


Sternberg Astronomical Institute
Lomonosov Moscow State University

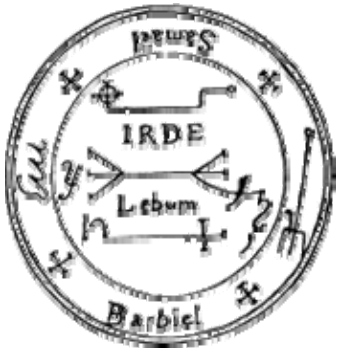
Time Series Prediction

Leonid Zotov

Fulbright Scholar 2008-2009

San Juan, Puerto Rico, 8 May 2009

FROM THE BEGINNING OF THE HISTORY PEOPLE TRIED TO PREDICT FUTURE



"We remember only past events, not future ones." *The Mystery of Time, John Langone*

"Good memory wherewith Nature has endowed us causes everything long past to seem present." *Leonardo da Vinci*

"The Past isn't dead. It's not even past." *W. Faulkner*

"But I knew, that only that which repeats itself can be grasped by study… The future is immanent in the present." *Citadelle, Antoine de Saint-Exupery*

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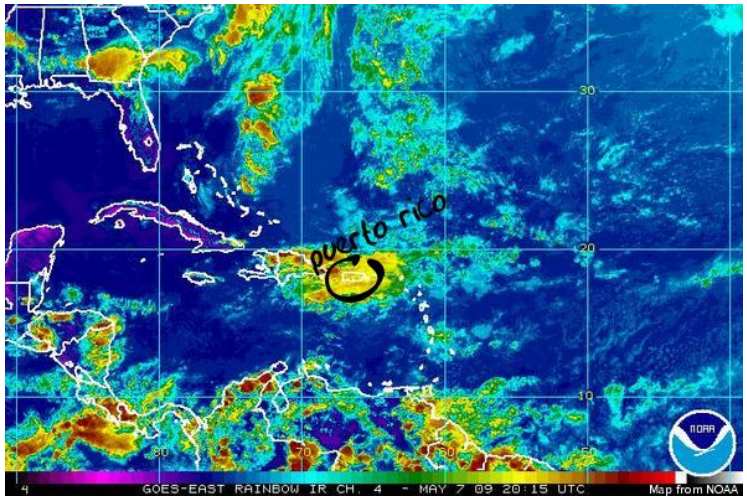
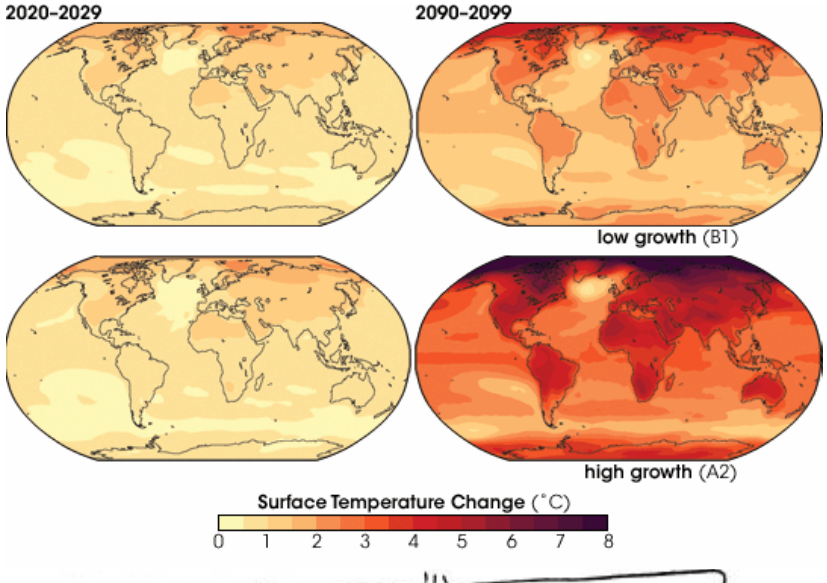
PREDICTIVE ABILITY IS ONE OF THE MOST IMPORTANT CHARACTERISTIC OF THE SCIENTIFIC THEORY

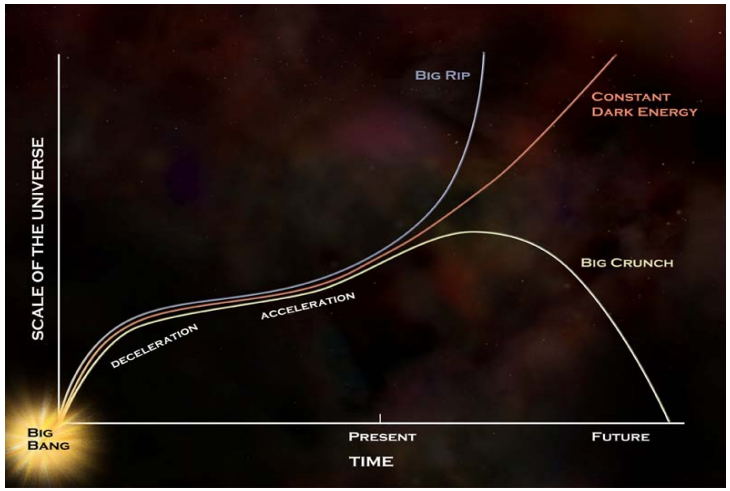
Weather forecast



Climate prediction



Future of the Universe



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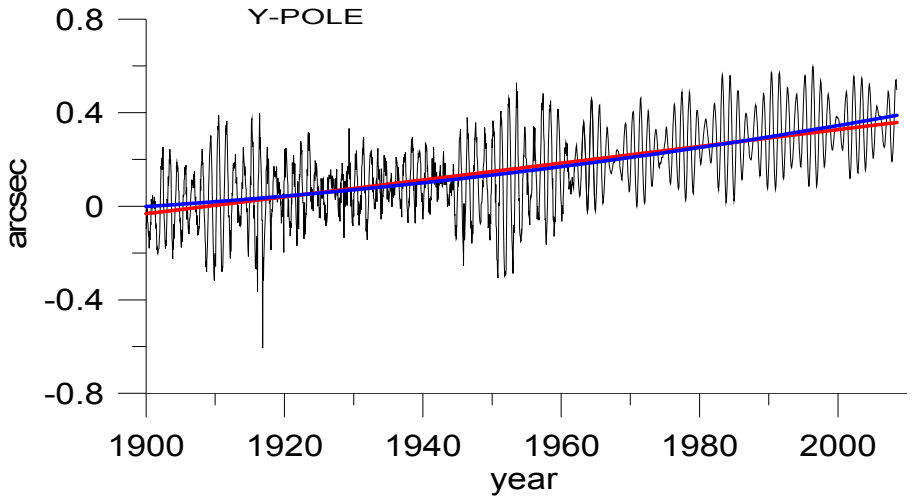
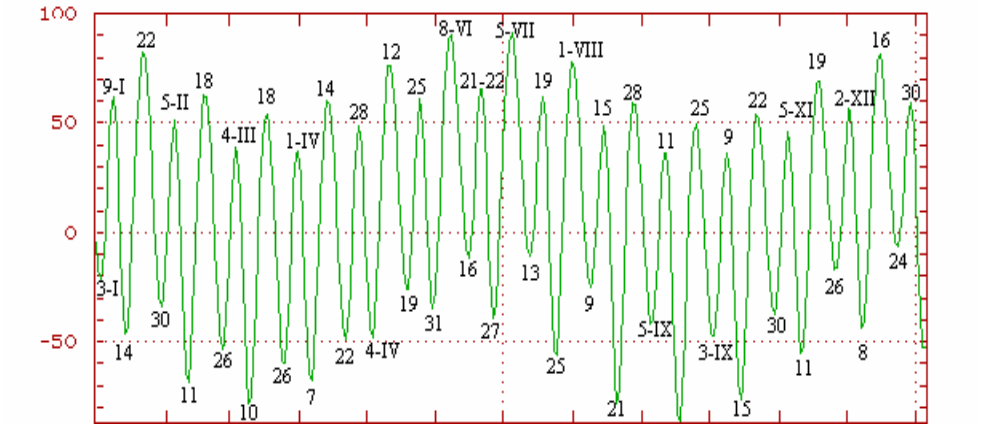
DETERMINISTIC AND STOCHASTIC COMPONENTS

