration, sec
MA1A(1,3)      = 1.72e+02, 4*0.0DO, $RA 4-th order polynomial
MA1A(6,3)      = 1.89e+01, 4*0.0DO, $DEC 4-th order polynomial
MA1F(1,3)      = 1.596568e+00, 4*0.0DO, $Thrust 4-th order polynomial
MA1M(1,3)      = 0.001e+00, 3*0.0DO, $mass flow 3-rd order polynomial
```



- **From SFF SIS ..**

The Small Forces File (SFF) provides to JPL's Orbit Determination Program (ODP) the cumulative delta-V effect of attitude thruster firings over one or more specified intervals of time. In some cases it also provides an estimate of the cumulative spacecraft mass loss due to the use of propellant in those attitude thrusters. The same format file is also produced to model predicted accelerations

- **Format**

- **ID, Flag, t_created, t_start, t_stop, dt, dmass, dv_x, dv_y, dv_z**
- **Comment fields are ignored by DPTRAJ/ODP**

- **Stardust Example**

7821, R, 2001-11-07 13:00:00, 2001-11-06 13:00:00.000, 2001-11-07 01:00:00.000, 43200.000, 0.003, 0.012, 0.006, 0.002

7822, R, 2001-11-08 01:00:43, 2001-11-07 01:00:43.560, 2001-11-07 13:00:43.560, 43200.000, 0.002, 0.002, 0.009, 0.001

- **P-file**

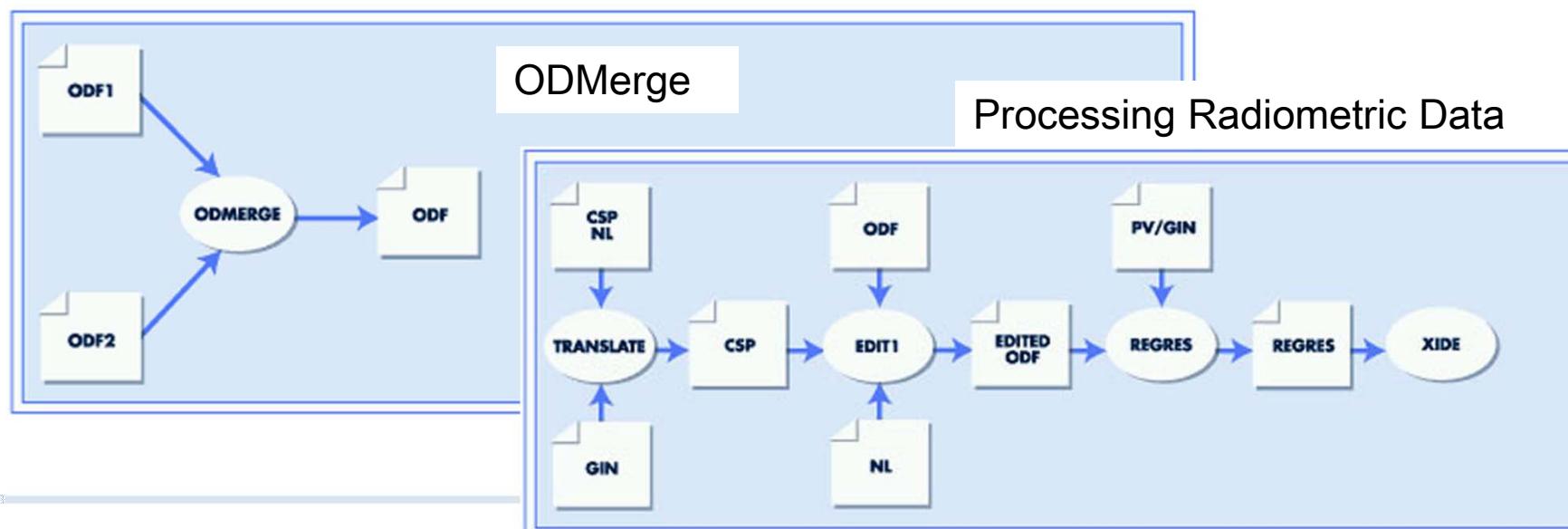
The **PFILE**, written by program **PDRIVE**, consists of records containing input data which characterizes the *spacecraft ephemeris* and records containing modified *difference arrays* which can be read and interpolated to produce the probe's position and velocity at any time. Program **FAST** can also write a **PFILE**.

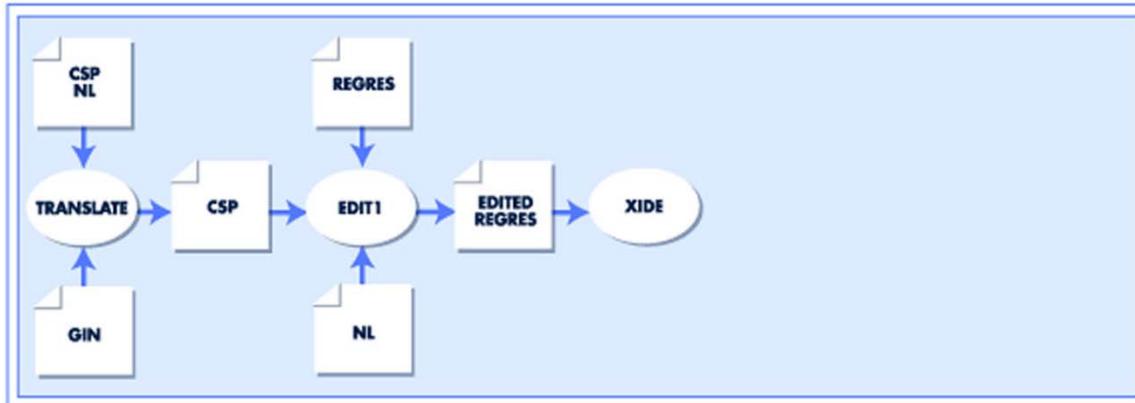
CONSTRUCT COMPUTED OBSERVABLES



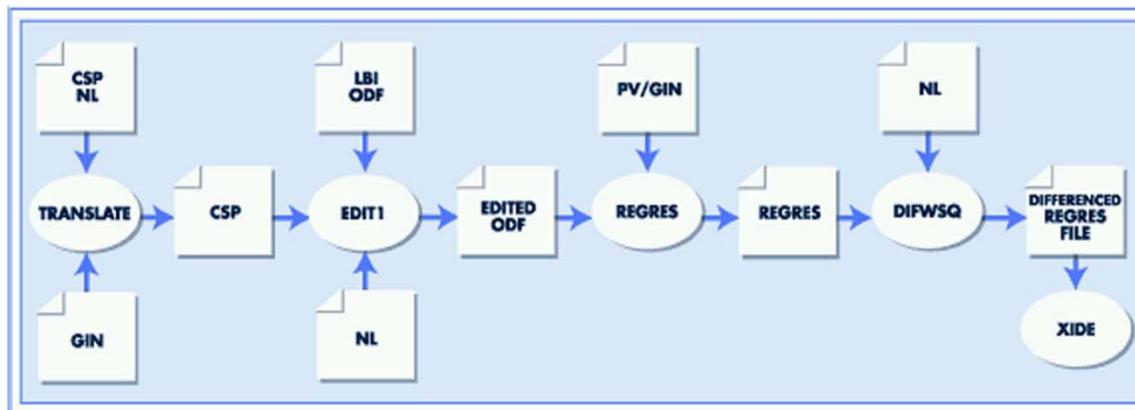
- Computed Observables:

- Using PVDRIVE data, ODF and CSP inputs, run REGRES to produce a file of observables, and all data partials of observables, at specified times.
- The observables and partials are with respect to estimated (solve for) parameters and regression partials, using the nominal or reference trajectory.

