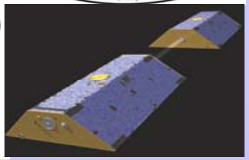

Singular spectrum analysis of GRACE observations

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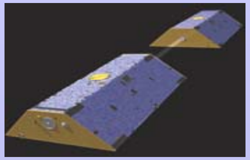
Gerald North symposium

Climate, Statistics, and Satellites, College Station, June 8-10 2009

MSSA IN APPLICATION TO GRACE MONTHLY DATA



Gravity Recovery and Climate Experiment (GRACE) satellites, launched 17.03.2002, provides an unprecedented set of Earth's temporal gravity field observations, whose signal manifests as Earth's mass redistribution within the Earth system. However, de-stripping/filtering to diminish the correlated high frequency errors is currently required to use the GRACE spherical harmonic data products.

PCA Principal Component Analysis → SSA Singular Spectrum Analysis → MSSA Multichannel SSA

We applied the MSSA to monthly GRACE gravity field solutions in both the spatial and spectral domains. The principal components (PC) were identified to be related to different physical processes, including seasonal to interannual water mass redistribution, ice/glacial melting, GIA, Sumatra-earthquake. Preliminary results indicate that one could separate noise and signals using the MSSA approach.

INITIAL DATA IN SPECTRAL DOMAIN

We used 79 CSR Level-2 RL04 monthly GRACE data since 2002 till 2009 with spherical harmonic coefficients complete to degree 60.

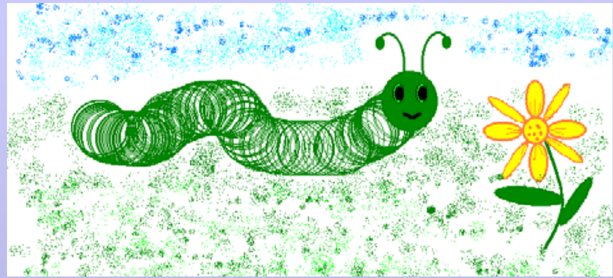
$$V(\rho,\theta,\lambda) = \frac{GM}{\rho}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{\rho}\right)^{n}(\bar{c}_{nm}\cos m\lambda + \bar{s}_{nm}\sin m\lambda)\bar{P}_{nm}(\cos\theta)\,.$$

$$\bar{P}_{nm}(\cos\theta) = \sqrt{\delta_m(2n+1)\frac{(n-m)!}{(n+m)!}}\sin^m\theta\frac{d^m P_n(\cos\theta)}{d(\cos\theta)^m}$$

Degree *n* changes from 2 for the appropriate selection of coordinates. C_{20} coefficients were excluded in processing. GGM01C model was used as a reference and subtracted. Results are represented in terms of equivalent water height (EWH) levels (mm).

$$\Delta h(\delta,\lambda,t) = \frac{a\rho_{ave}}{3\rho_w}\sum_{n=2}^{60}\sum_{m=0}^{n}\frac{2n+1}{1+k_n}W_n(\Delta\bar{c}_{nm}(t)\cos m\lambda + \Delta\bar{s}_{nm}(t)\sin m\lambda)\bar{P}_{nm}(\cos\theta)$$

"CATERPILLAR" – SSA METHOD



1. Trajectory matrix

$$X = \begin{bmatrix} X_1 & \dots & X_K \end{bmatrix}$$

$$[L \times K]$$

$$X = [x_1, x_2, \dots x_K] = \begin{pmatrix} f_0 & f_1 & \dots & f_{N-L} \\ f_1 & f_2 & \dots & f_{N-L+1} \\ \vdots & \vdots & \ddots & \vdots \\ f_{L-1} & f_L & \dots & f_{N-1} \end{pmatrix}$$

2. Singular value decomposition SVD

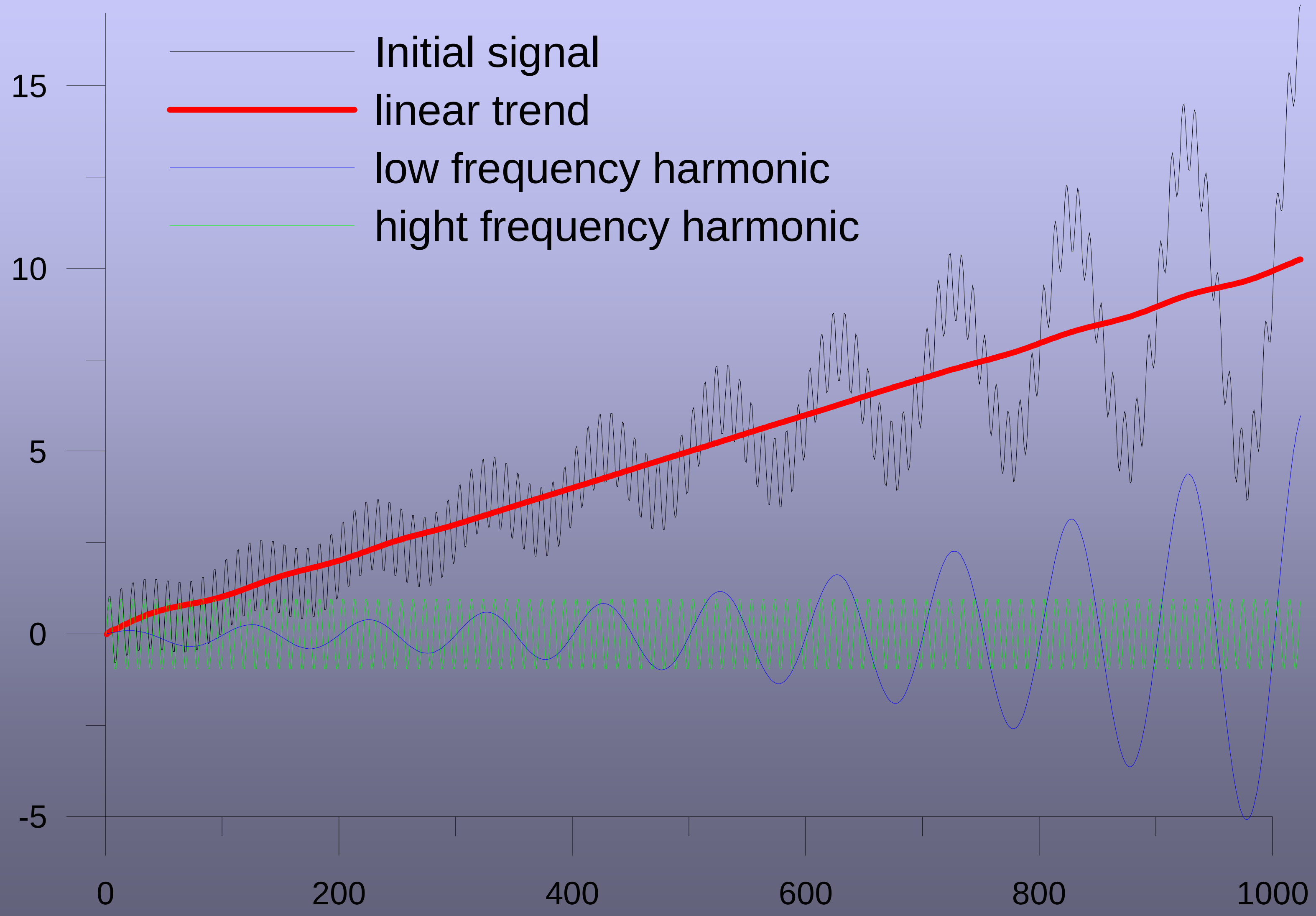
$$X = USV^T = \sqrt{\lambda_1} U_1 V_1^T + \dots + \sqrt{\lambda_d} U_d V_d^T$$