Functional modeling

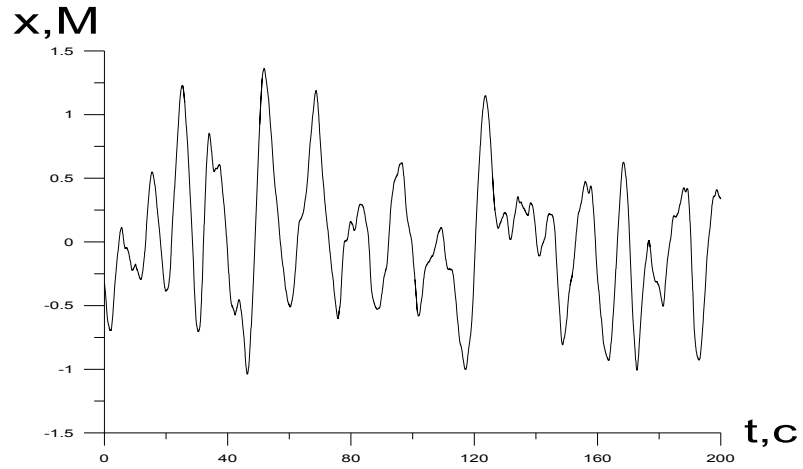
Polynomial trends

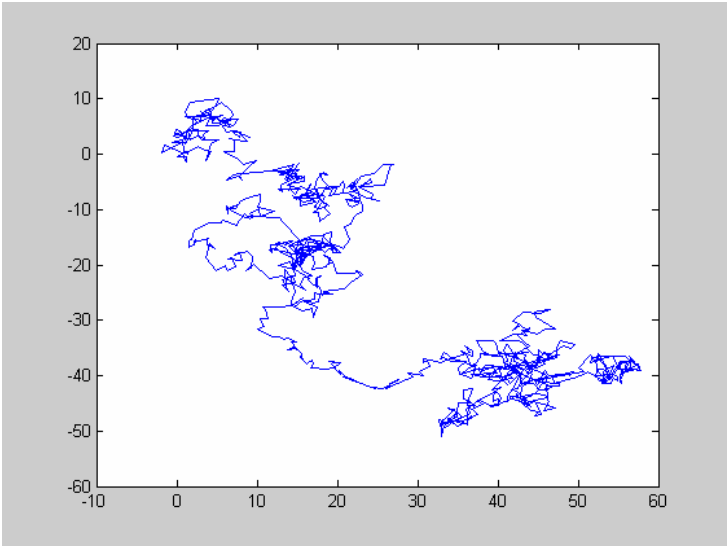


Harmonic trends



Probabilistic modeling





N.S. Sidorenkov 2009

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MATHEMATICAL AND PHYSICAL MODELING

Mathematical approximations

Least Squares Method

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{u},$$
$$< \mathbf{u} >= \mathbf{a}, \qquad \mathbf{cov}(\mathbf{u}) = \mathbf{Q},$$

$$\bar{\mathbf{x}} = (\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{Q}\mathbf{z},$$

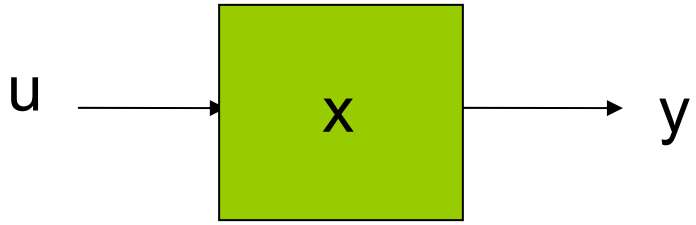
Auto Regression with Moving Average

$$x_i = -\sum_{k=1}^{p} \alpha_k x_{i-k} + \sum_{k=0}^{q} \beta_k n_{i-k},$$

Least Squares Collocation

Neural networks

Dynamic modeling



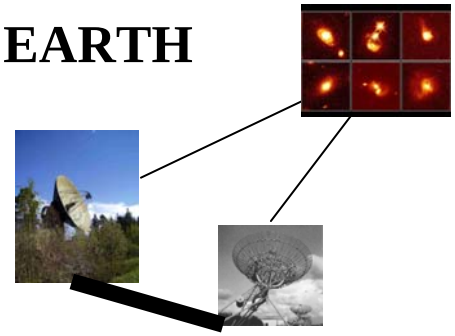
$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{G}(t)\mathbf{x}(t) + \mathbf{F}(t)\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{r}(t),$$

$$< \mathbf{u}(t)\mathbf{u}^T(\tau) >= \mathbf{Q}(t)\delta(t-\tau),$$
$$< \mathbf{r}(t)\mathbf{r}^T(\tau) >= \mathbf{R}(t)\delta(t-\tau).$$

“the flow of cause-effect relationships from the past to the future”