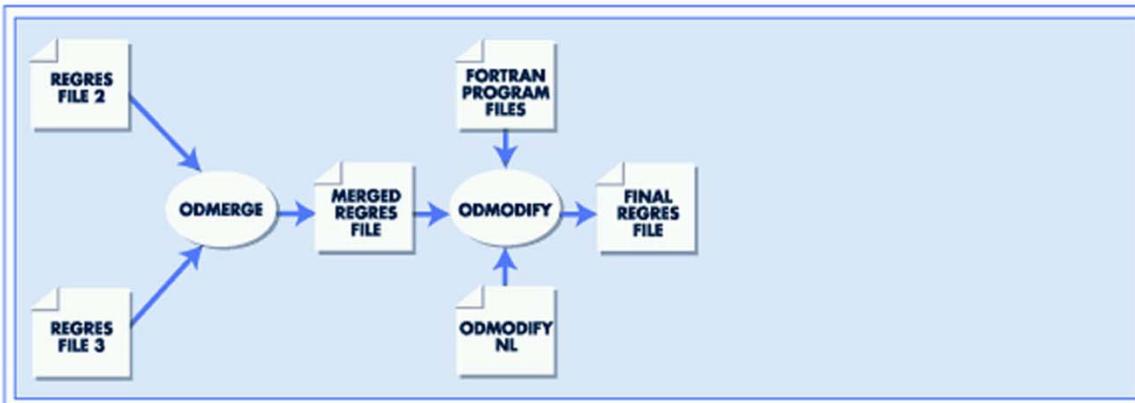




Processing
Differentiated
Data



Processing
Optical Data



Merging and
Modifying
Data



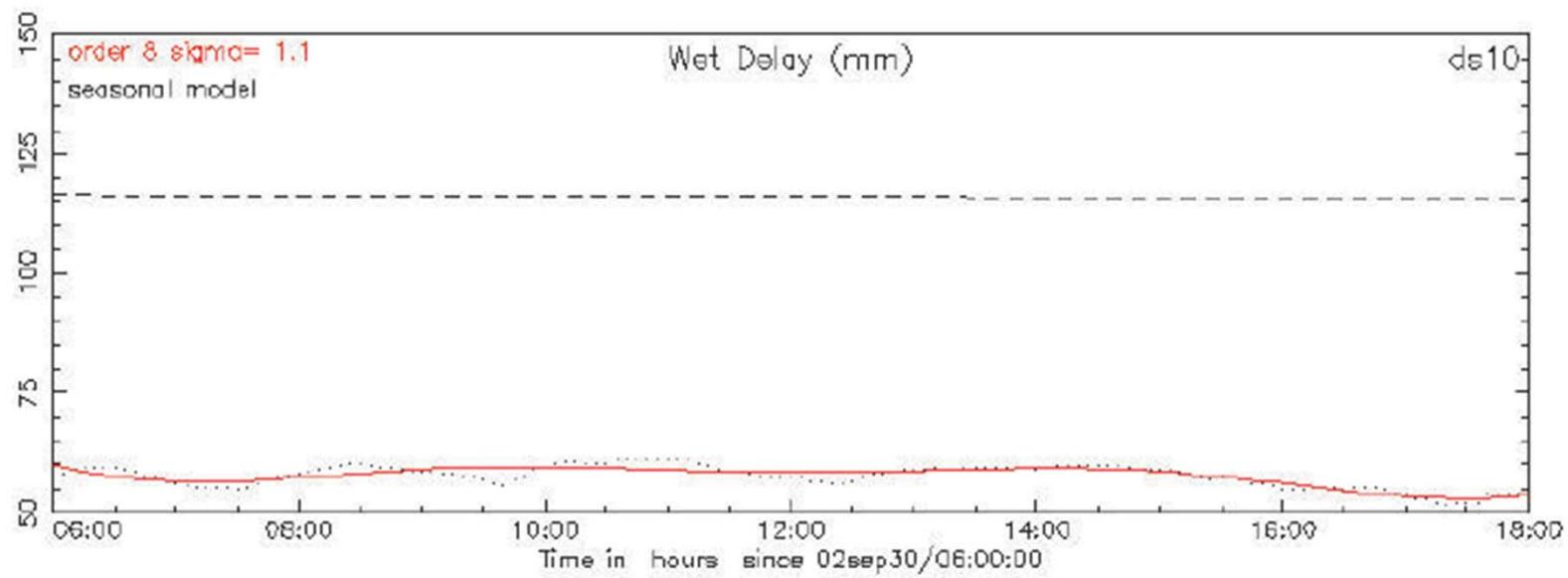
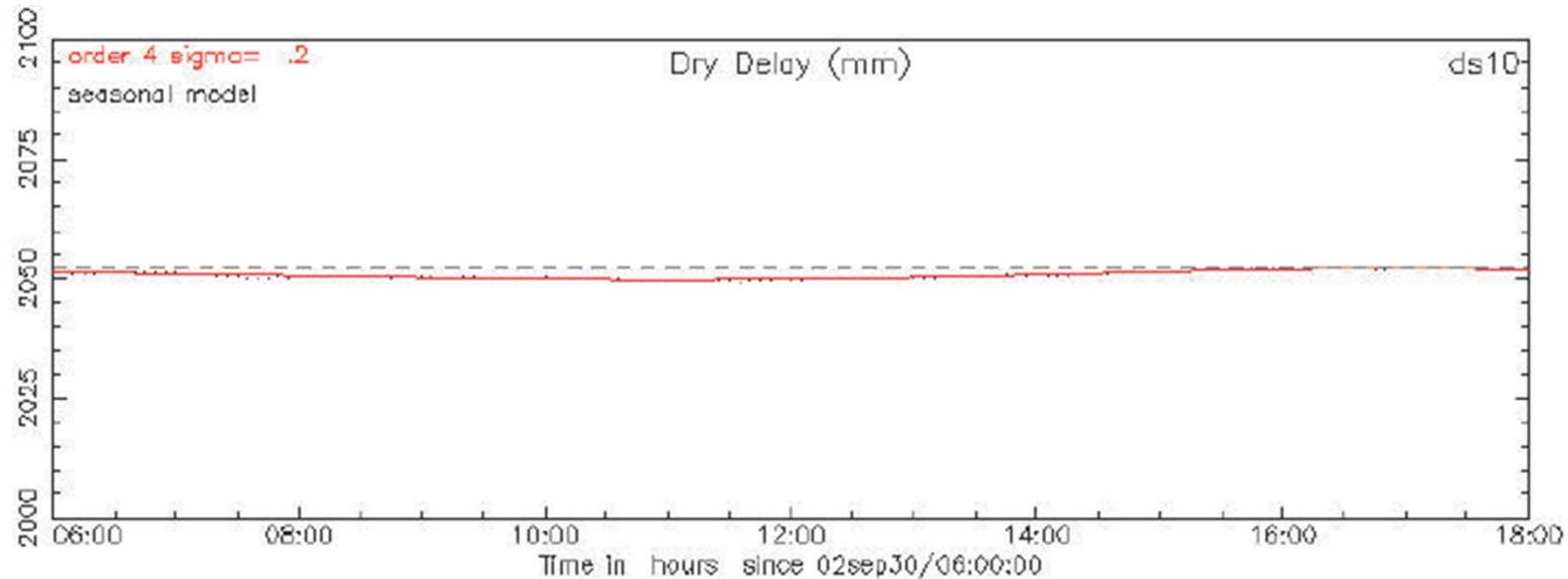
- **Earth Orientation File**
 - Earth Orientation Parameters File (EOP) by the Kalman Earth Orientation Filter (KEOF).
 - Used to tie the measurements recorded at the tracking stations in the International Earth Rotation Service (IERS) terrestrial ref frame (ITRF93) to the celestial ref (radio) frame (ICRF93).
- **What is it?**
 - Gin inputs for Earth’s orientation in the ICRF93 frame. Contains:
 - x, y polar motion updates (milliarcseconds),
 - TAI - UT1 (sec), TAI - UTC (sec)
 - nutation corrections (milliarcseconds)
 - One file for all missions, all DSN stations

Major Inputs to the ‘Regres’ Software



- Media Calibrations:
 - Ionospheric & Tropospheric Calibrations are computed by the Tracking System Analytic Calibration (TSAC) group.
 - Used to calibrate radio metric data (Doppler, Range, DDOR) using measurements of media based on GPS survey at DSN complexes.
- What is it?