3. Principal components grouping

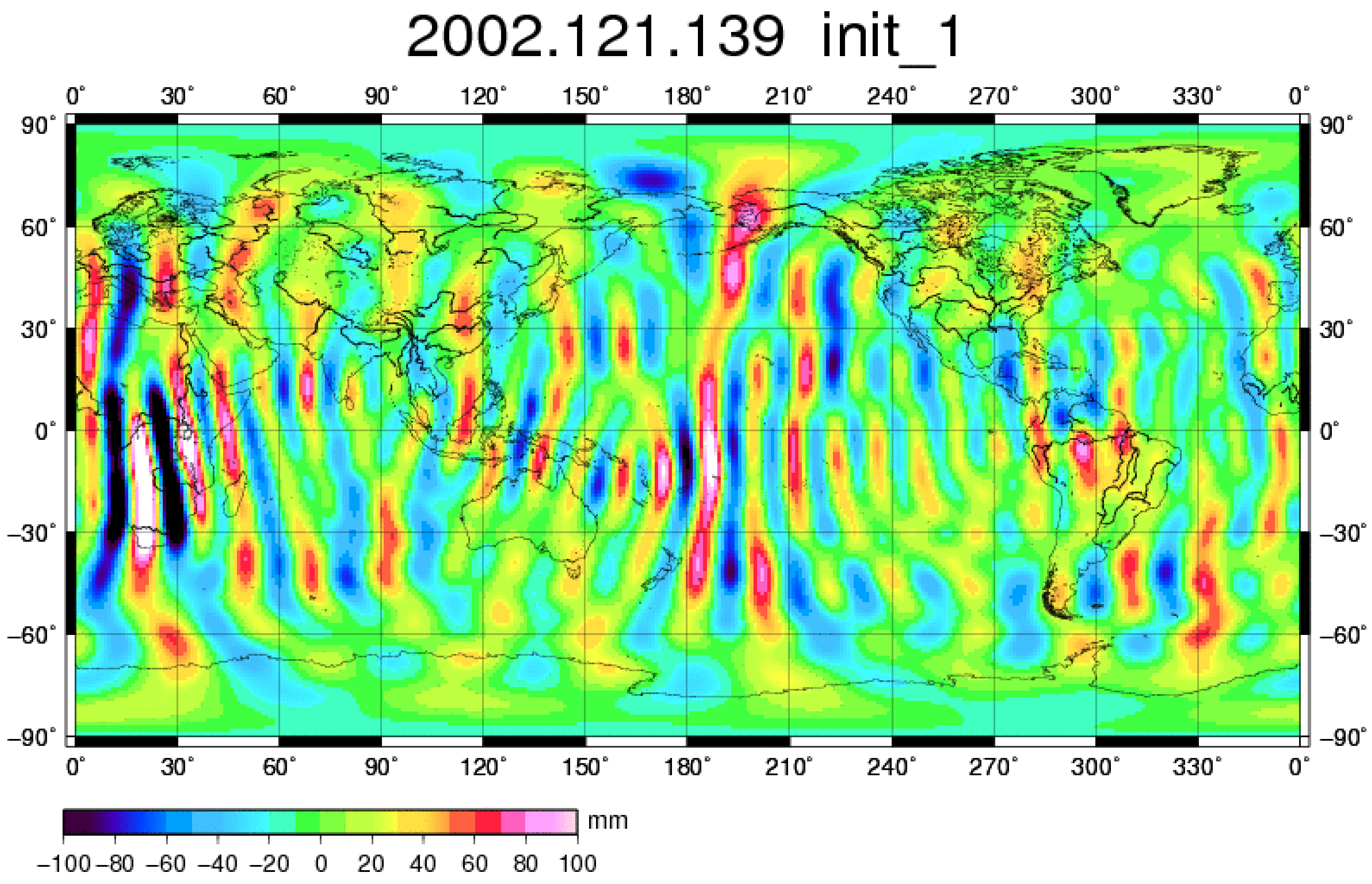
4. Hankelization

$$g_k = \begin{cases} \dfrac{1}{k+1} \sum_{m=1}^{k+1} y_{m,k-m+2}, & 0 \le k < L^* - 1 \\ \dfrac{1}{L^*} \sum_{m=1}^{L^*} y_{m,k-m+2}, & L^* - 1 \le k < K^* \\ \dfrac{1}{N+K} \sum_{m=k-K^*+2}^{N-K^*+1} y_{m,k-m+2}, & K^* \le k < N \end{cases}$$

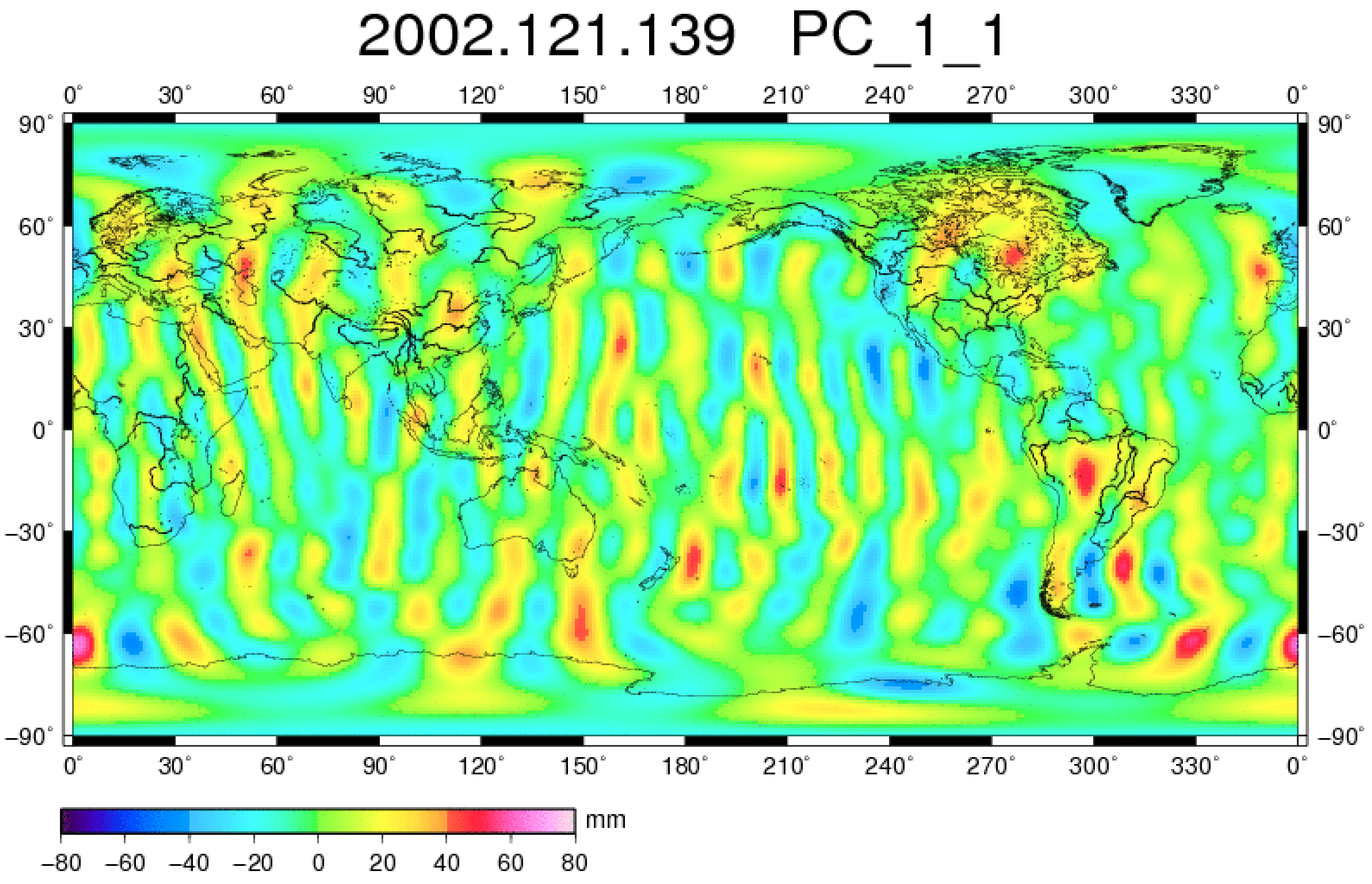
SIMPLE EXAMPLE OF 1D SSA



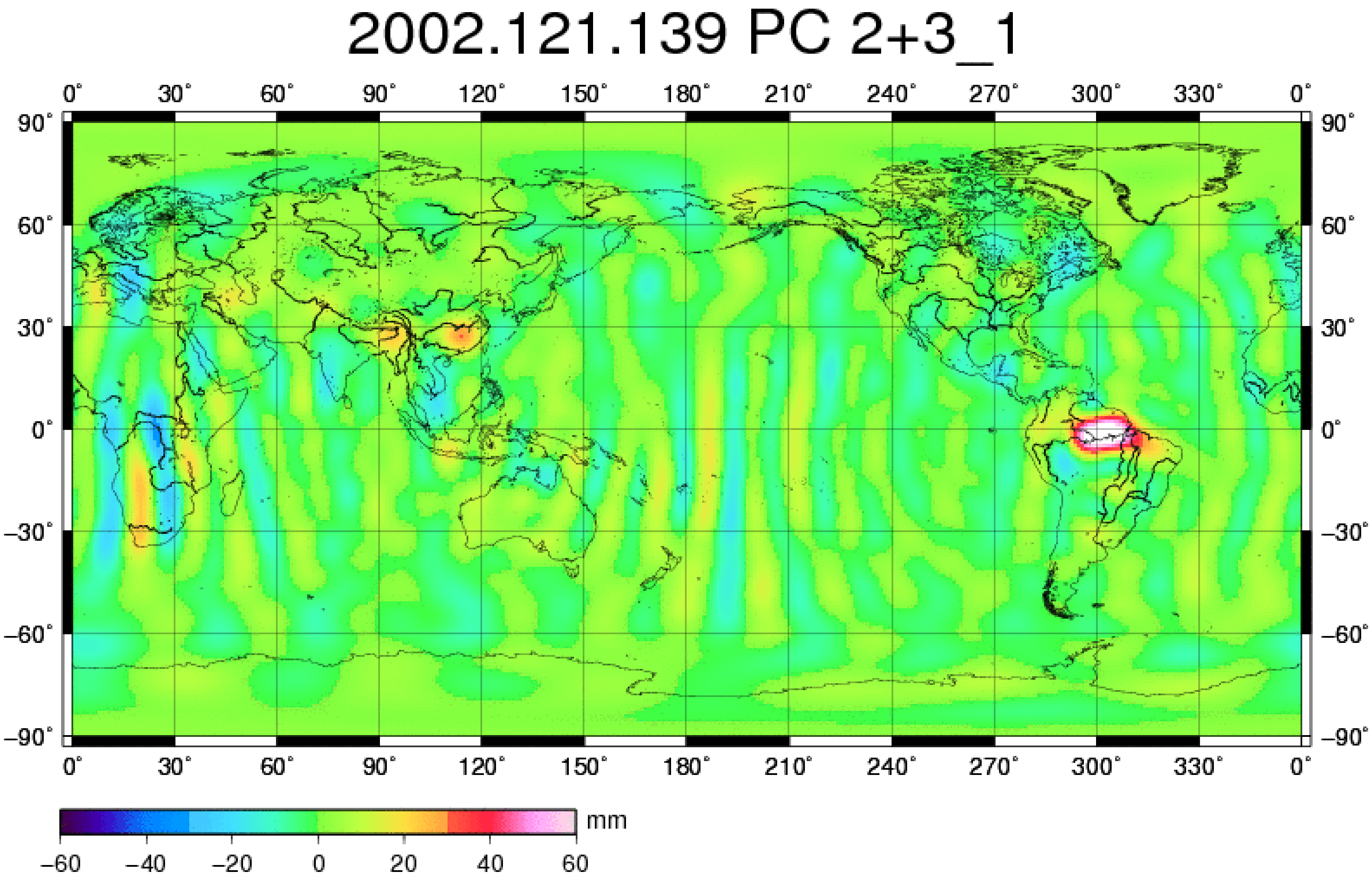
INITIAL UNFILTERED GRACE DATA (SPATIAL MAP)