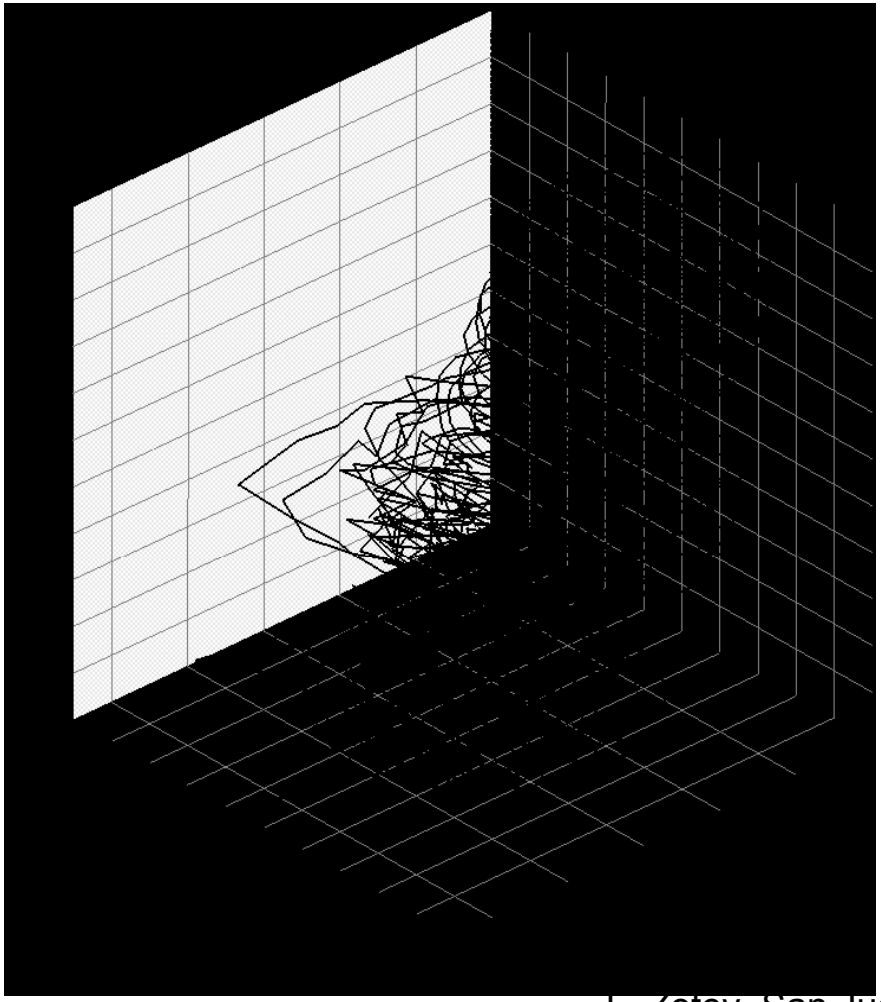
Kalman filtering

PREDICTION OF THE ROTATION OF THE EARTH



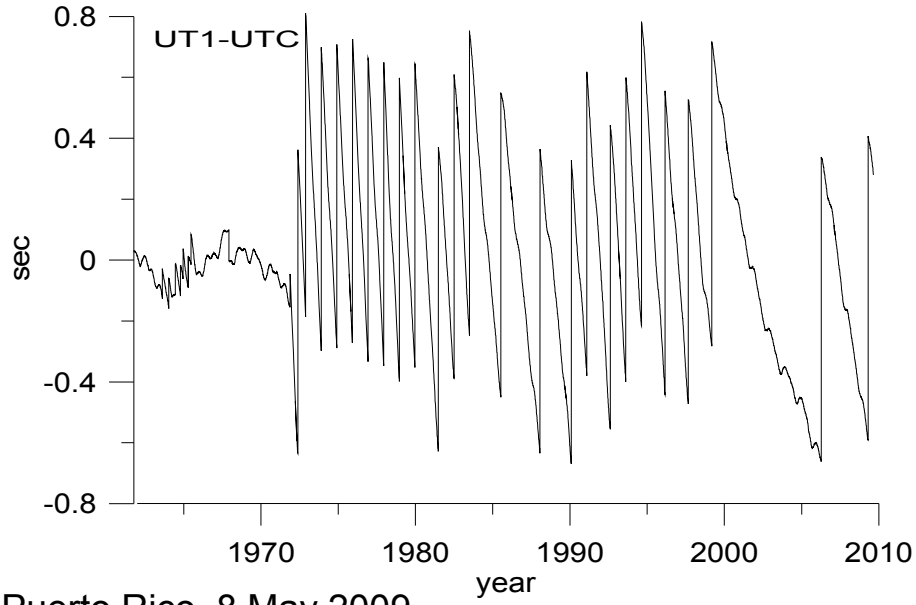
Initial data

- Series EOP C04 with 1-day step since 1962 г.
- Series EOP C01 with 0.05-year step since 1890 г.



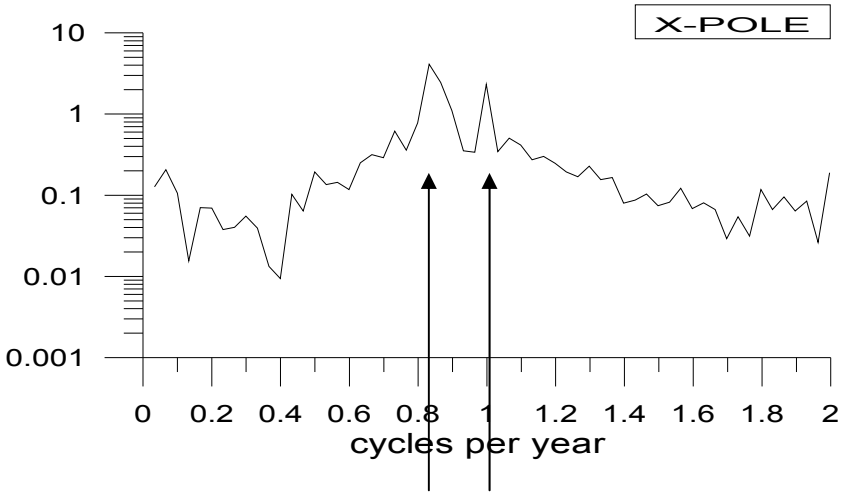
Observations from

- IVS VLBI
- Laser ranging of Moon and Satellites
- IGS GPS
- DORIS

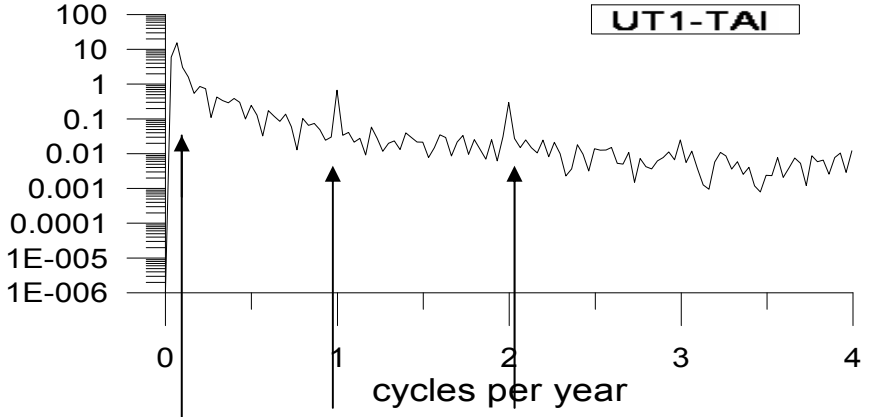


ANALYSIS OF THE EARTH ORIENTATION PARAMETERS (EOP)