 - Individual files of coefficients written for each month that describe radio signal delay as a function of time in meters
 - Troposphere/Zenith delay coefficients
 - Seasonal (Trigonometric, wet and dry, good for all missions/time)
 - Station Altitude (Constant, dry, 1 per station, good for all missions/time)
 - Seasonal Correction Delay (Normalized power series, wet and dry, 1 per complex, good for all missions)
 - Made with weather data from the complexes and GPS data
 - Ionosphere delay coefficients (Normalized power series)
 - Doppler/range (1 per complex per spacecraft per pass)
 - Delta-DOR (1 per complex per observation sequence)
 - Made with GPS data

Tropospheric Delay example



Major Inputs to the ‘Regres’ Software



- Station Locations
 - Used to relate data received at Deep Space Station antennas to S/C ephemeris (position, velocity) given with respect to some center of integration.
Covariance used to apply station location errors OD filter estimates.
- What are they?
 - Gin namelist inputs of
 - all DSN stations in cylindrical coordinates relative to the ITRF93 frame,
 - also includes all pertinent station complex parameters: i) antenna offsets, ii) linear plate motion parameters, iii) horizon masks,
 - regres model inputs (STNMOD)
 - solid tides, pole tide, ocean loading, polar motion, plate motion, ET-TAI vector conversion, etc
 - Gin namelist inputs for ‘old ODP filter’ for
 - Correlated cylindrical position covariance for all DSN stations
 - Need to be converted to newer sigma inputs



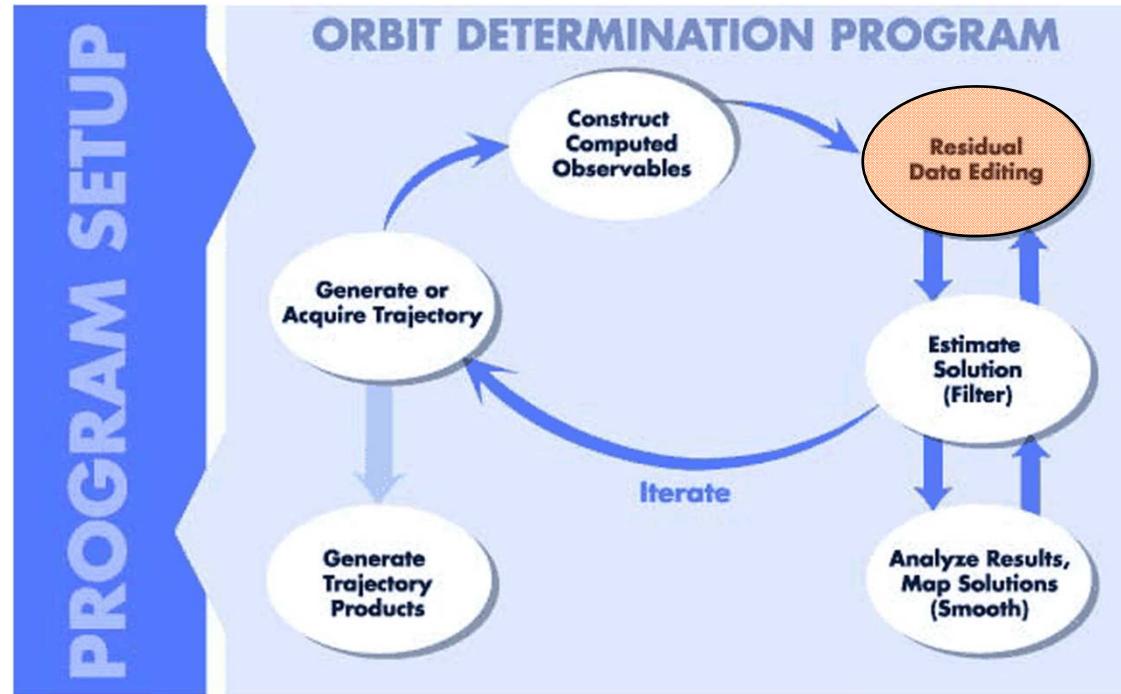
- Orbit data files (ODFs) or Tracking data files
 - Used as observables for determining S/C ephemeris using least squares filter.
 - 1,2,3-way Doppler, 2-way -3-way Doppler, SRA ranging, DDOR, optical, etc
- What is it?
 - Bit-packed files with tracking data observables (all data types) and some relevant ancillary information
 - Time, Data type, Bands, Station participants, Reference frequencies, etc.
 - Built from DSN data, along with configuration information from complexes
 - Note: Most navigators convert ODFs directly to the NAVIO Tracking Data File (NTDF) using ‘odfconvrt’ and use ODP tools to compress, inspect, change, etc.

Tracking Data Types (1)

- **One-way doppler (F1)**
 - F1 measurements are made when the S/C is the source of the signal's reference frequency. The frequency reference is usually provided by an auxiliary oscillator or an ultra-stable oscillator (USO). The signal frequency is doppler shifted on the downleg path.
 - Measures the line-of-sight component of the S/C velocity
 - F1 is prone to irregular biases and ramps
 - Units in the ODP are in hertz (Hz)
 - 1 Hz = 35.6 mm/s for X-band downlink
- **Two-way coherent doppler (F2)**
 - F2 measurements are made when a single tracking station radiates a signal to a S/C which in turn multiplies the received signal by a constant (turn-around ratio) and sends the signal back to the transmitting station. The signal frequency is Doppler shifted on both the upleg and downleg paths.
 - Measures the line-of-sight component of the S/C velocity
 - Units in the ODP are in hertz (Hz)
 - 1 Hz = 17.8 mm/s for X-band uplink/downlink
- **Ranging (SRA)**
 - Range observations are obtained at a single tracking station by measuring the time delay between the transmission and reception of a ranging signal
 - Measures the line-of-sight component of the S/C position
 - Units in the ODP are “range units” or RUs
 - 1 RU = 0.142 meter
 - **SRA** is an acronym for Sequential Ranging Assembly

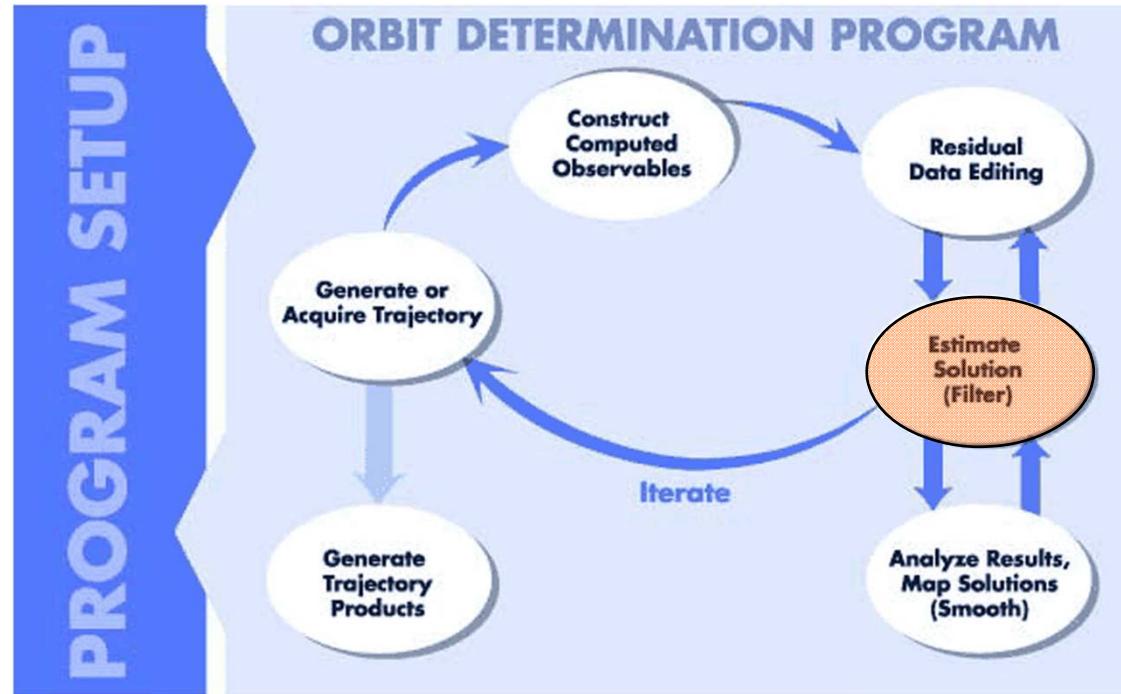
Tracking Data Types (2)

- **Delta-differential One-way Ranging ΔDOR**
 - A measurement technique that uses two widely separated tracking stations to simultaneously view a S/C and then simultaneously view an angularly nearby natural radio source (e.g. a quasar) to provide an angular position determination. For both the S/C and natural radio source, the difference in signal arrival time between the stations is measured. This time delay, coupled with the knowledge of the baseline joining the stations, provides a direct determination of the angle between the baseline and signal source.