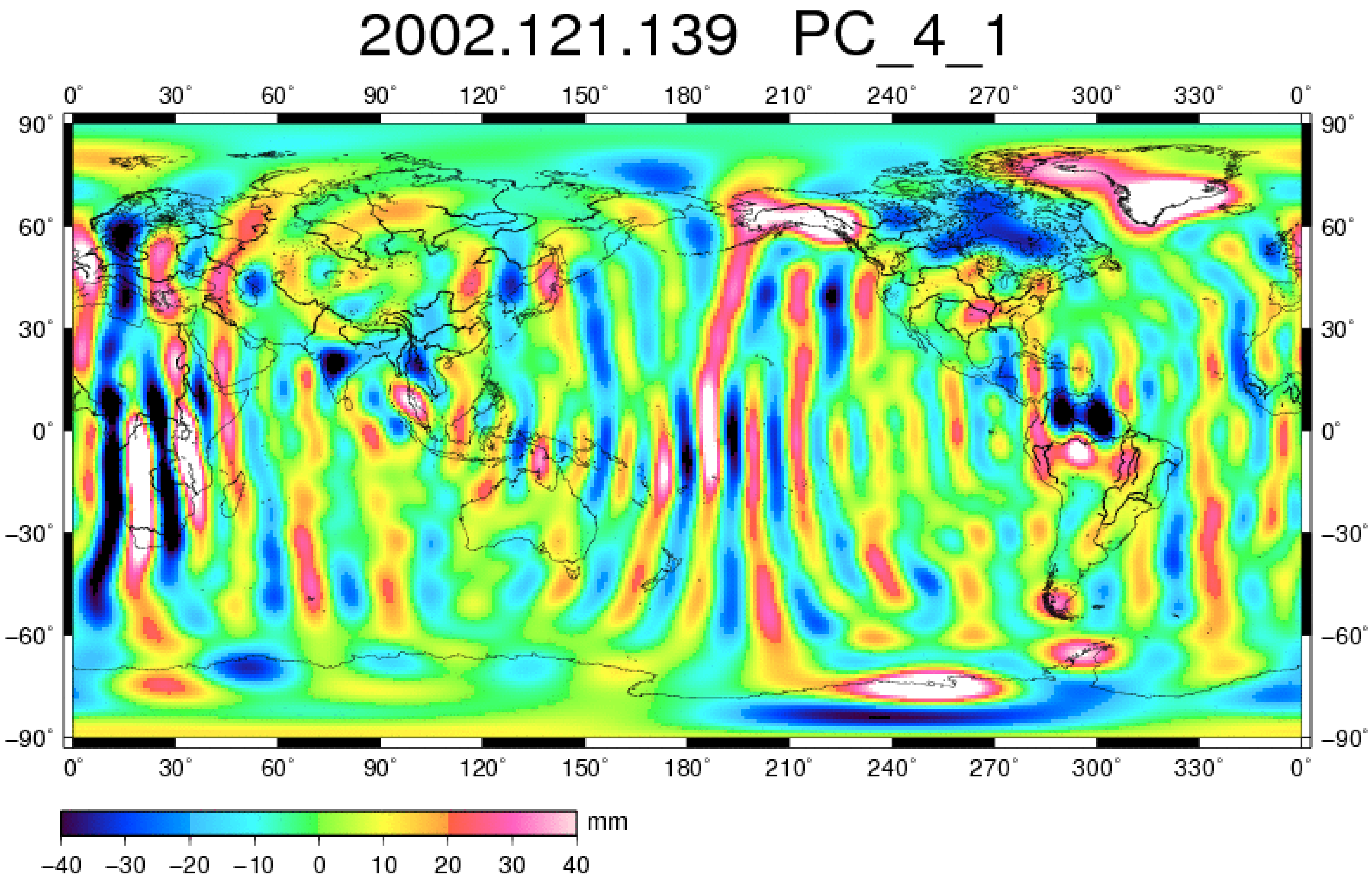
Principal Component N1 for MSSA with L=10 in spectral domain