Fourier-spectrogram



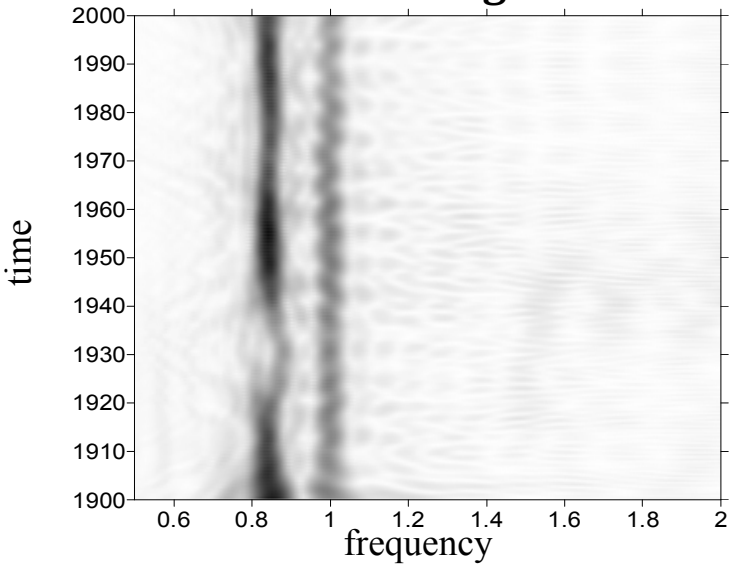
Chandler and annual oscillations



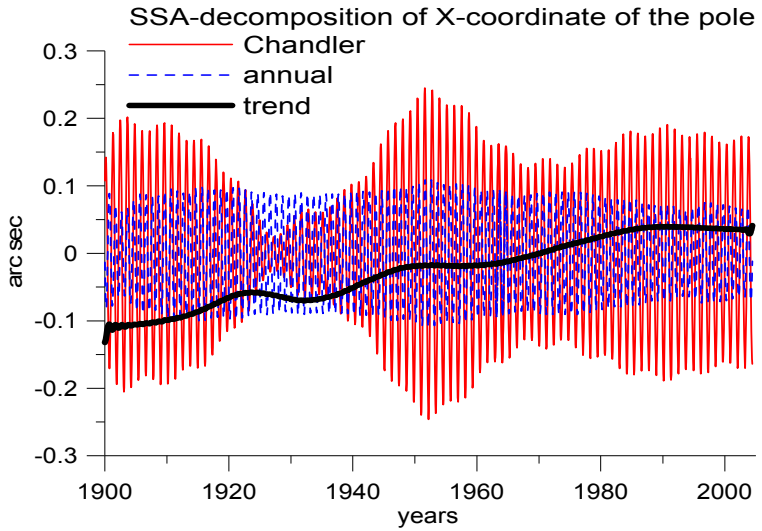
Annual and semiannual modes

18-year harmonic

Wavelet-scalogram



Singular spectral analysis



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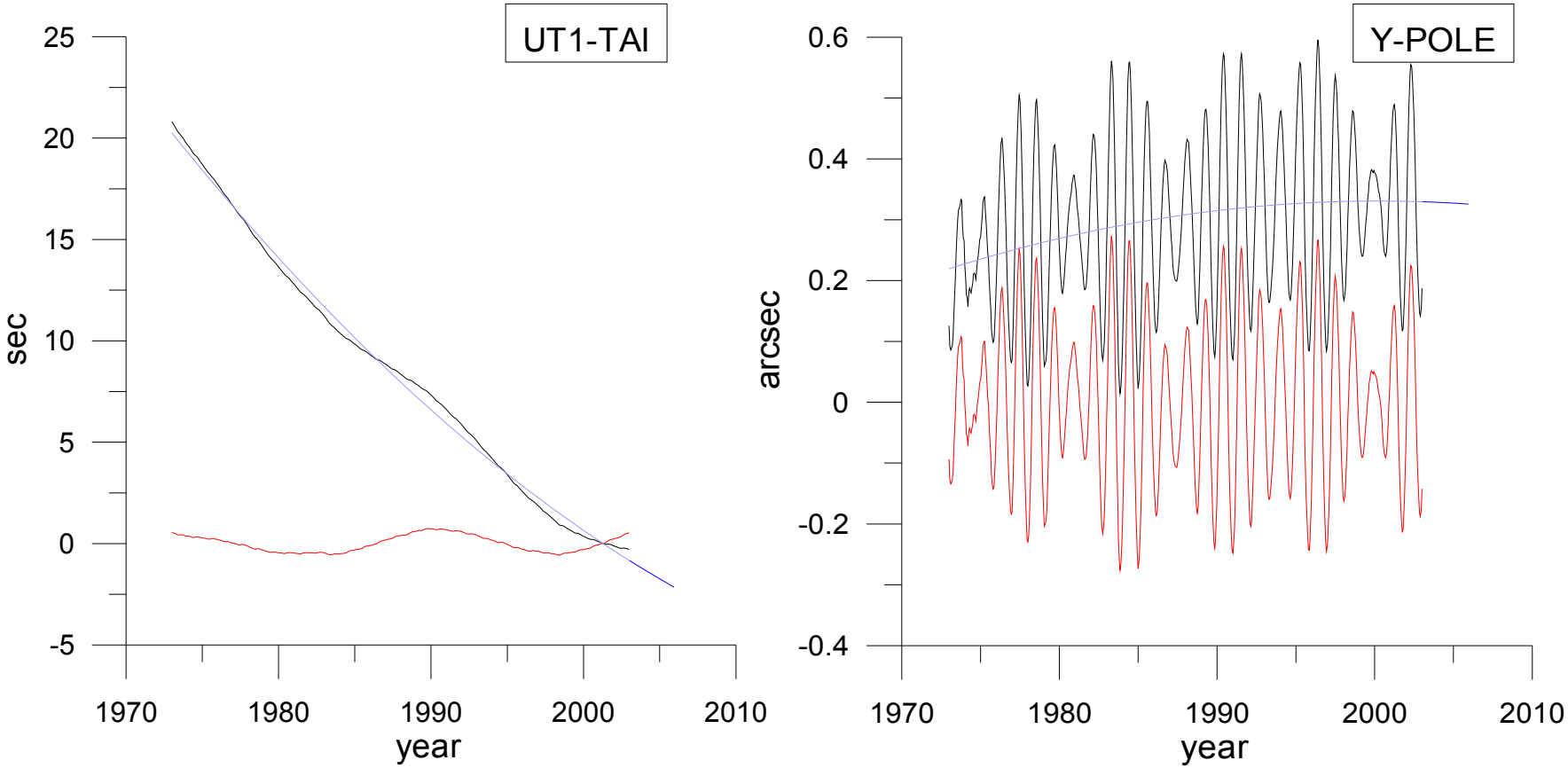
TREND MODELING