 - Measures the plane-of-sky angular position of the S/C
 - Units in the ODP are nanoseconds
 - Common error sources (e.g. media, station, clock) cancel
 - ΔDOR is one type of Very Long Baseline Interferometric (VLBI) measurement
- **Some Other Common Tracking Data Types in the ODP**
 - Three-way coherent Doppler **F3**
 - Two-way minus three-way Doppler **F2MF3**
 - Delta-differential one-way Doppler **ΔDOD**

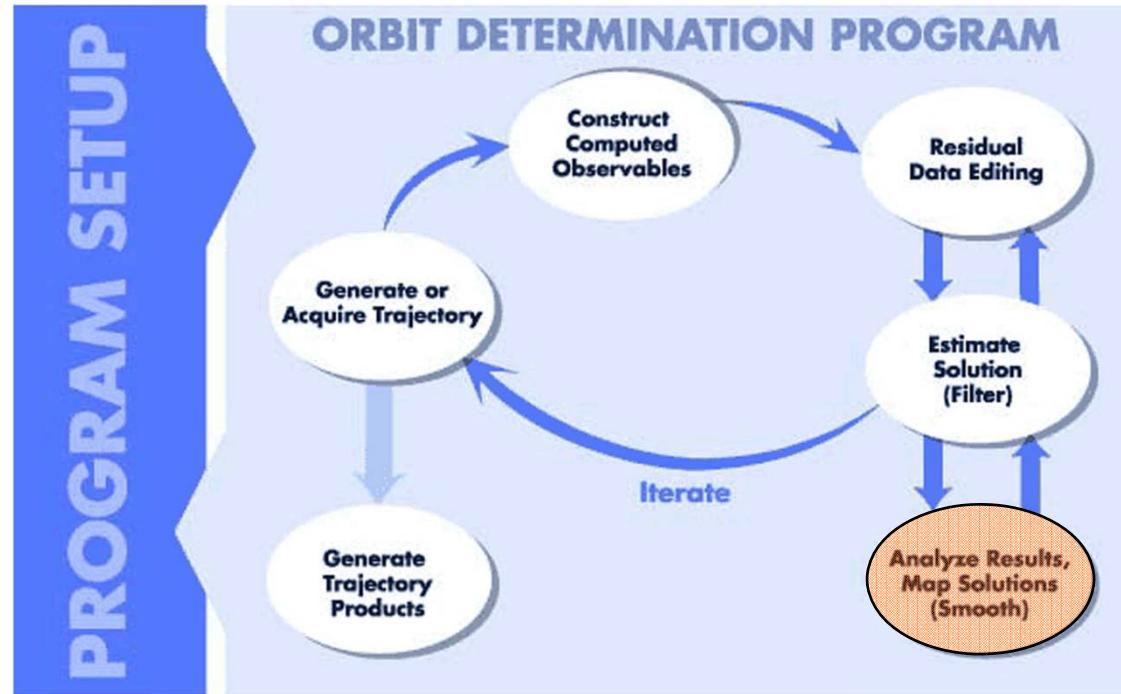


- **Estimate**

- Residual data editing may be necessary. The raw tracking data may exhibit corrupt data points or anomalies that should not be included in the final solution.
- If the residual values do not fall within the expected tolerances, then the CSP file may be used to exclude the questionable data during the next REGRES run.

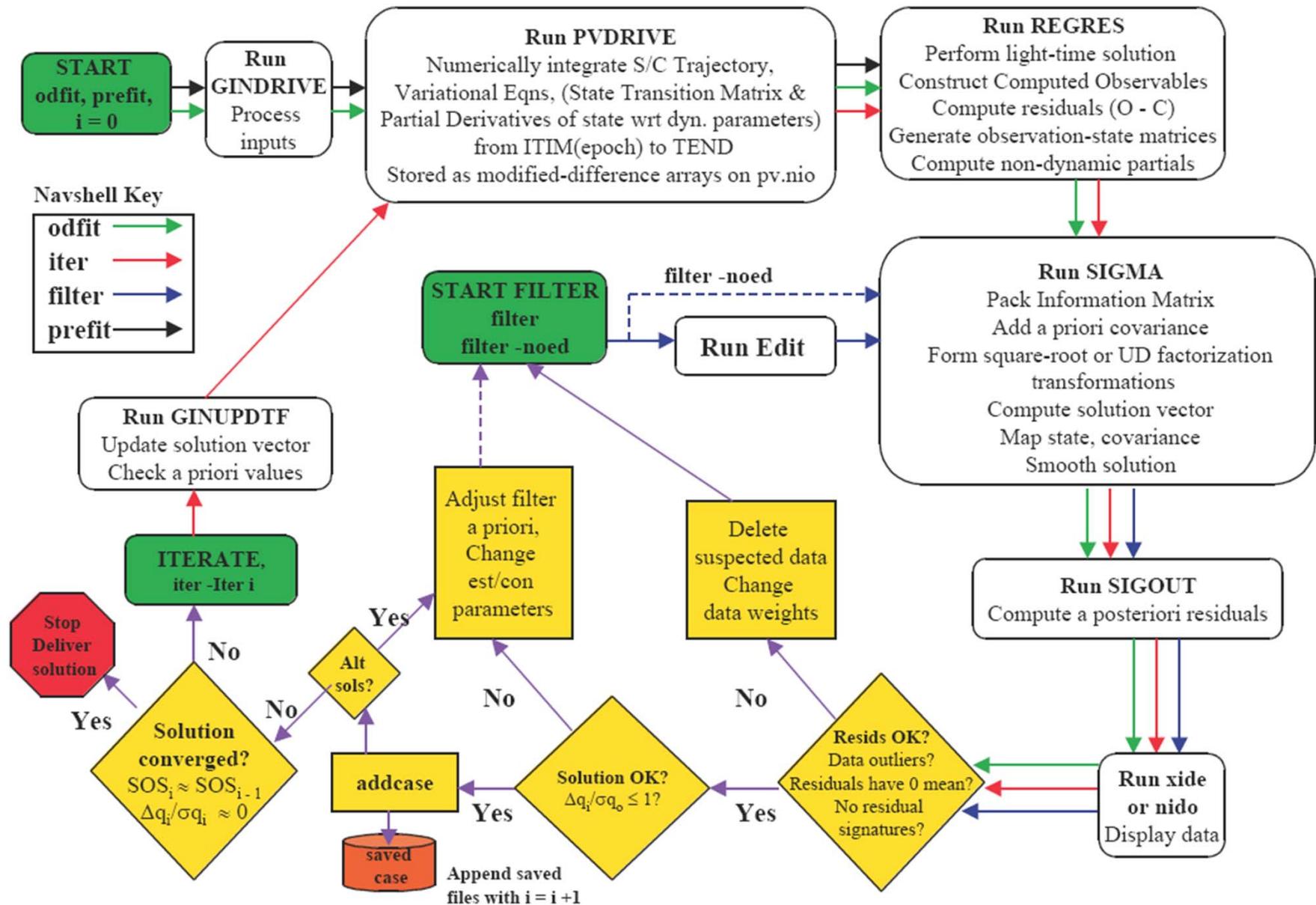


- **Estimate**
 - The SIGMA program generates solutions and covariances. SIGMA packs weighted data partials and residuals, using the epoch/pseudo-epoch state filtering information and using either the SRIF or U-D algorithms.
 - Solutions are automatically available for output or may be derived by back substitution.



- **Analyze Data**

- SIGMA maps the solutions and their associated data as well as landmarks and the spacecraft, planet, and satellite ephemerides and their partials, are mapped forward or backward in time to a variety of coordinate sets.