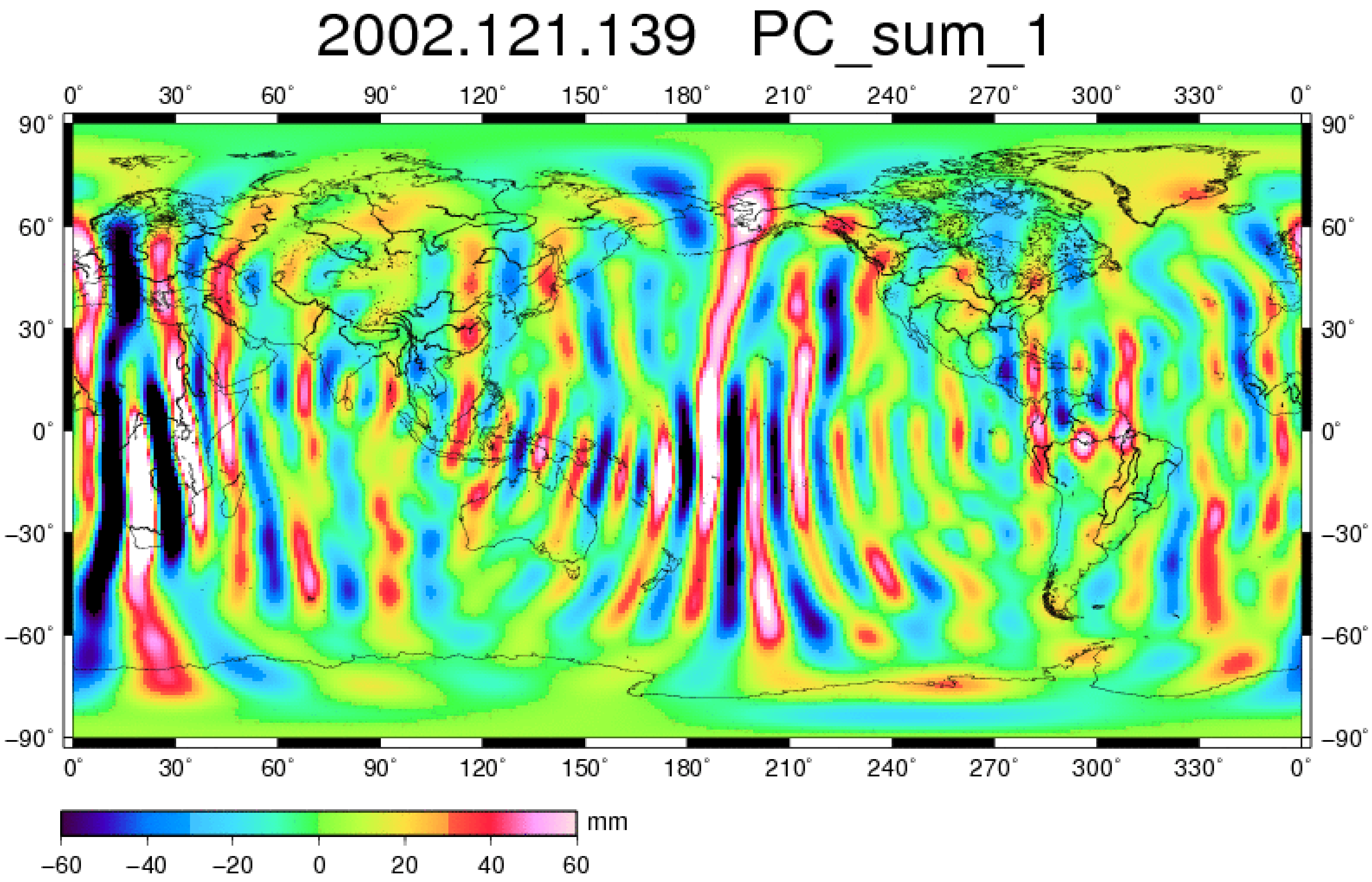
PC 2+3 spectral MSSA, L=10



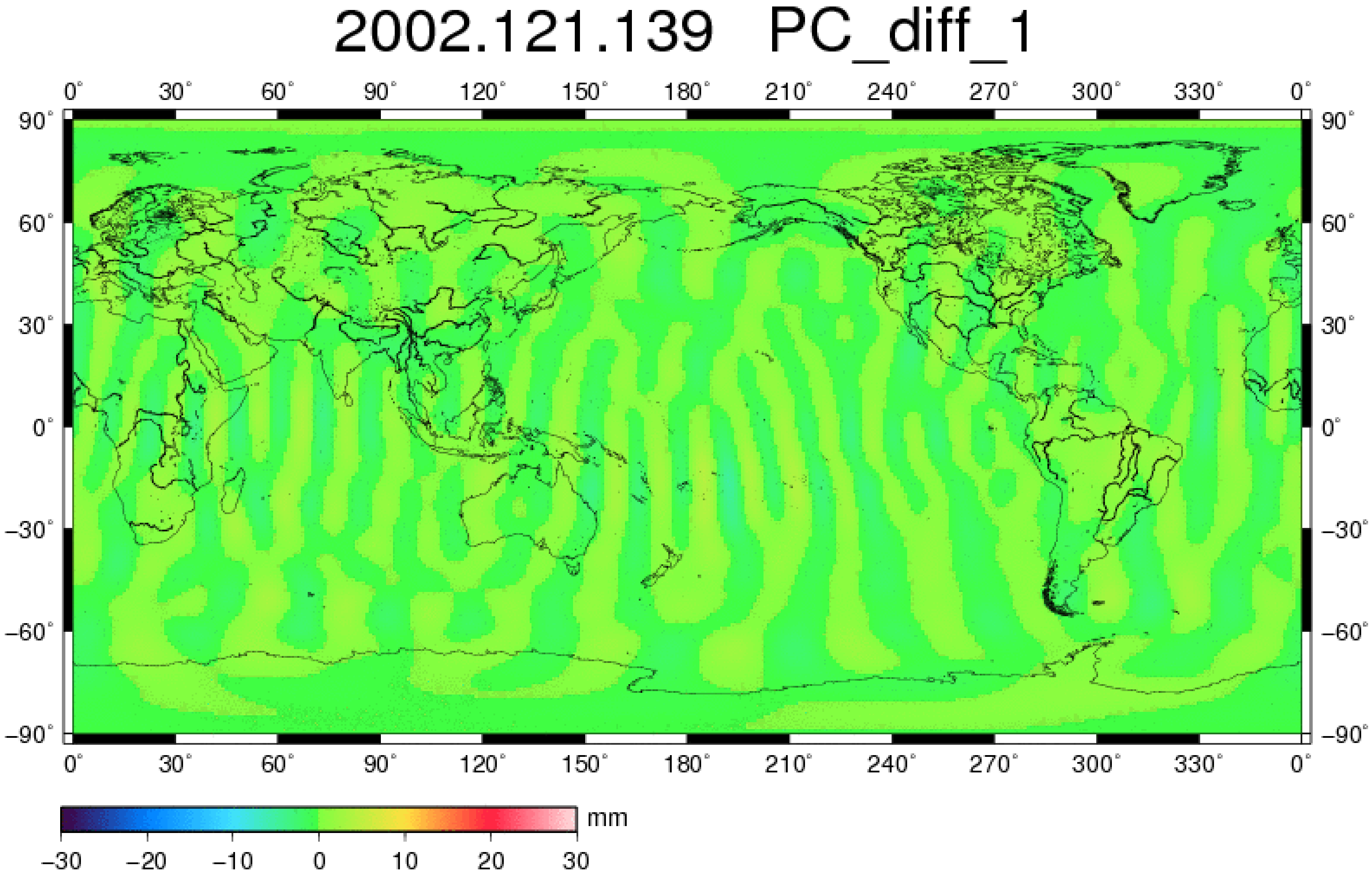
PC4, spectral MSSA, L=10



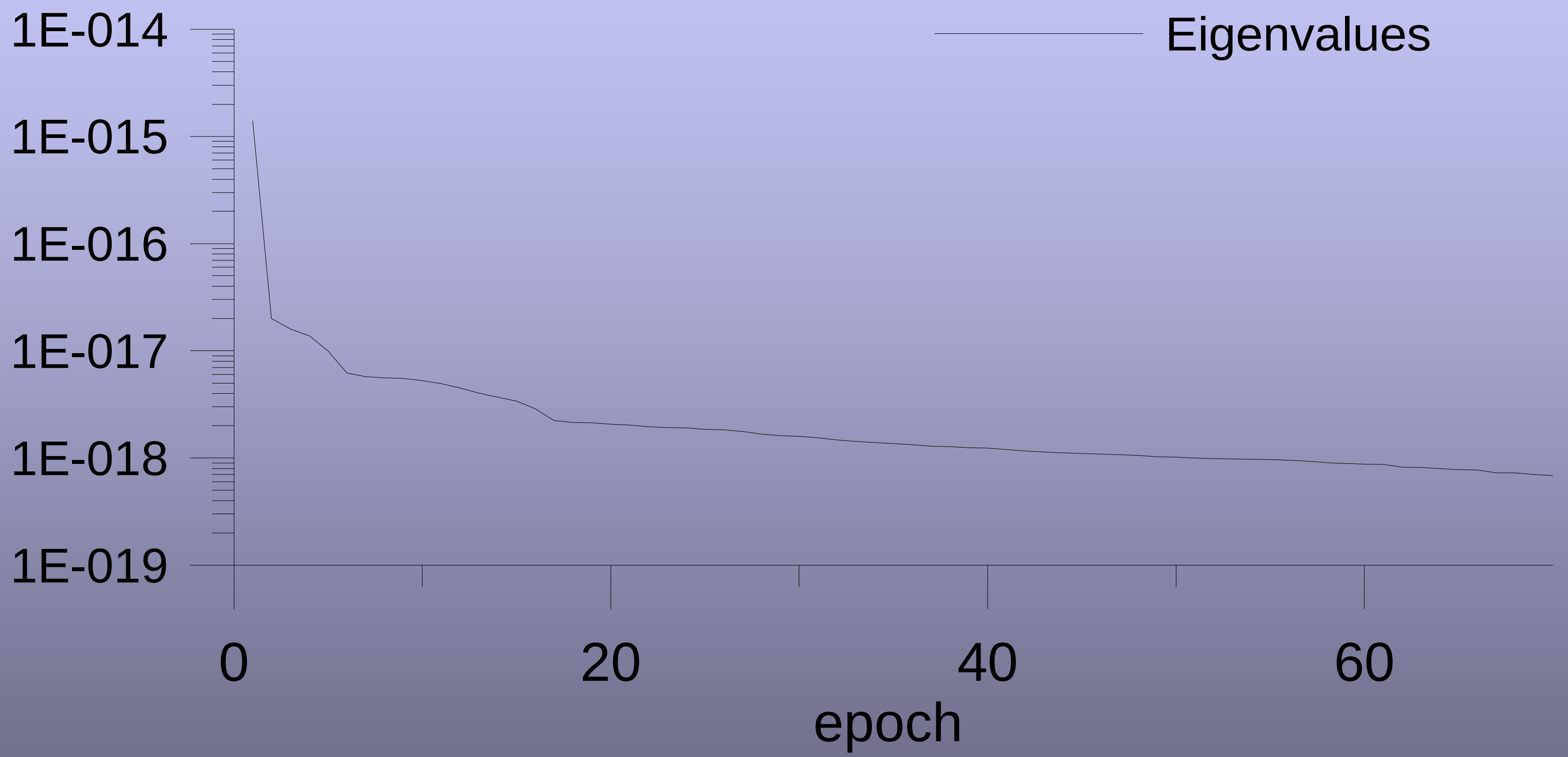
Sum of PC 2-10, spectral MSSA L=10



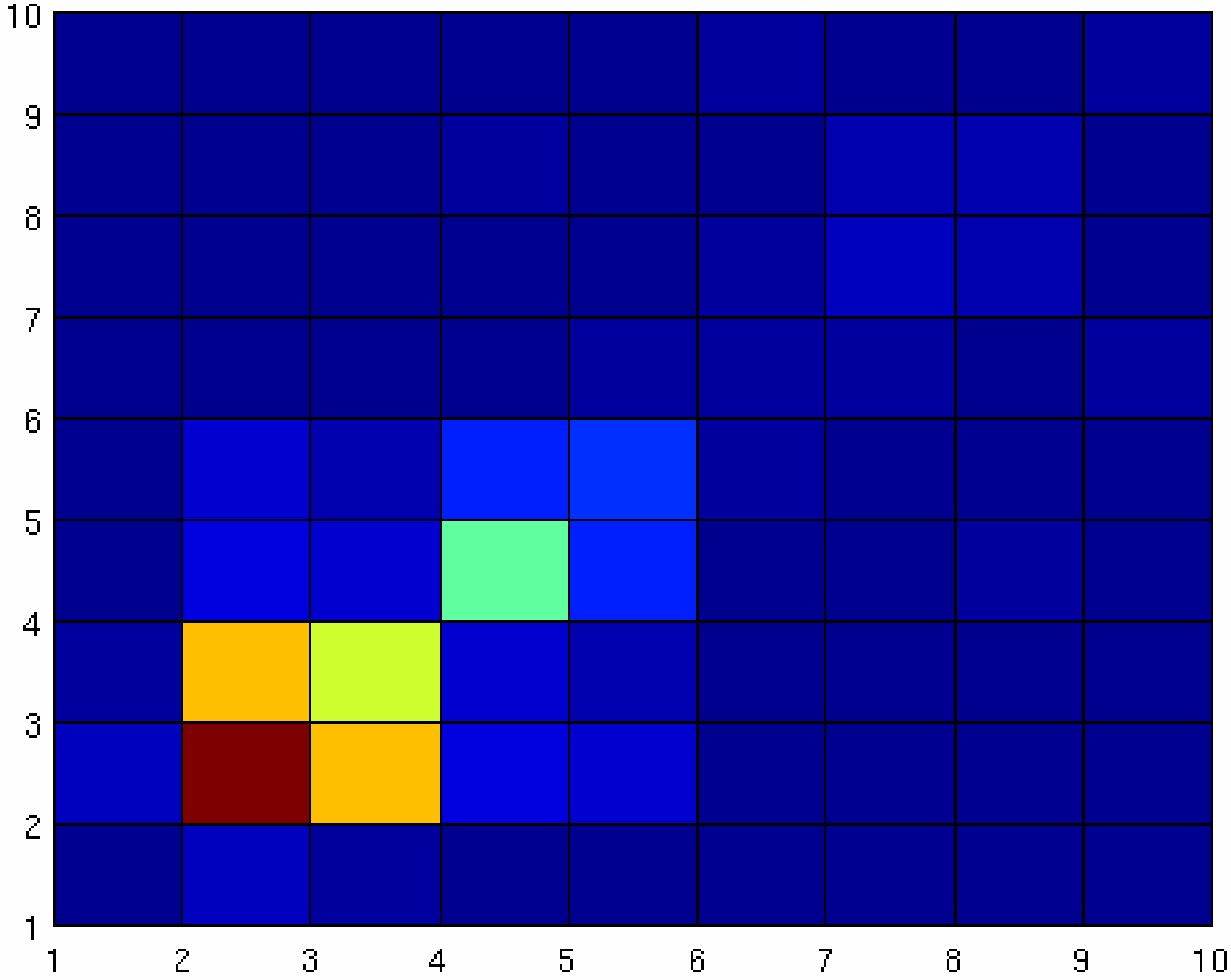
Difference between initial signal and sum of PC 1-10, spectral MSSA L=10