Smooth trend – polynomial of the 2-d order

LS-based solution

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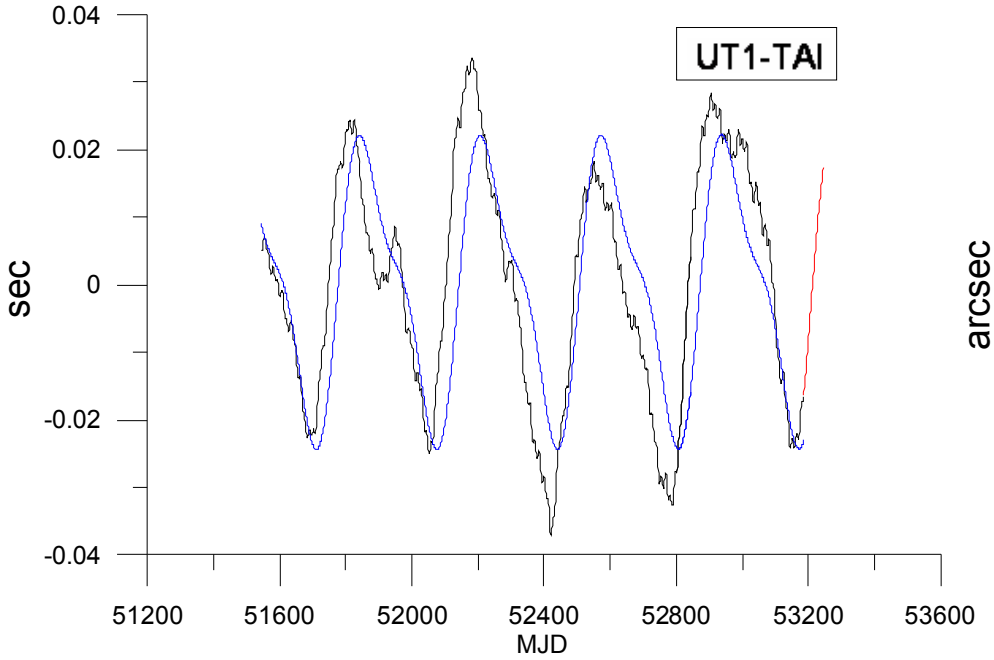
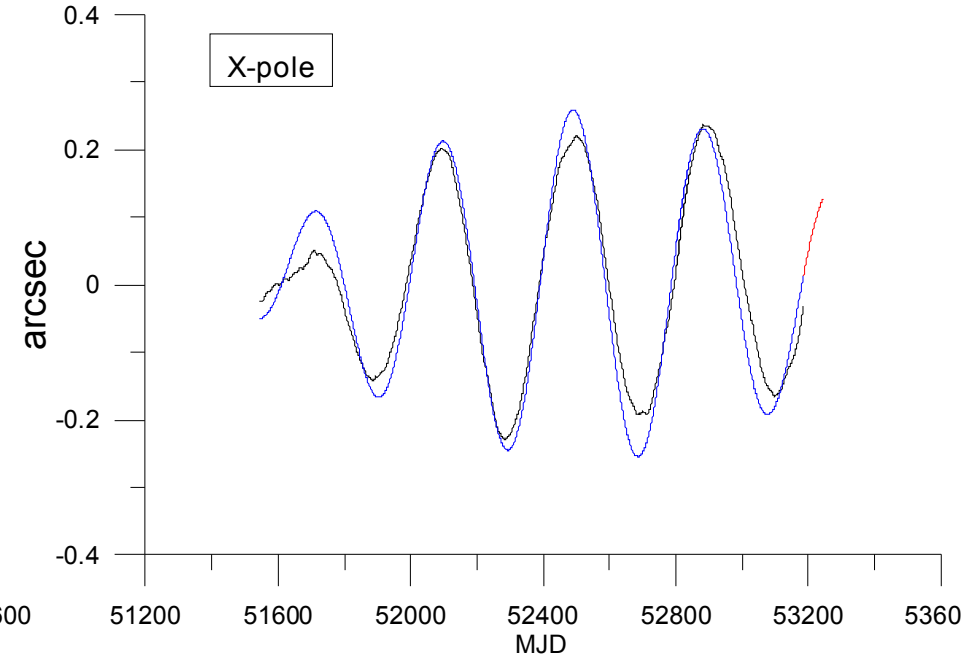
PERIODIC COMPONENTS MODELING AND PREDICTION





Period, years	Amplitude, sec
18.6	0,51
9,3	0,14
1	0,015
0,5	0,009

$$f_c(t) = \sum_{i=1}^{n} A_i \cos(\omega_i t + \varphi_i)$$

Period, years	Amplitude, arcsec
1,19	0,15
1	0,09

Nonlinear LS for phases and frequencies adjustment

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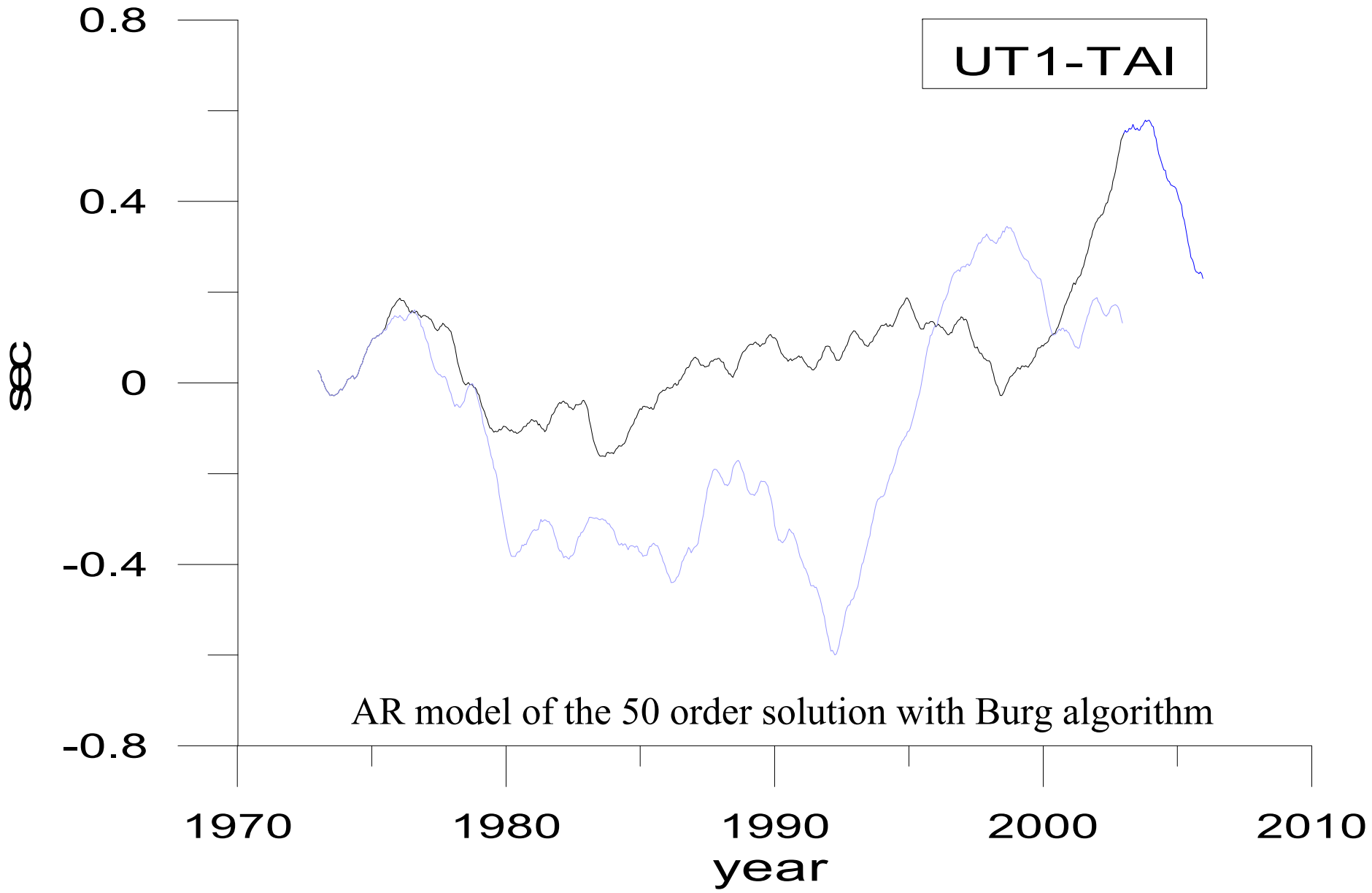
AUTOREGRESSION MODELLING



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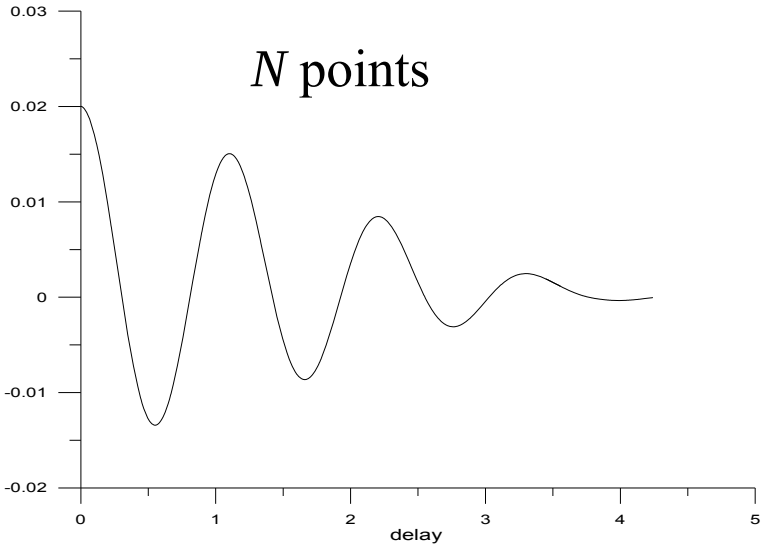
LEAST SQUARES COLLOCATION