- The mapping function uses the filter model of the dynamic process noise to properly reflect the modeled stochastic effects, while mapping either forward beyond the end of a data arc, or back into its interior.



Review of OD Processes



- **Run PVDRIVE**

- Integrate S/C Trajectory:
 - Variational Eqns, State Transition Matrix &
 - Partial Derivatives of state wrt estimated/considered dynamic parameters
 - from ITIM (epoch) to TEND
 - Stored as modified difference arrays on pv.nio

$$\dot{\mathbf{X}}^* = F(\mathbf{X}, t), \text{ where } \mathbf{X} = \begin{bmatrix} \mathbf{x} \\ \mathbf{q} \end{bmatrix}, \mathbf{x} = \text{state vector}, \mathbf{q} = \text{dynamic model constants}$$

$$\dot{\Phi}(t, t_{k-1}) = A(t)\Phi(t, t_{k-1}), \Phi(t_{k-1}, t_{k-1}) = I$$

$$A(t) = \frac{\partial F(\mathbf{X}^*, t)}{\partial \mathbf{X}}$$

- **Run REGRES**

- Process all radio-metric observations:
 - Perform light-time solution for each radio-metric measurement
 - Interpolate and converge participant (S/C, station) states at times, t_1 (transmitted), t_2 (reflected (transmitted for 1-way)) and t_3 (received)
 - Compute partials of observations wrt non-dynamic parameters (station locations, media cals, timing and polar motion)
 - Construct computed observables, $\mathbf{G}(\mathbf{X}_i, t_i)$
 - Generate observation-state matrices

$$\tilde{\mathbf{H}}_i = \frac{\partial \mathbf{G}(\mathbf{X}^*, t_i)}{\partial \mathbf{X}}, \quad \mathbf{H}_i = \tilde{\mathbf{H}}_i \Phi(t_i, t_k)$$

- Compute residuals ($O - C$), where y_i is the observation at time, t_i
$$\varepsilon_i = y_i - \mathbf{H}_i \mathbf{x}_k$$

Review of OD Processes

- **Run SIGMA**
 - Compute estimate vector that minimizes the weighted sum of squares of residual errors between observed and computed quantities using least squares:
 - Map solution and covariance
 - Pack information matrix:
 - Add a priori covariance
 - Propagate covariance
 - Accumulate tracking observations
 - Perform Householder orthogonal transformations to convert information matrix to upper triangular
 - Perform solution, back-substitution method
 - Smooth batch solutions

Typical Filter Set-up

- **Parameters that are typically estimated**

- S/C State: `X#scid`, `Y#scid`, `Z#scid`, `DX#scid`, `DY#scid`, `DZ#scid`
- Solar pressure (flat plate, cylindrical, bus models)
 - Area scale factors: `AREAii#scid`
 - Surface optical properties, specular, diffuse : `SPECii#scid`, `DIFFii#scid`
 - Overall scale factor: `SOLCOF#scid`
- Maneuvers (finite)
 - $|\Delta V|$: `DVBii#scid` or
 - Force: `F0ii#scid`
 - Pointing (RA, Dec wrt EME2000 or S/C): `ALP0ii#scid`, `DLT0ii#id`
 - Duration, Start time: `DRBii#scid`, `STBii#scid`
- Maneuvers ('impulsive', 'small forces') AMDs, Turns performed on thrusters (ACS events)
 - ΔV_x , ΔV_y , ΔV_z (wrt EME2000 or S/C) :
`IDLXii#scid`, `IDLYii#scid`, `IDLZii#scid` or `SDXiii#scid`, `SDYiii#scid`, `SDZiii#scid`
- Small Forces File parameters
 - Overall scale factor: `DVFSCL#scid`
 - Scale factor for x, y, z components (S/C or EME2000): `DVFX#scid`, `DVFY#scid`, `DVFZ#scid`
- Drag when S/C's altitude is low enough to planet/satellite with atmosphere
 - Atmosphere scale factor: `SCL0_i` for Mars-Gram atm model
 - Density: `D0i00j` for exponential atm model
 - Area scale factors `DSCLI`
- Spherical Harmonic Gravity coefficients
 - Zonal, Sectorial, Tesseral coefficients: `Jnii`, `Cnijj`, `Snijj` where n = body no., i = deg, j = order
- Nongrav acceleration (gas leak) (typically solve for constant terms)
 - Quadratic: `ATARii#scid`, `ATAXii#scid`, `ATAYii#scid` must have SAAT, SAAP set in gin inputs
 - Exponential decay: `GLARii#scid`, `GLAXii#scid`, `GLAYii#scid` must have STREXP, STPEXP, AR, AX, AY, BB set in gin inputs

Typical Filter Set-up

- **Parameter errors that are typically considered**

- Station locations: CU14, CV14, LO14, etc where 14 = DSS14, etc
- Media
 - Ionosphere day & night: IONOND1, IONON1, etc where 1 = Goldstone, 4 = Canberra, 6 = Madrid
 - Troposphere dry & wet: TROPD1, TROPW1, etc where 1 = Goldstone, 4 = Canberra, 6 = Madrid
- Earth orientation
 - Timing UT1, Polar motion: UT1, XPOLE, YPOLE
- Quasar locations when Δ DOR measurements are included
 - RA, Dec wrt EME2000: QR020, QD020, where 020 is quasar number
- Planet Ephemerides
 - Earth-Moon barycenter should be included with correlated EM_Bary-Target_planet covariance:
DMWi, DPI, DQi, EDWi, DAI, DEi where i = planet no. or B for Earth-Moon Barycenter
- Planet GMs
 - Earth, Moon, Target planet: GM3, GMM, GM4

- **Parameters that are typically estimated stochastically**

- Media
 - Ionosphere day & night: IONOND1^, IONON1^, etc where 1 = Goldstone, 4 = Canberra, 6 = Madrid
 - Troposphere dry & wet: TROPD1^, TROPW1^, etc where 1 = Goldstone, 4 = Canberra, 6 = Madrid