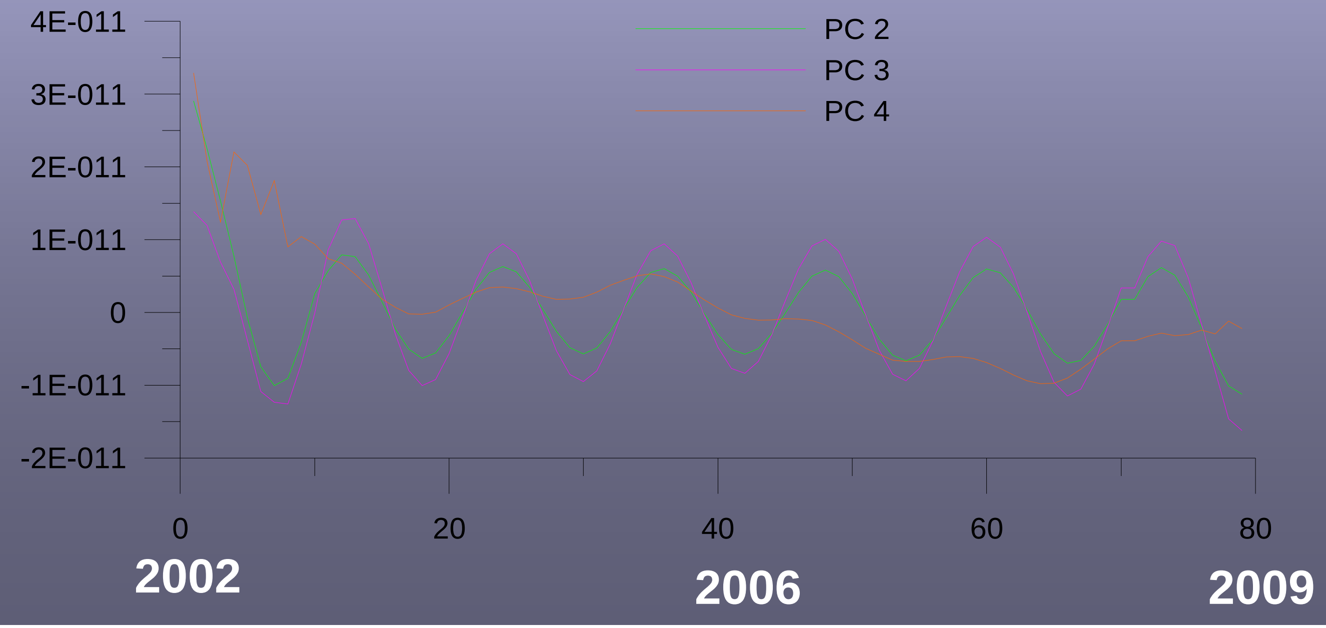
EIGENVALUES



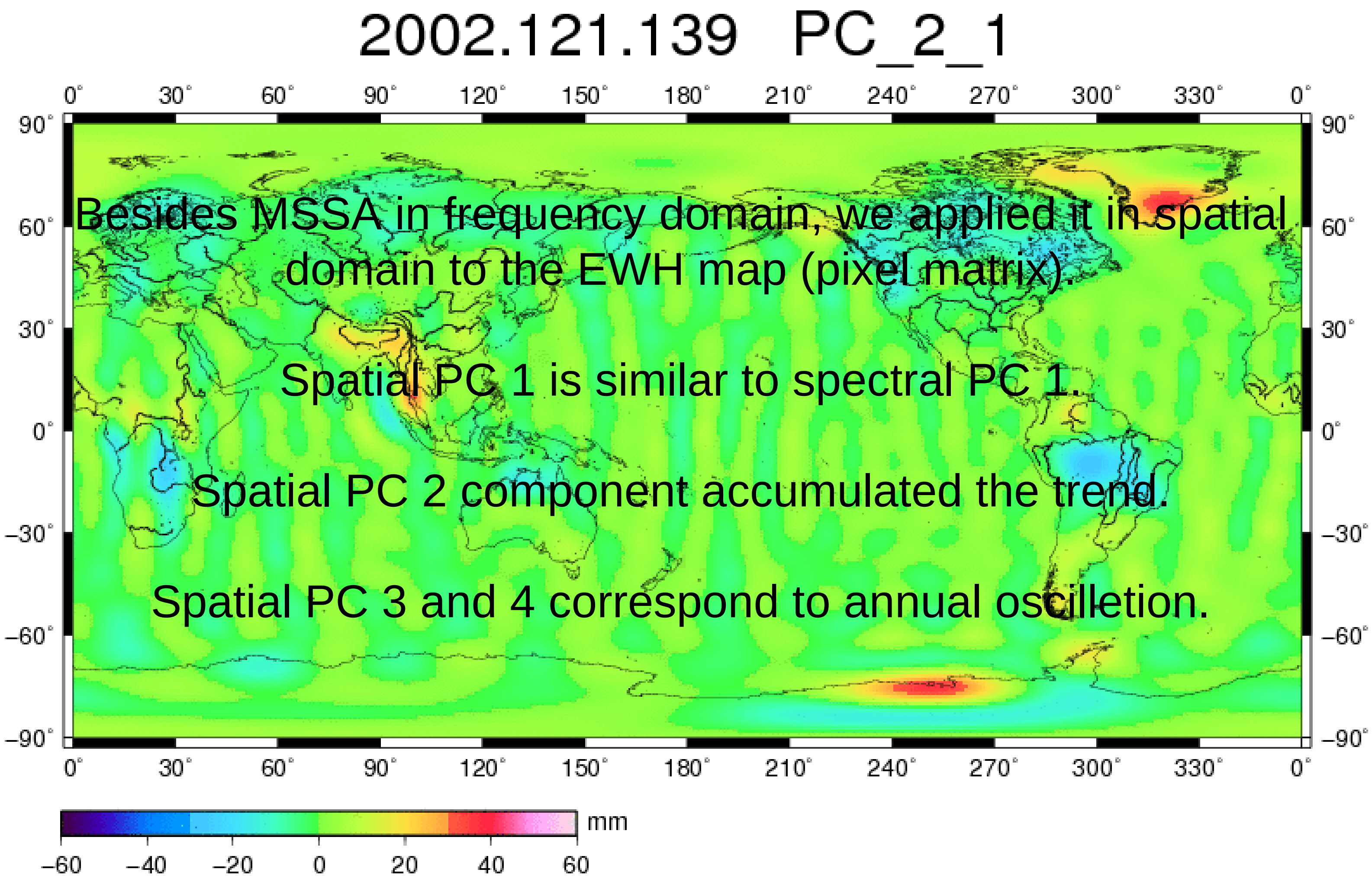
ω-CORRELATION



Approximation of Stokes coefficient S_{21} by principal components of MSSA, L=10



PC 2 for MSSA in spatial domain, L=10



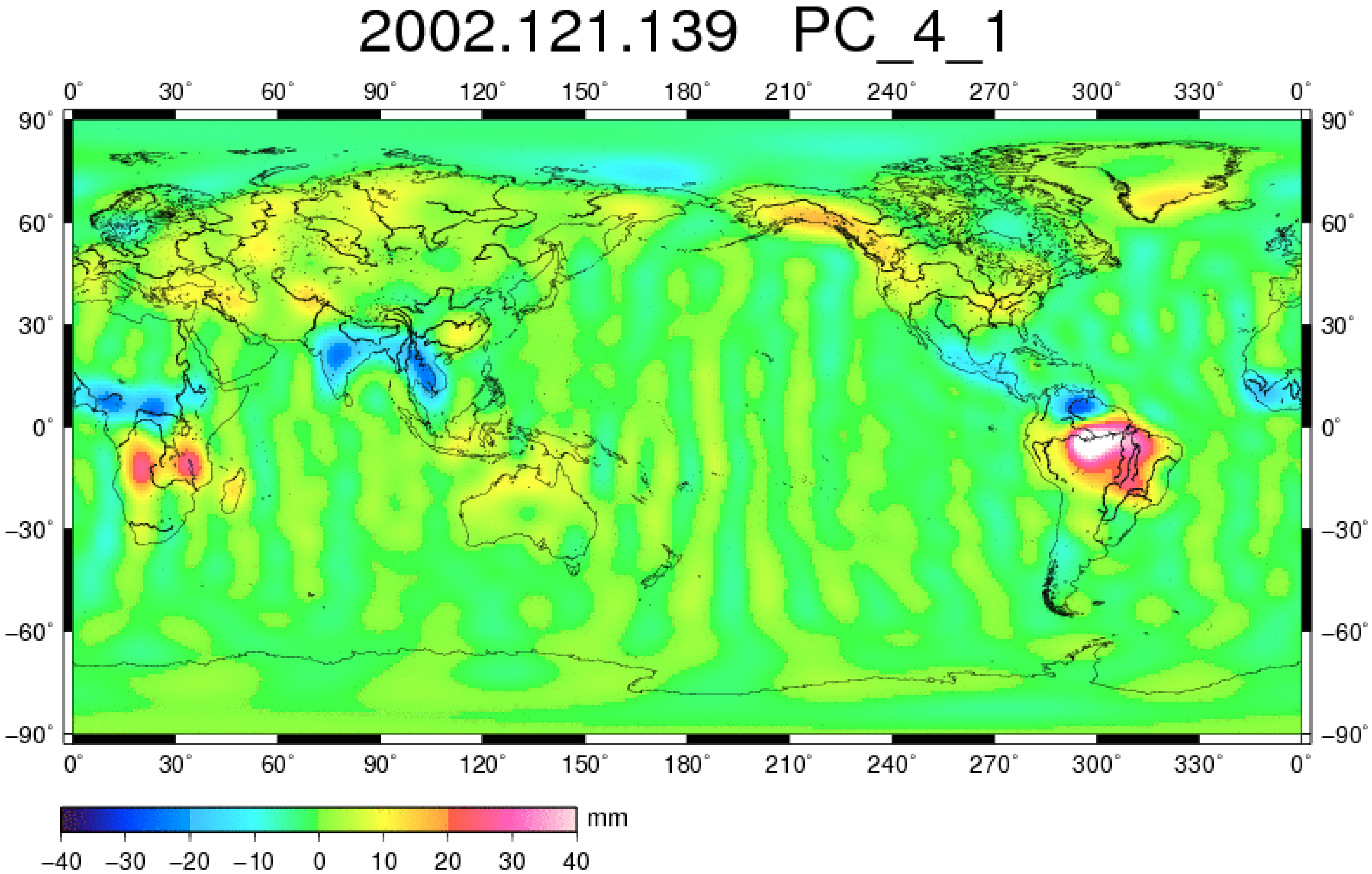
Besides MSSA in frequency domain, we applied it in spatial domain to the EWH map (pixel matrix).

Spatial PC 1 is similar to spectral PC 1.