Signal ACF and its forecast



N points

N_1 $N\text{-}N_1$

Q_{xx} Q_{fx}

$$l = x + n$$

$$x = Hl$$

- Interpolation
- Filtering
- Prediction

$$f = Q_{fl} Q_{ll}^{-1} l$$

$N\text{-}N_1$ points

$$Q_{xx}^{ext} = \begin{bmatrix} Q_{xx} \\ Q_{fx} \end{bmatrix} \begin{matrix} N_1 \\ N\text{-}N_1 \end{matrix}$$

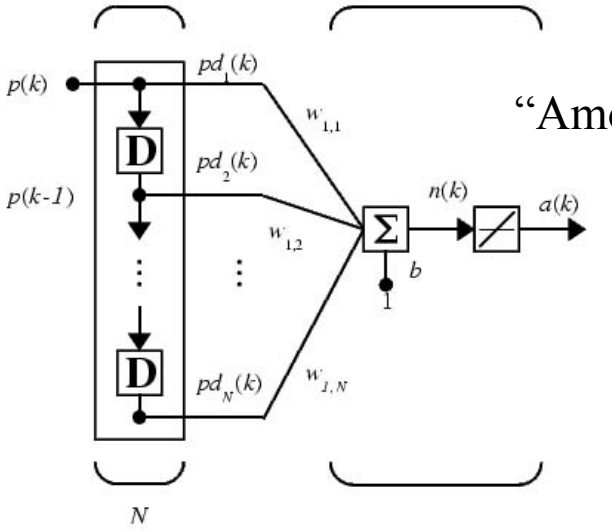
N_1

White noise assumption

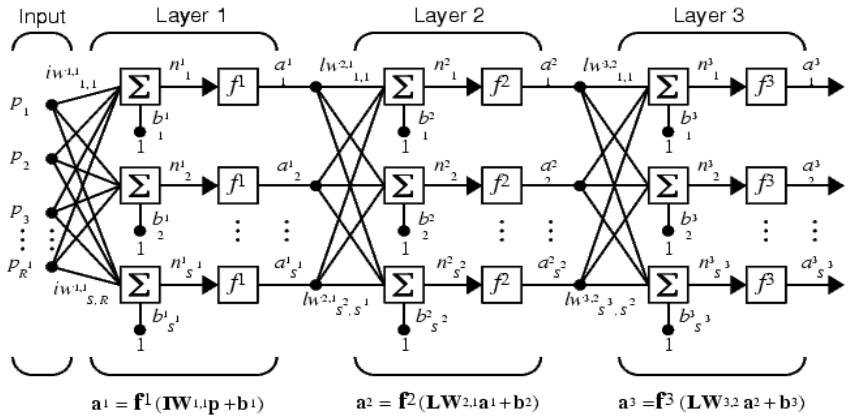
$$Q_{nn} = \sigma_n^2 I \quad Q_{xx}^{ext} = Q_{ll}^{ext} - Q_{nn} \quad f = Q_{fx} Q_{ll}^{-1} l$$

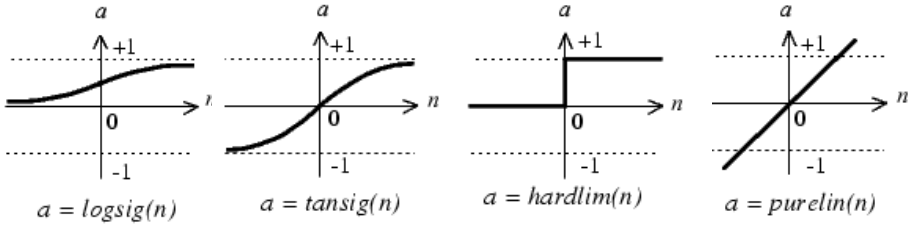
NEURAL NETWORK

- Signal *p(k)* sequentially comes to the Time Delay Line
- At every iteration vectorial signal *pd(k)* comes to the neurons of the input layer
- Neuron with the linear activation function predicts the next value of the signal
- Neural network is trained with use of the signal of comparisons (taken from the past interval)
- Levenberg-Marquardt algorithm is used for weights *W* and bias *b* tuning



"Amoeba"