- Earth orientation
 - Timing UT1, Polar motion: UT1^, XPOLE^, YPOLE^
- Data Biases (typically 1 per tracking pass)
 - Range: TYP37#scid\$dss^
 - Doppler: TYP12#scid\$dss^
- Nongrav acceleration (gas leak) (typically solve for constant terms)
 - Quadratic: ATARI#scid^, ATAXii#scid^, ATAYii#scid^ must have SAAT, SAAP set in gin inputs
 - Exponential decay: GLARI#scid^, GLAXii#scid^, GLAYii#scid^ must have STREXP, STPEXP, AR, AX, AY, BB set in gin inputs

Typical filter inputs

- Weights
 - X-Band 2-way Doppler is nominally weighted at 0.1 mm/s (60 sec count) or less 1σ
 - noise structure discourages weighting at RMS
 - X-Band Range is typically weighted 1 - 4 meters
 - Δ DOR weighted at 0.12 nsecs (4.5 nrad)
 - Deweighting data showing persistent noise larger than nominal
- Tracking Data Deletions
 - Nominally data is deleted < 10 deg elevation
 - Obvious outliers
 - noisy data $> 3\sigma$
 - usually delete data during maneuvers if not modeled in detail
 - data showing unusual/unexpected behavior
 - data just before and after planet occultations where signal passes through planet's atmosphere
 - whole tracking pass exhibiting very large noise $>> 3\sigma$
 - whole pass exhibiting obvious bias
- A priori uncertainties
 - epoch state can be set to large uncertainties (makes apriori values unimportant)
 - Maneuver errors are generally S/C-specific with fixed and proportional errors (see NAV plan)
 - Turn, AMD errors are generally constrained to project requirements
 - see MER example
- A priori values
 - used to tie (center) covariance to the nominal value
 - set to nominal values
 - can be set to different value to 'drive' the estimation of the parameter without changing nominal in gin/pv
 - Maneuver parameters may be updated from telemetry

Typical stochastic parameters



- Range Biases
 - pass-to-pass uncertainty ~ 2 - 3 meters
 - 0 time correlation
 - updates per pass
- Stochastic Accelerations
 - spherical uncertainty ~ 2 - 20% of solar pressure acceleration
 - 0 time correlation
 - or
 - few hours - several days time correlation
 - updates range from hourly to several days
- Media corrections
 - Ionosphere (use S-Band numbers)
 - day: 55 cm (1σ)
 - night :15 cm (1σ)
 - Troposphere
 - dry: 1 cm (1σ)
 - wet: 1 - 2 cm (1σ)
 - 0 correlation
 - updates \geq 6 hrs
 - see MER example
- UT1, polar motion
 - see MER example

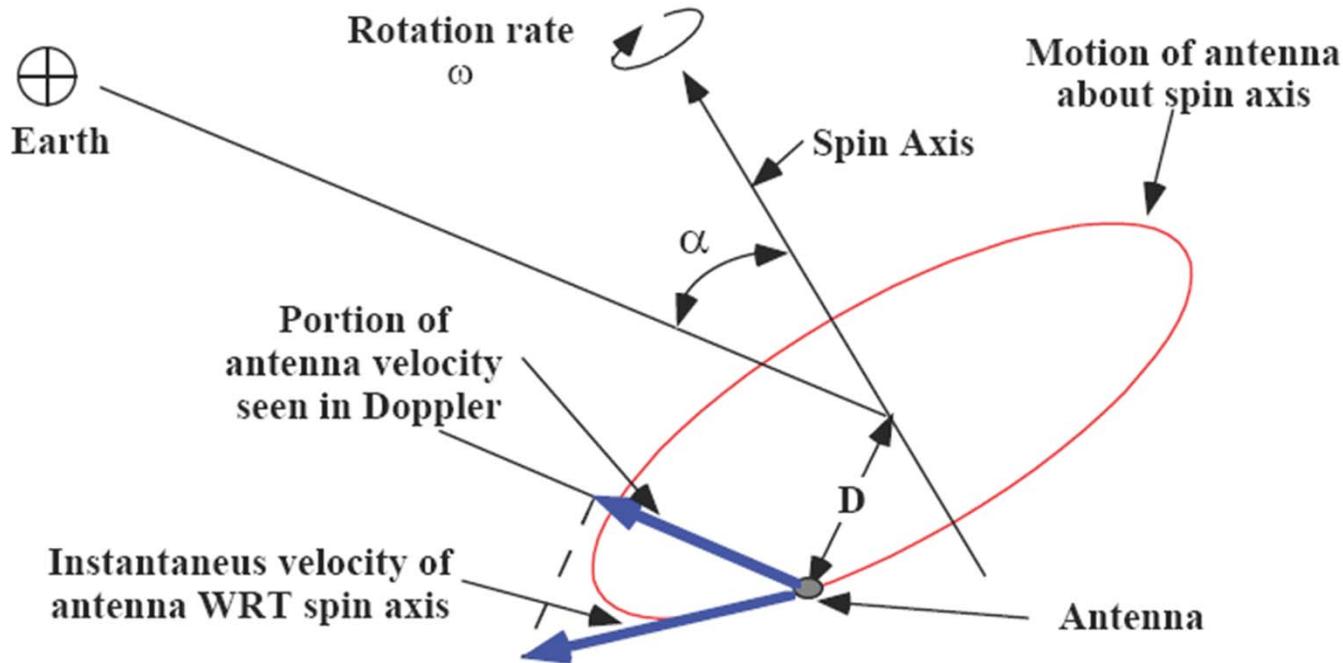
ORBIT DETERMINATION AND PARAMETER ESTIMATION

MER Baseline OD Filter Assumptions



Error Source	Estimated?	A Priori Uncertainty (1σ)	Correlation Time	Update Time	Comments/References
2-way Doppler (mm/s)	–	0.075	–	–	~4.5 mHz
Range (m)	–	4	–	–	29 range units
Δ DOR (nrad)	–	4.5	–	–	0.12 ns
Δ DOR Schedule	Level 2 [*]			Level 2 (DSN request) with last point of final baseline pair no later than 2 days before data cutoff.	
Epoch State					
Position (km)	✓	1000	–	–	
Velocity (km/s)	✓	1	–	–	
Range Bias (m)	✓	2	0	Per pass	Estimated per pass.
Doppler Bias (mm/s)	✓	0.005	0	Per pass	Estimated per pass.
Mars & Earth Ephemerides	DE405+				
Station Locations (cm)	3				
Pole X, Y (cm)	✓	2 → 10	0	6 hrs	*Use lower value up to 7 days before data cutoff; then ramp up to higher value at data cutoff. (For UT1, 0.256 ms ⇒ ~10 cm.)
UT1 (cm)	✓	2 → 10	0	6 hrs	
Quasar Locations (nrad)	2				
Ionosphere – day (cm)	✓	55	0	6 hrs	S-band values.
Ionosphere – night (cm)	✓	15	0	6 hrs	
Troposphere – wet (cm)	✓	1	0	6 hrs	
Troposphere – dry (cm)	✓	1	0	6 hrs	
Solar Pressure					
Area (%)	✓	5	–	–	
ACS Event ΔV (mm/s)	Every 8 days				
Line-of-Sight Comp.	✓	3	–	–	
Lateral Comp.	✓	3	–	–	
Normal Comp.	✓	3	–	–	
TCMs					
TCM-1	✓	422, 440	–	–	MER-A Open Melas (VM53A), MER-B Open Hematite (TM20B) TCM-4 at E - 8 days TCM-5 at E - 2 days TCM-6 at E - 6 hrs (no TCM-5) 5% (3σ) proportional error (per axis) 8 mm/s (3σ) fixed error (per axis)
TCM-2	✓	17, 15	–	–	
TCM-3	✓	3, 5	–	–	
TCM-4	✓	3, 3	–	–	
TCM-5	✓	3, 3	–	–	
TCM-6	✓	7, 7	–	–	
Non-gravitational Accelerations (km/s ²)	✓	2.0×10^{-12}	10 days	1 day	Spherical covariance. Estimated daily (1 day batches).

Doppler Spin Geometry



$$\dot{\rho}(t) = \frac{\Delta\rho(t)}{\Delta t} = D \sin\alpha \left(\frac{\sin(\omega(t + \frac{\Delta t}{2} - t_0)) - \sin(\omega(t - \frac{\Delta t}{2} - t_0))}{\Delta t} \right)$$

$$Amp(\dot{\rho}(t)) = 2D \sin\alpha \sin\left(\frac{\Delta\theta}{2}\right), \quad \Delta\theta = \text{mod}(\omega\Delta t, 2\pi)$$

D = distance from the spin axis to the electric center of the antenna

ω = spacecraft spin rate

α = angle between the spin axis and S/C to Earth line (Earth Look Angle, or Earth Aspect Angle)

t_0 = reference time

Doppler Spin Signatures

- Spinning s/c can produce spin effects in Doppler data
 - Circular polarization produces Doppler bias
 - for 1-way: rps (Hz is Hz...)