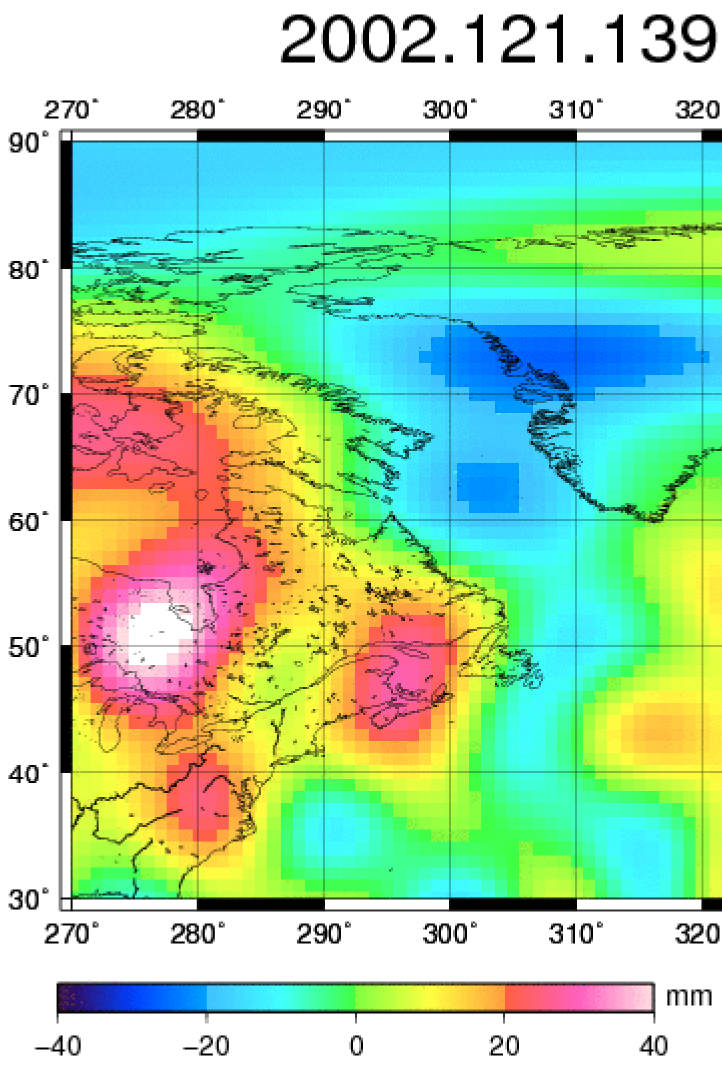
Spatial PC 2 component accumulated the trend.

Spatial PC 3 and 4 correspond to annual oscilletion.

