PANTELEEV'S CORRECTIVE FILTERING AS THE REGULARIZING ALGORITHM FOR THE EARTH'S CHANDLER WOBBLE EXCITATION RECONSTRUCTION

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The resonant Chandler wobble of the Earth's pole with ~ 0.2 arcsec amplitude, accompanied by the low-frequency trend and annual motion, is provided by some process, hidden in the atmosphere and ocean variability. The problem can be described by the Euler-Liouville equation

$$\frac{i}{\sigma_c}\frac{dp(t)}{dt} + p(t) = \chi(t), \tag{1}$$

where $p = p_1 + ip_2 \approx x_p - iy_p$ is the complex pole trajectory (x_p, y_p – coordinates of the pole); $\sigma_c = 2\pi f_c(1+i/2Q)$ is a complex Chandler frequency, depending on real frequency f_c and quality factor Q (standard values are $f_c = 0.843$ cycles per year, $Q = 100$); $\chi = \chi_1 + i\chi_2$ is an effective angular momentum function, or excitation. If to denote the direct problem (1) as $Ap = \chi$, with linear operator A, our goal is to extract Chandler wobble and reconstruct its excitation χ from the noisy observations p, i.e. to solve the inverse problem. Unregularized solution $A^{-1}p$ is improper way to do this.

To extract Chandler wobble we used Panteleev's filter, with impulse response

$$h(t) = \frac{\omega_0}{2\sqrt{2}} e^{-\left(\frac{\omega_0|t|}{\sqrt{2}} - i2\pi f_c t\right)} \left(\cos\frac{\omega_0 t}{\sqrt{2}} + \sin\frac{\omega_0\,|t|}{\sqrt{2}}\right), \tag{2}$$

and frequency response

$$L_h(f) = \frac{f_0^4}{(f-f_c)^4 + f_0^4}\ , \tag{3}$$

centered at the Chandler frequency f_c with half-width parameter $\omega_0 = 2\pi f_0$. This filter with parameter $f_0 = 0.04$ years^{-1} allows to extract Chandler wobble (CW) of the Earths pole with less then 10% error from the 50 milliseconds of arc (mas) noise, as shown by modelling.

It is well known, that for the inverse problem solution it is required to build an algorithm, converging to the exact pseudo-solution when the errors in operator and observations tend to zero [1]. If to use the filter (3) together with inverse operator, the Chandler excitation can be obtained

$$\hat{\chi}(f) = A^{-1}L_h\hat{p}(f) = W_{corr}\hat{p}(f), \tag{4}$$

where we use $\hat{}$ to denote Fourier transform (spectrum) and spectral form of inverse operator A^{-1}. By operator $W_{corr} = A^{-1}L_h$ we mean such an algorithm, called Panteleev's corrective filtering, which allows to obtain the inverse problem solution not corrupted by noises in the frequency band, where A^{-1} has large frequency response and amplifies noises. This method was applied in [2] to the problem of Chandler wobble excitation reconstruction.

To be sure, that such filtering is a regularizing algorithm, we should check, that filter parameters depend on the data and operator noise and the obtained solution would converge to the exact pseudo-solution, when errors tend to zero.

Firstly, it is easy to show, that if the errors in operator, i.e. in values f_c and Q, would tend to zero, the bias of the solution, obtained with corrective filter W_{corr}, would tend to zero, due to the fact that the filter L_h is centered at f_c.

On the other side, the obtained one-parametric family of solutions depends on the half-width of the filter $T_0 = 1/f_0$. To select this parameter in accordance with the minima of discrepancy principle, we performed simple modelling, described below. The Chandler excitation χ model was generated to produce $p(t) = A\chi$ similar to the real Chandler wobble, filtered out from observations. Then we obtained p from χ using the operator parameters model $m_2(T_c = 1/f_c = 433$ years, $Q = 100)$ as unperturbed (the true one) and models $m_1(T_c = 436$ years, $Q = 130)$, $m_3(T_c = 430$ years, $Q = 70)$ with reasonable errors-in-operator A introduced through parameters $f_c = 1/T_c$ and Q. Treating model m_2 as an exact one, we obtained mean error for $m_{1,3}$ at the level $h||\chi|| = 70$ mas. After that, Gaussian noise was added to observations p over all the time period 1846-2017. The mean amplitude of noise $\delta = 50$ mas was selected a bit less than early astrometric noise expected before 1900 yr, but much larger than the observational noise of cosmic geodesy epoch (0.1 mas), began in 1970s.