 - for 2-way: $(1 + d/u) * \text{rps}$, where d/u is 880/749 for X-up/X-down
 - sign hard to predict -- estimate Doppler bias or fit range post-launch to determine sign, then apply as *a priori*
 - Offset antenna can produce periodic signature
 - Magnitude a function of Earth aspect (or look) angle (α , EAA or ELA) of spin axis, antenna offset distance (D), spin rate (ω), sample time (Δt)
 - Not a function of instantaneous LOS antenna velocity wrt cm, except for very short count times

$$\dot{\rho}(t) = \frac{\Delta\rho(t)}{\Delta t} = D\sin\alpha \left\{ \frac{\sin(\omega(t + \frac{\Delta t}{2} - t_0)) - \sin(\omega(t - \frac{\Delta t}{2} - t_0))}{\Delta t} \right\}$$

$$Amp(\dot{\rho}(t)) = 2D\sin\alpha \sin\left(\frac{\Delta\theta}{2}\right), \quad \Delta\theta = \text{mod}(\omega\Delta t, 2\pi)$$

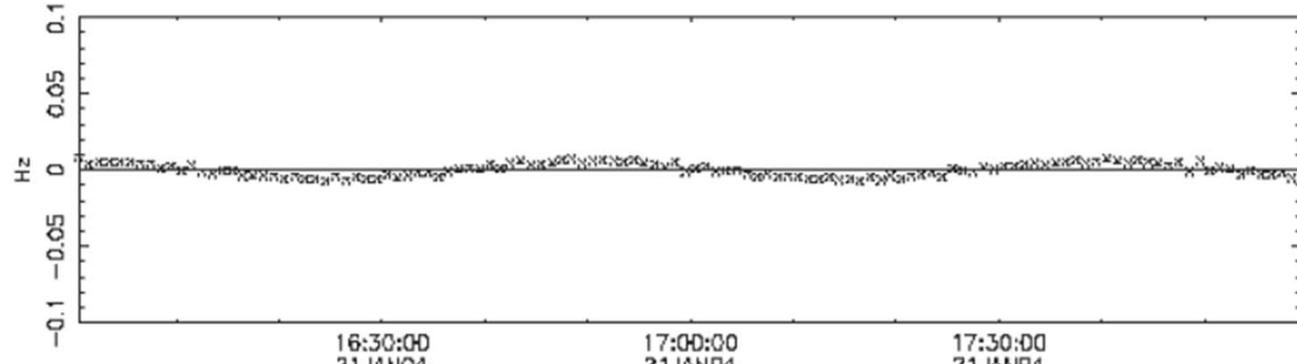
- Number of "braids" is $n2\pi/\Delta\tau$, where n is chosen such that the remainder is ~small
- Small delta-theta can produce a signature that aliases into geocentric angle estimates, if $\Delta t * 2\pi / \Delta\theta$ is close to pass length
 - otherwise could more easily choose Δt to minimize amplitude

Examples of Doppler Spin Signatures

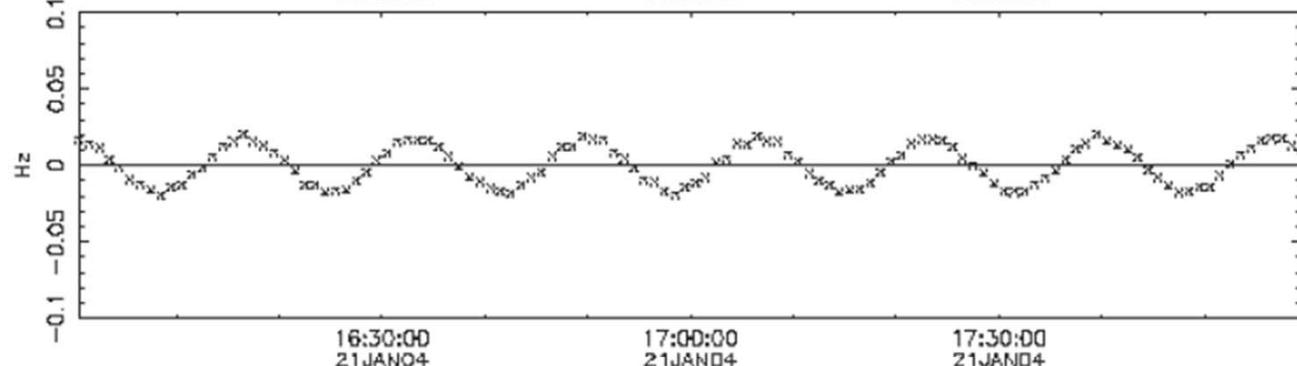
all $\Delta t = 60\text{s}$, $D \sin(\alpha) = 50\text{ mm}$ (typical MER late cruise), X/X band, 0.017 mm/sec noise

F2 profits

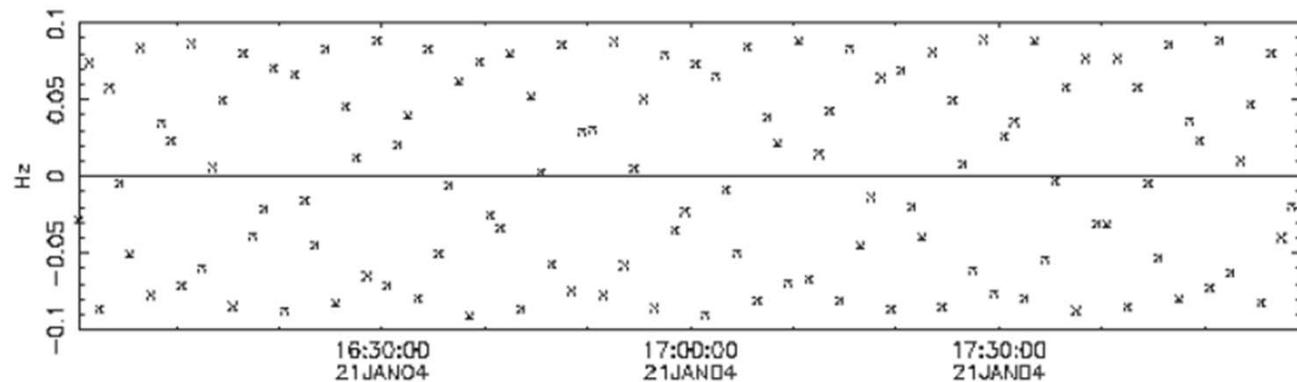
$\omega = 2.02\text{ rpm}$



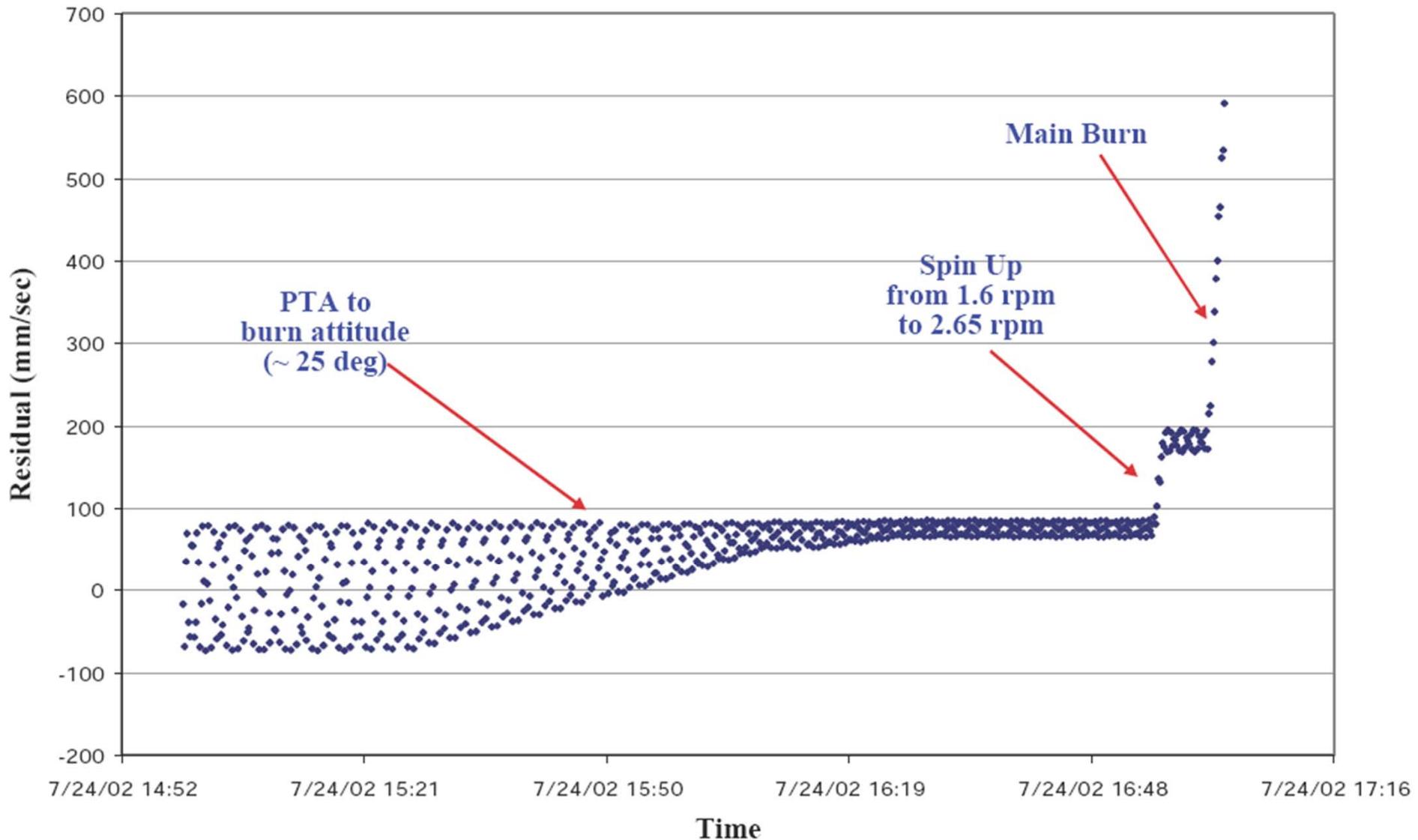
$\omega = 2.06\text{ rpm}$

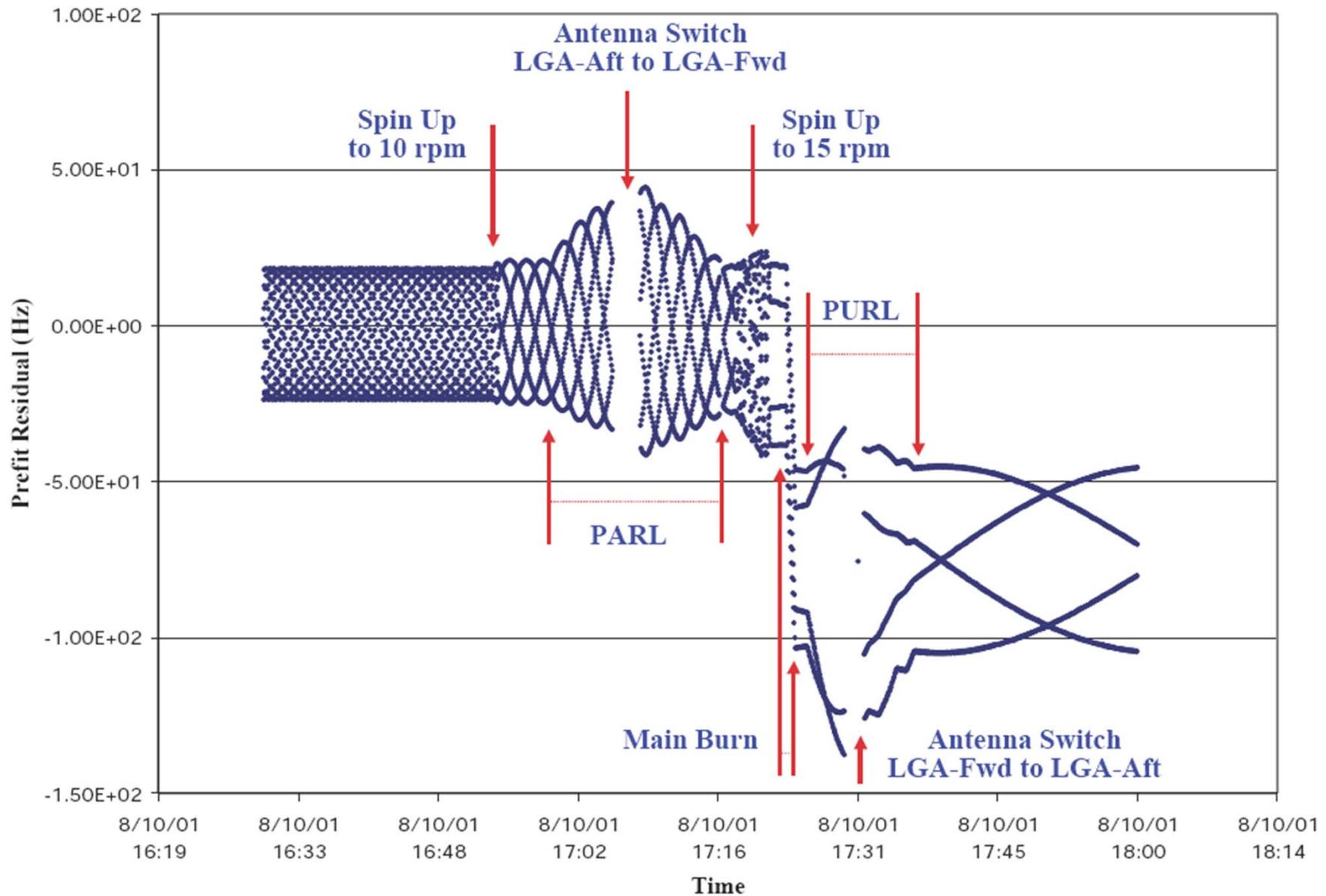


$\omega = 1.61\text{ rpm}$
(Genesis)



Genesis SKM-2B Doppler Prefit Residual





Parameter estimation strategies

- Orbit determination procedure:
 - First, it determines the s/c's initial position and velocity in a data interval;
 - Then estimates the magnitudes of the orientation maneuvers, if any.
- Modern navigational software often uses Kalman filter estimation since it allows determination of the temporal history of stochastic process than does the weighted least-squares estimation.
 - ▷ JPL's ODP used both the batch-sequential (BSF) with filter smoothing algorithm and the weighted least-squares estimation (WLS) approaches.
 - ◊ Different constant batch sizes: 0, 1, 5, 10, 30, and 200 day batch sizes. In each batch one estimates the same number of desired parameters
 - ◊ Experimented with different expected correlation time for the underlying stochastic processes that may be responsible for the anomaly
 - ▷ CHASMP software uses only the weighted least-squares approach. χ^2 test used as indicator of the quality of the fit.