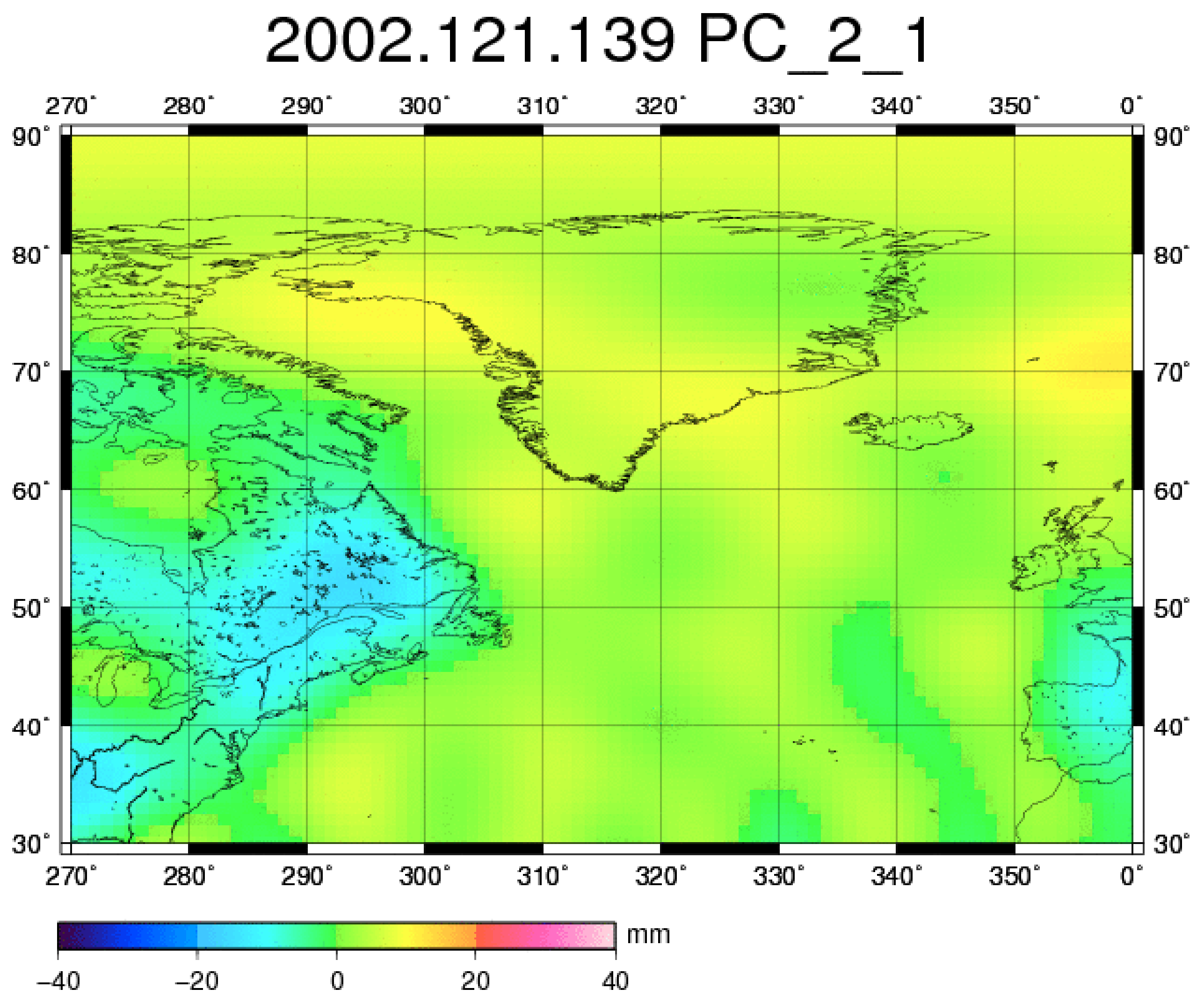
PC 3,4 for spatial MSSA, L=10



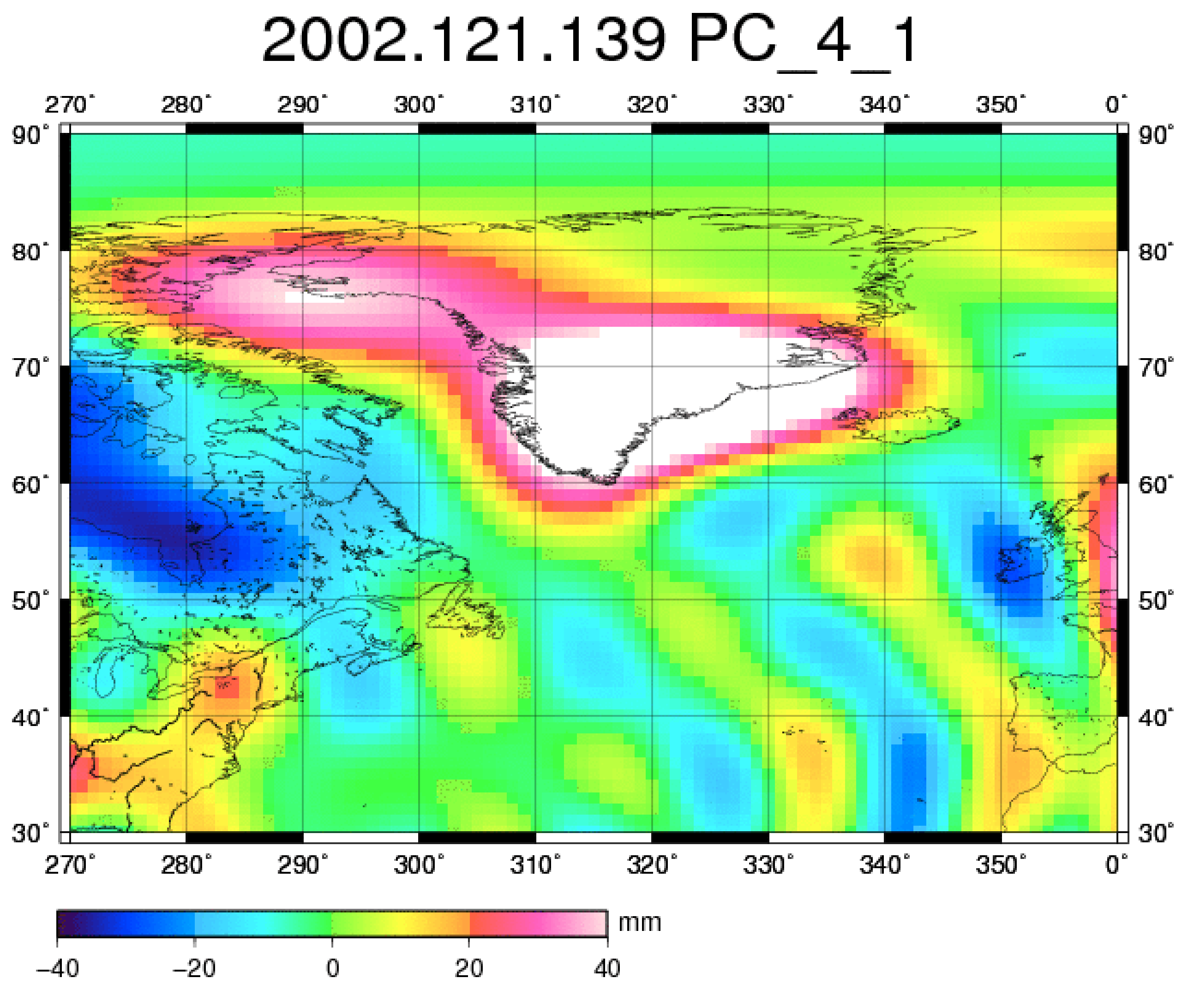
Greenland PC 1 spatial MSSA, L=10



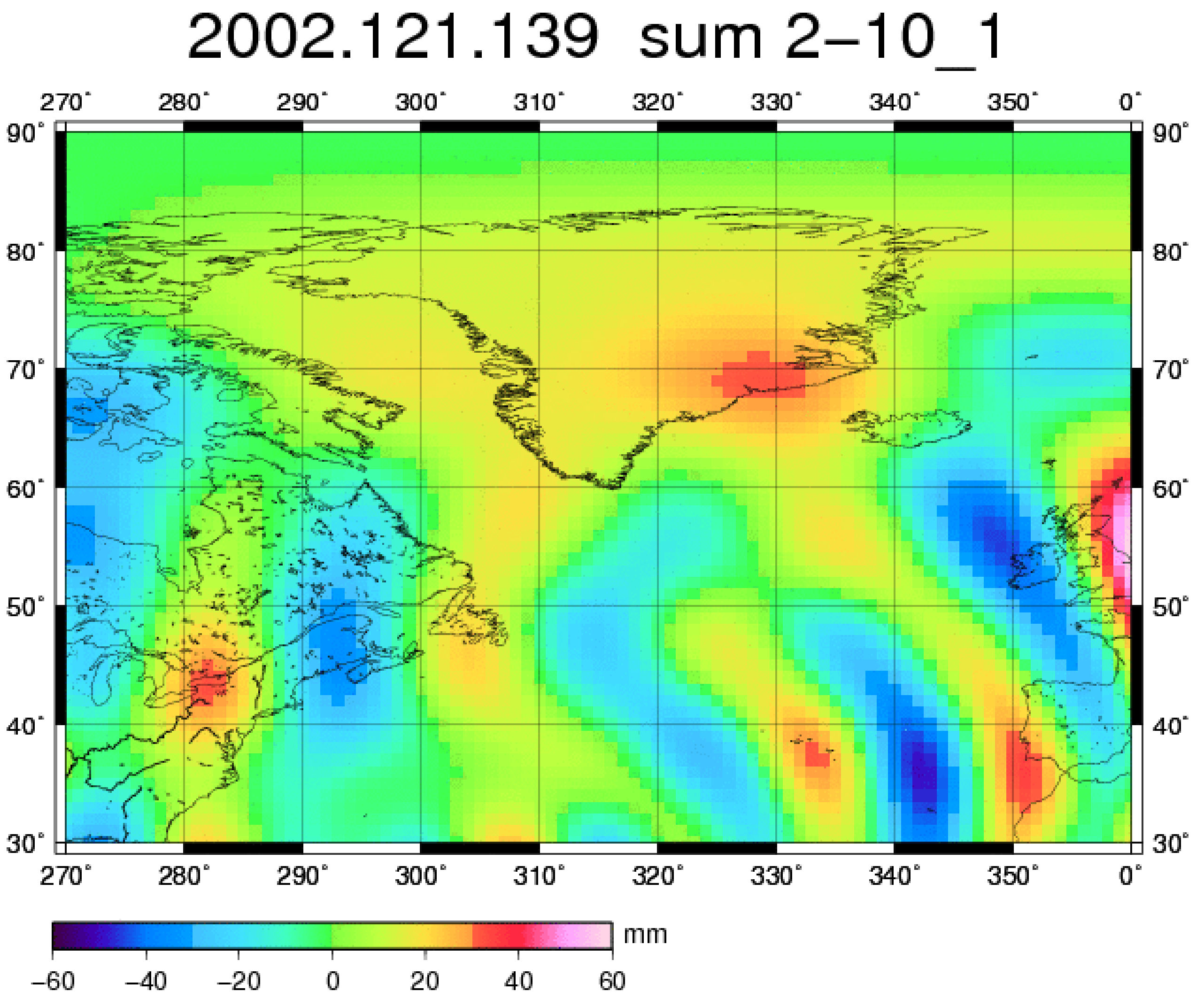
Greenland PC 2 spatial MSSA, L=10



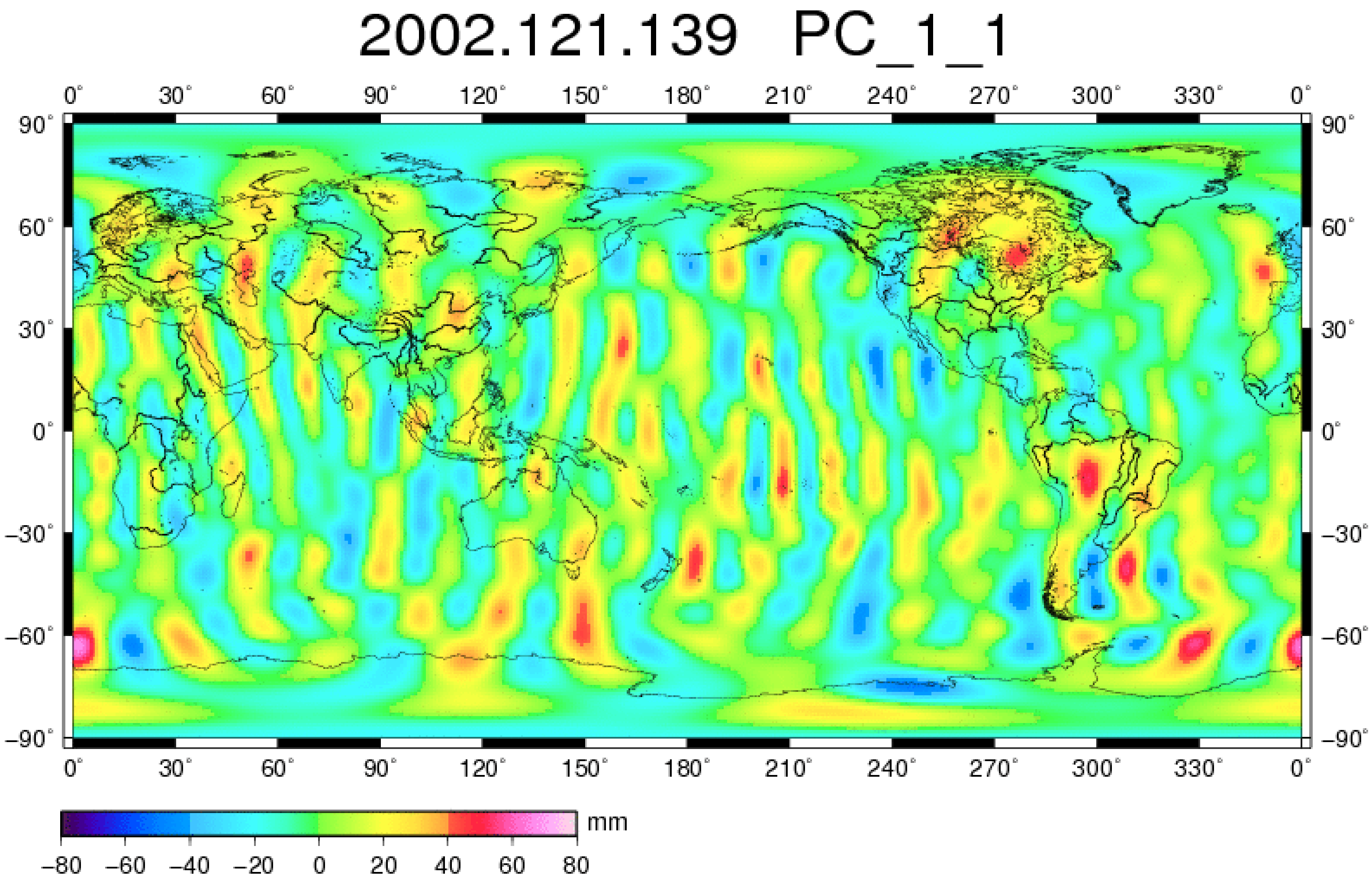
Greenland PC 3,4 spatial MSSA, L=10



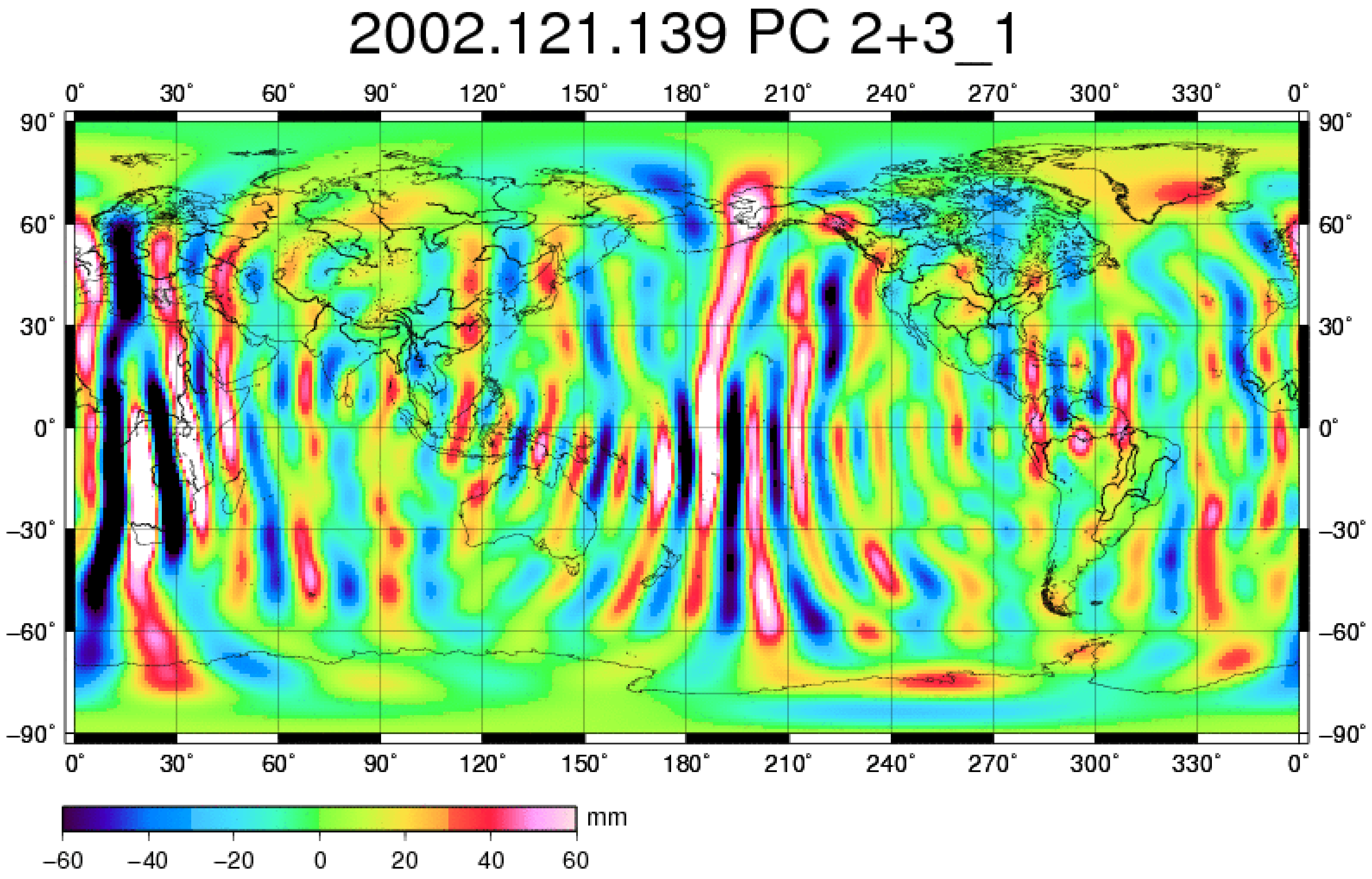
Greenland PC 2-10 spatial L=10



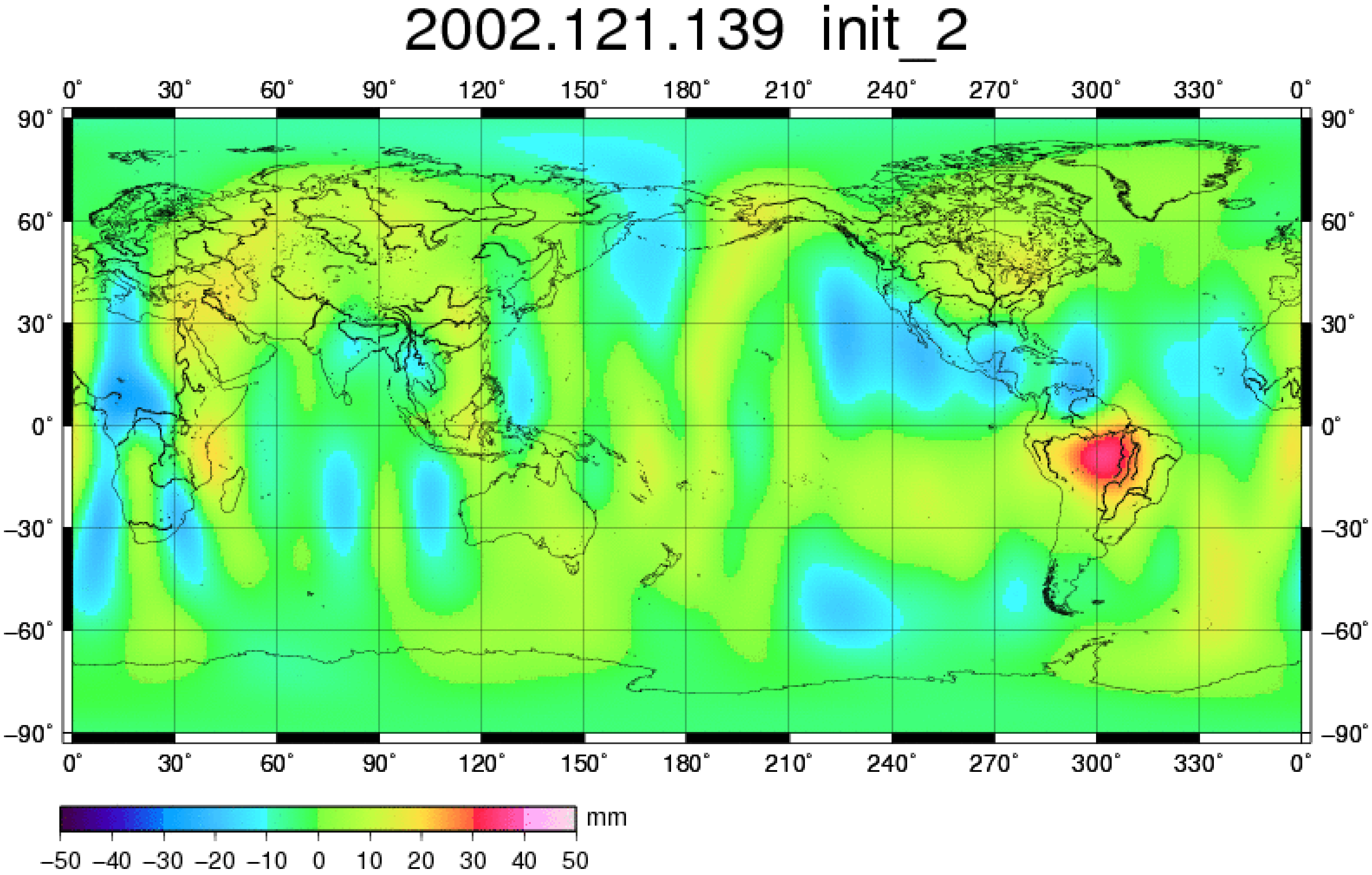
PC 1 spatial PCA, L=1



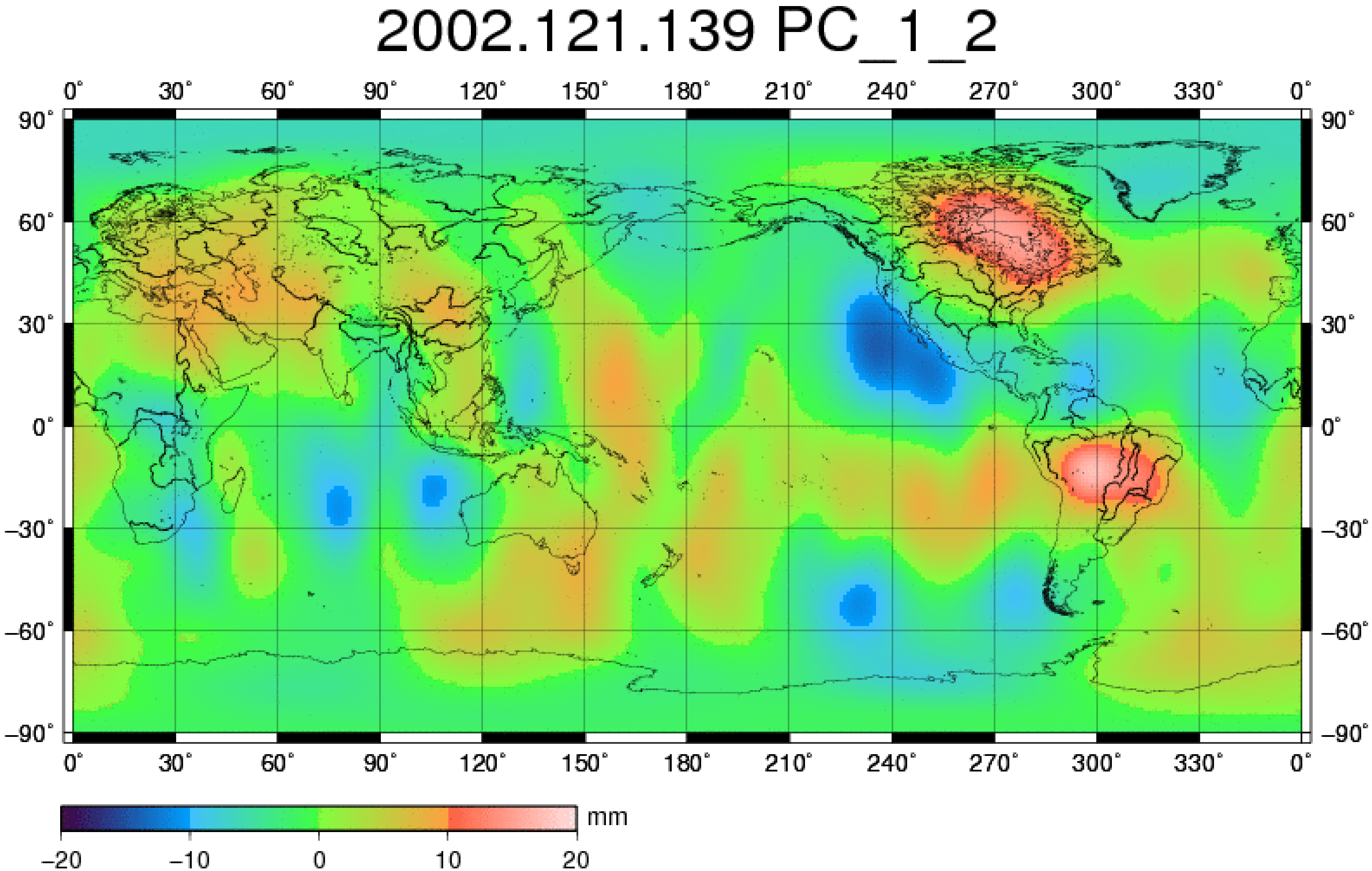
PC 2,3 spectral PCA, L=1



INITIAL FILTERED DATA (Gaussian filter R=800km)



PC 1 spatial filtered L=10



Panteleev corrective smoothing

If we have signal *u,* which passes through the dynamic system (equipment) with transfer function *W* and gives us observations *f* together with noise, we can reconstruct *u* applying inverse operator together with smoothing operator *Wsmooth.* Smoothing is very important to prevent amplification of noises. If dynamic system is linear, we can write down the corrective smoothing in spectral domain (hat means Fourier transform) as

$$\hat{u}_{corrected} = \frac{W_{smooth}(i\omega)}{W(i\omega)} \hat{f}(\omega) = W_{Pantelev}(i\omega)\hat{f}(\omega)$$

Method can be used instead of regularization. We can extract useful information from the signal only if we are able to find basis (functional, vectorial, etc.), where it is separable from the undesirable systematic errors and noise. If we can use SSA basis for denoising and realize *Wsmooth* in this basis, we would be able to reconstruct the signal *u* successfully. At that, we can apply *Wsmooth* before or after inversion. if we have been able to decompose Level-2 signal, as it’s shown, we should be able to decompose also the initial unprocessed signal and use this approach for inversion.

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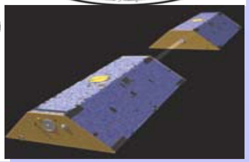

Acknowledgements

To perform this work Ohio Supercomputer Center GLENN cluster was used. Presentattion at the confernece “Climate, Statistics, and Satellites” was sponsored by NSF.

We thank Zhenwei Huang, who helped on initial GRACE data processing. This work is sponsored by RFBR grant 08-02-00256, NASA OSTST, and OSU’s Climate, Water Carbon Program.

Thanks to Jerry North for invitation and hospitality!