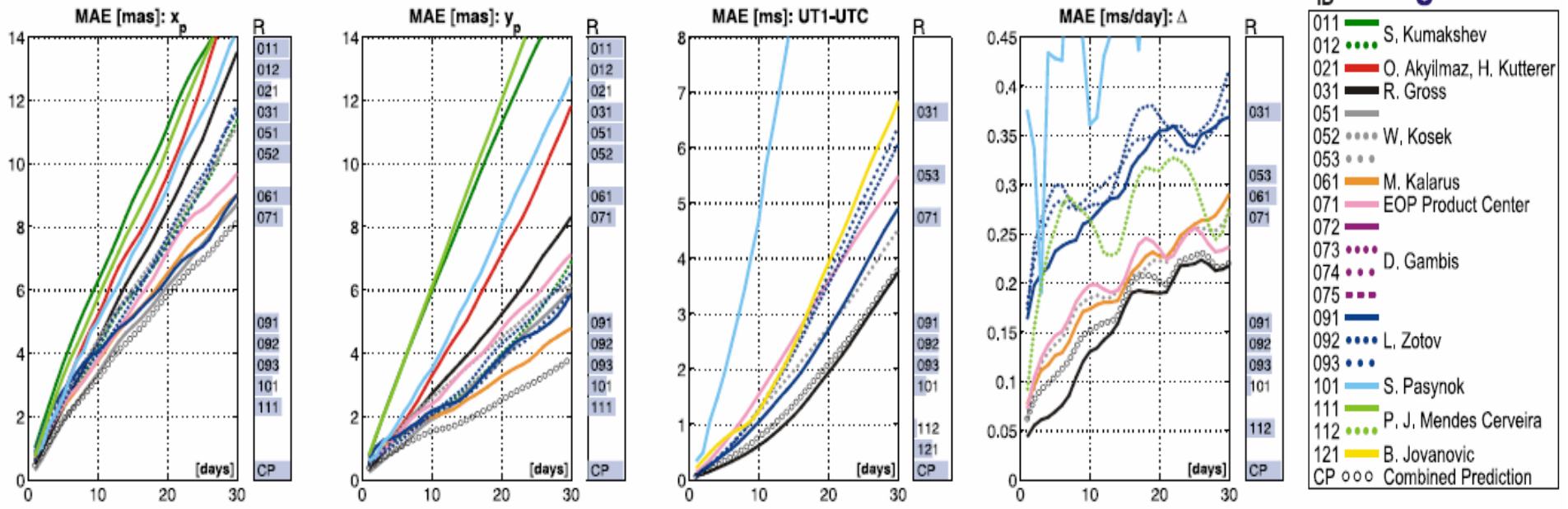




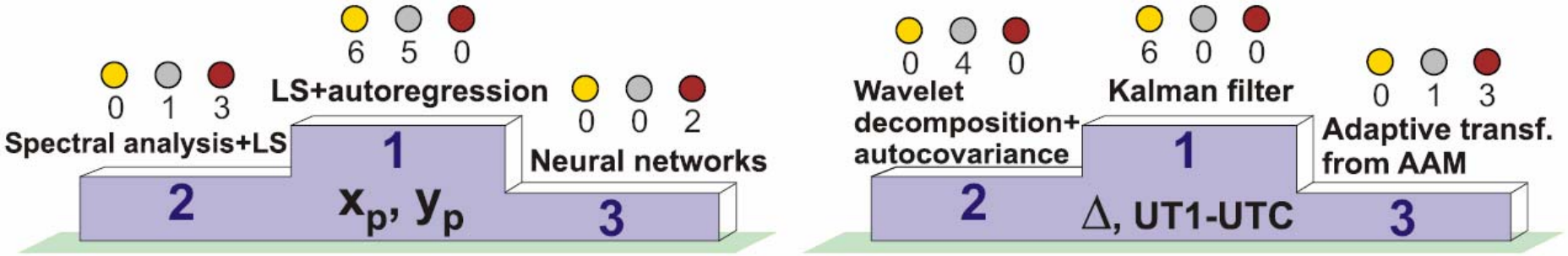
RESULTS OF EOP PREDICTION CAMPAIGN

RESULTS

short-term



The Best Prediction Techniques



PHYSICAL MODELLING - INPUT EXCITATION RECONSTRUCTION

$$\frac{i}{\sigma_c}\frac{dm}{dt} + m = \chi$$

Jefferson-Wilson filter

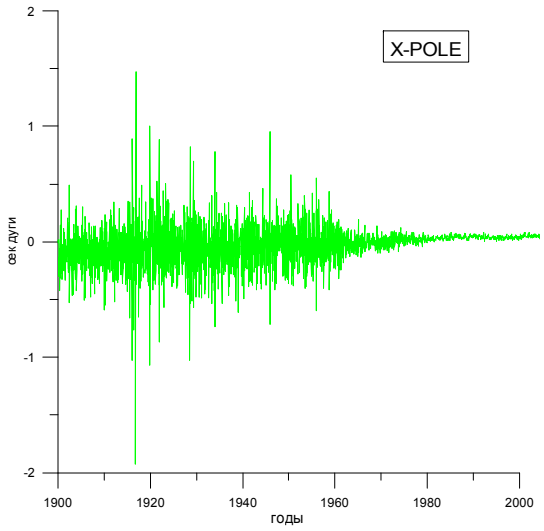
$$\chi(t) = \frac{ie^{i\pi f_c \Delta t}}{\sigma_c \Delta t}\left(m_{t+\frac{\Delta t}{2}} - e^{i\sigma_c \Delta t} m_{t-\frac{\Delta t}{2}} \right)$$

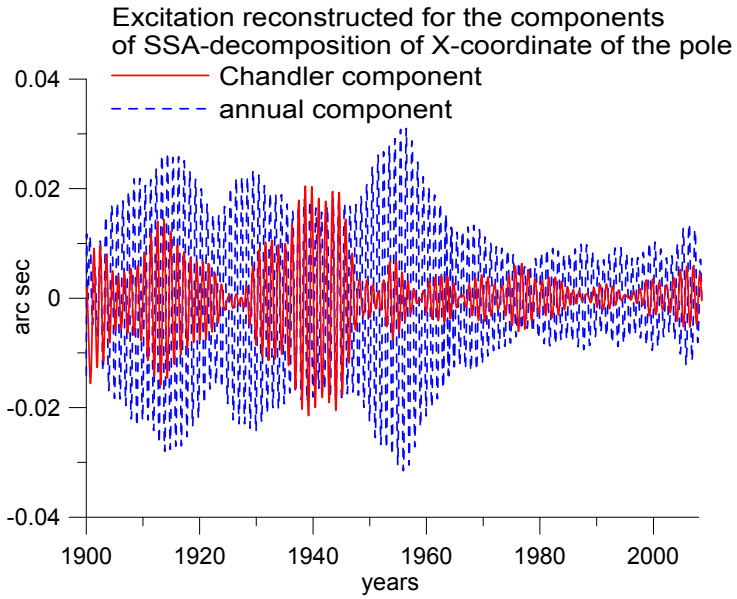
Corrective smoothing, regularization

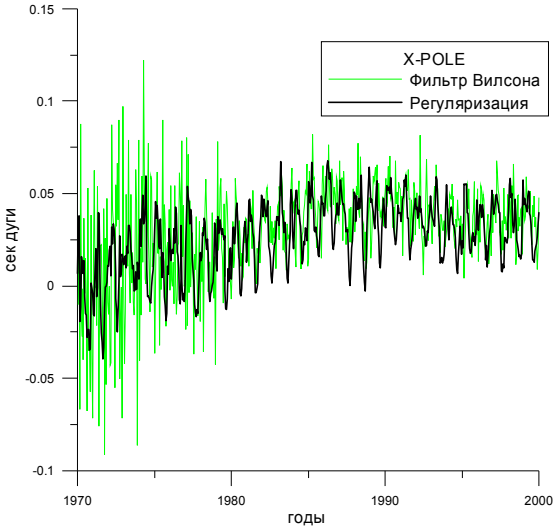
$$\chi = F^{-1}\left(F(W_{reg}) \cdot F(m)\right)$$

$$\chi = h_{reg} * u = \int h_{reg}(\xi - t)u(t)dt$$

$$h_{reg}(t,\alpha) = \frac{1}{\alpha t} e^{\frac{it}{\sigma_c}} \cos\left(\frac{-i\sigma_c}{\sqrt{\alpha}}\right)$$