- Both programs yielded very similar results. The differences between them can be mainly attributed to (other) systematics.

<u>Error Source</u>	<u>Current Modeling Accuracy</u>
<u>Station Locations</u>	
Crust-relative	5 cm
Pole location	5 cm
Timing (UTC)	0.5 ms
<u>Media</u>	
Ionosphere (X-Band, 8.4 GHz)	5 cm
Troposphere	4 cm
<u>Ground Instrumentation</u>	
Station oscillator	10^{-14}
Hardware range delays	0.5 - 1 m
<u>Dynamics</u>	
Non-gravitational acceleration of spacecraft	$10^{-12} - 10^{-11}$ km/s ²



ORBIT DETERMINATION AND PARAMETER ESTIMATION



BUCK-UP SLIDES

- Determination of the state of orbiting objects whose motion can be described approximately by a set of equations of motion and monitored by discrete observations which are subject to both random and systematic errors
 - Time systems
 - Coordinate systems
 - Spacecraft force models
 - Tracking measurement models
 - Parameter estimation

- Solve the geodetic problems by the use of precise measurements to, from or between spacecrafts to
 - determine the site position/velocity to define the terrestrial reference frame;
 - determine the gravitational constant of the PCB;
 - determine the gravity field of the PCB;
 - determine of secular and tidal variations of the gravity field of the PCB;
 - determine the orientation of the PCB.



- Time Systems

- Sidereal time & universal time: time scales directly related to the Earth rotation
- TAI & UTC: atomic time scales
- TDT & TDB: dynamical time scales

- Coordinate Systems

- J2000.0 barycentric & geocentric reference frames
- Body-fixed terrestrial reference frame of Earth & PCB

- Local parameter
 - S/C dependent arc parameters
- Global parameter
 - GM's
 - Spherical harmonics of the gravity field model
 - Love number of the solid tides
 - Parameters of the rotation model
 - Planetary/satellite/small body ephemeris

- Compute reference orbits for each data arc (batch filter).
 - Adjust nominal value for arc dependent parameters.
 - Fix nominal value for global parameters.
- Generate SRIF equations of arc and global parameters for each data arc using converged reference orbits.
- Merge SRIF equations of global parameters from grouped data arcs for each spacecraft.
- Combine the SRIF equations derived from all orbiting space crafts.
 - Apply the empirical *apriori* information equations for estimated parameters.
 - Determine the relative data weight for each of the SRIF equations to take into consideration of measurement and dynamic model errors.