If to apply inverse operator A^{-1} directly to noisy data without additional filter L_h, the mean error in reconstructed excitation would have enormous level $\sigma_\chi = 550$ mas. Using corrective filtering with fixed f_c

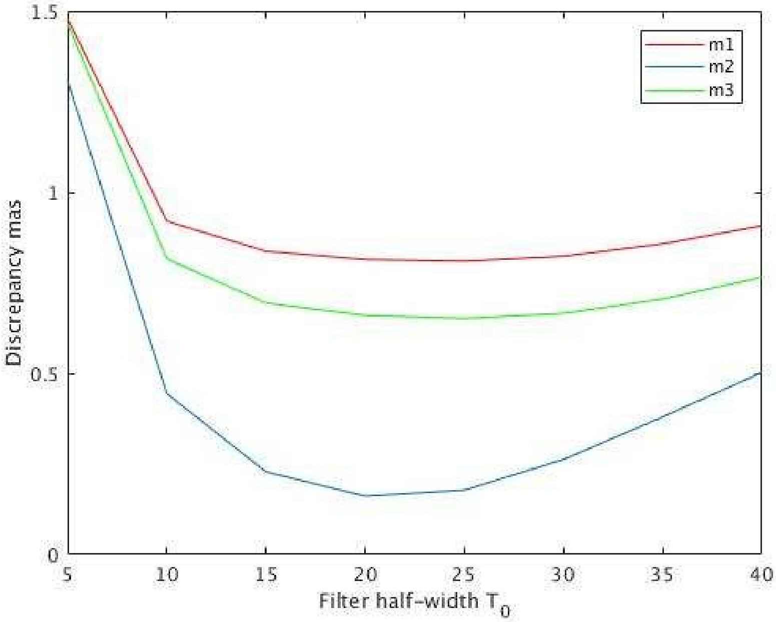

Figure 1: Mean desacrepancy $||(\hat{\chi} - W_{corr}\hat{p})||$ as a function of filter (2) half-width $T_0 = 1/f_0$ for the case of observational noise (m_2), and both observational and operator noises (m_1, m_3).

and Q as in m_2, and parameter f_0 changing from 1/40 to 1/5 years^{-1}, we estimated the level of agreement of reconstructed excitation with the initial one (modelled). The value of discrepancy $||(\hat{\chi} - W_{corr}\hat{p})||$ is shown in Fig. 1 for the case of error-in-observations only (m_2, blue line) and two cases of errors in observations and operator ($m_{1,3}$, red and green). If the filter (3) width f_0 is too narrow, it damps useful component of the solution. On the other side, if its band-pass is too wide, to much noises would pass, and the agreement would be worse. In the absence of operator noise, the optimal filter parameter obtained from the minima of the curve in Fig. 1 is $f_0 = 1/20$ years^{-1}. Standard error in excitation reconstruction is 0.2 mas. When the noise present in both observations and operator parameters f_c, Q, the optimal half-width is $T_0 = 25$ years ($f_0 = 0.04$ years^{-1}) and the modelled reconstruction error 0.7-0.9 mas, two to three times less than the pike amplitudes (2-3 mas) of the excitation reconstructed from the real observations [2].

Thus, we believe, that excitation obtained for the real Chandler wobble [2] (not shown here) with quasi 20-year amplitude modulations is real, it maintains Chandler wobble and explains its amplitude changes [3]. We conclude that Panteleev's filtering is a regularizing algorithm with filter (2) playing the role of a stabilizer, helping to reject high and low frequency noises in the band, where direct operator amplitude response is small. For the excitation we obtained the one-parametric family of solutions, with a filter width as a parameter. Using the model of the Chandler excitation, we show, that the optimal filter parameter f_0 1/25 years^{-1} [2] can be selected in accordance with the generalized discrepancy principle [1].

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