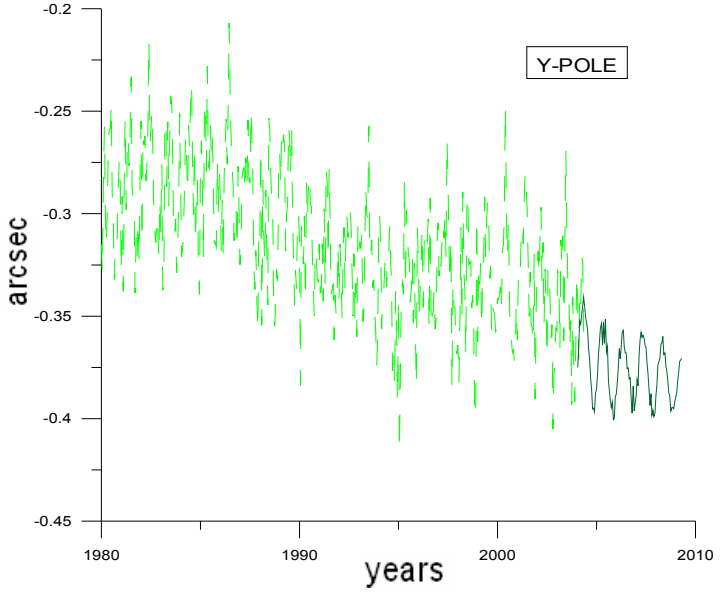






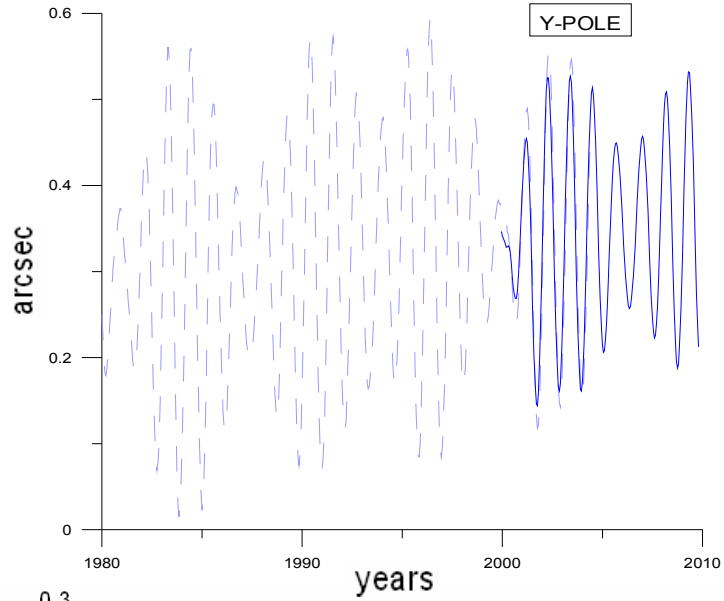
EXCITATION AND POLE TRAJECTORY FORECAST

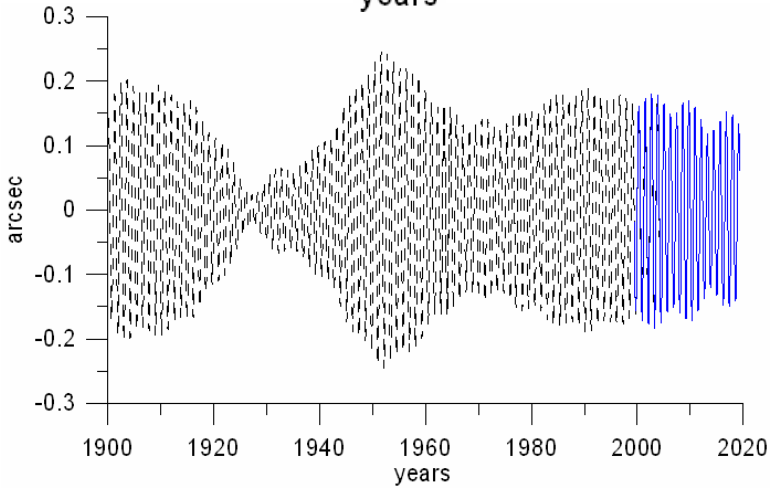
Excitation forecast



$$m^+(t_j) = m^-(t_j) + K(t_j)(z(t_j) - C(t_j)m^-(t_j)),$$
$$K(t_j) = P_m^-(t_j)C^T(t_j)[C(t_j)P_m^-(t_j)C^T(t_j) + R(t_j)]^{-1},$$
$$P^+(t_j) = P^-(t_j) - K(t_j)C(t_j)P_m^-(t_j),$$
$$m^-(t_{j+1}) = \Phi(t_{j+1}, t_j)m^+(t_j) + F\int_{t_j}^{t_{j+1}} \xi(t_{j+1}, \tau)v(\tau)d\tau.$$

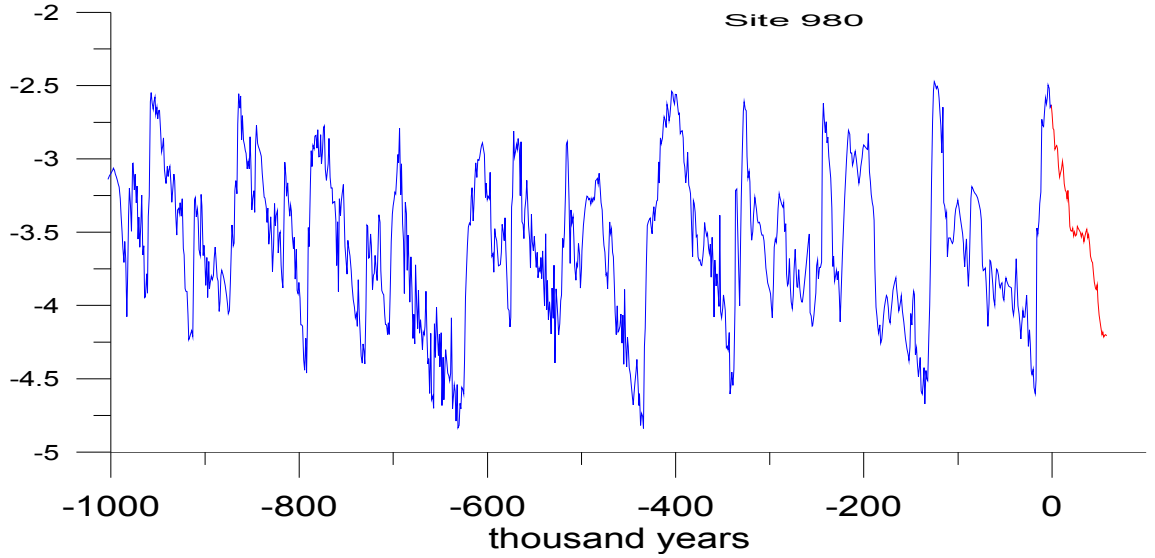
Trajectory forecast by Kalman filter



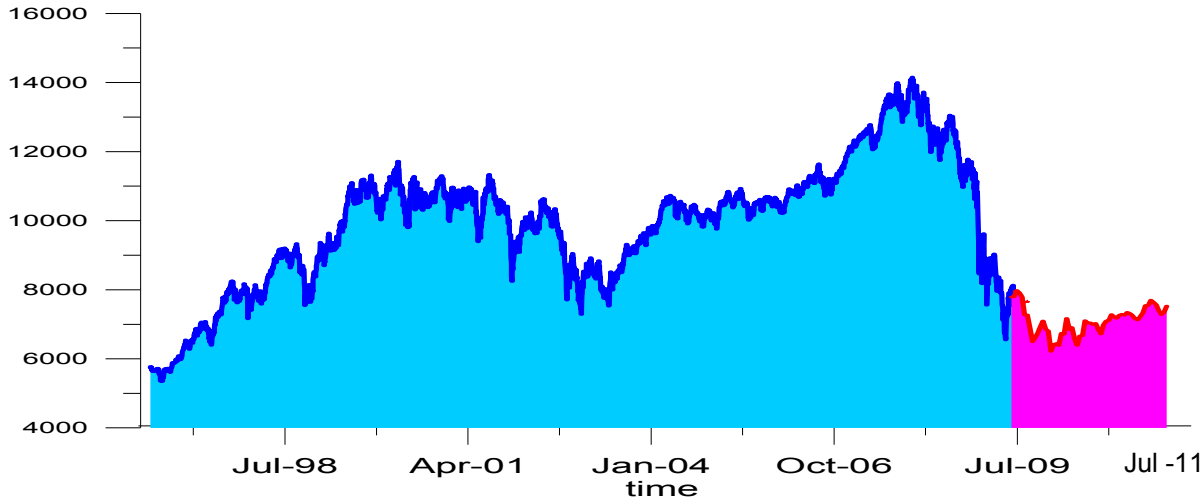


15-NEURON BRAIN FORECASTS

North Atlantic foraminiferal oxygen isotope data



Dow Jones Industrial Average



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Muchas gracias
para atencion

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