The analysis of discrepancies of the nutation theories MHB2000 and ZP2003 from VLBI observations.

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Abstract

Spectral and structural analysis of the discrepancies of the nutation theories for the nonrigid Earth ZP2003 and IAU2000 from VLBI observations was performed.

As a result of structural analysis it was shown that the linear parts of the equations of moments in these theories are modeled well enough and the improvement in the future is possible only by the perfection of the models for the nonlinear parts.

As a result of spectral analysis it was shown that the main differences between ZP2003 and IAU2000 are at semiannual and main nutation 18.6–year periods. The reasons of these discrepancies are under discussion. The major part of deviations of the theories from observations is determined by the free core nutation, which needs to be studied and monitored in the future. Spectral investigation proved the existence of an unknown process, which compensates the influence of atmosphere at semiannual frequency.

With use of SVD least squares method the empirical corrections to the main harmonic oscillations for ZP2003 were estimated.

Introduction

The position of the Earth rotation axis in the inertial space changes under the influence of the tidal potential of the celestial bodies. The name precession and nutation of the Earth axis is used for this phenomenon. The parameters defined by the theory of precession and nutation are necessary for the transformation from the average celestial system of coordinates to the true one for epoch of observations. The theory MHB2000 [1] was adopted by the International Astronomical Union (IAU) as the new nutation theory IAU2000 instead of the nutation theory IAU1980 at the XXIV assembly and it came into force on the 1 January 2003. A strong requirements stands in front of the modern nutation theories – their departures from observations must be at the level of several tenths of millisecond of arc. The theory ZP2003 [2,3] developed in Russia satisfies this criteria. This theory differs from IAU2000 by the method of the calculation of atmosphere and liquid core effects. Besides in ZP2003 the lows of conservation of energy and moment are taken into account during the determination of the parameters of the Earth internal structure, which are not known precisely from observations.

The position of the point on the celestial sphere, which the instantaneous axis of rotation of the Earth passes through, may differ a little from the position, predicted by the theory. The differences derived from observations are described by two parameters: $d\varepsilon$ (correction to the nutation in inclination) and $d\psi$ (correction to the nutation in longitude). Analysis and modelling of the discrepancies of the modern theories of nutation for inelastic Earth from observations are essential for improvement of the nutation theory in the future with a goal to reach a microarcsecond level. Here we present the results of spectral and structural analysis of discrepancies for the theories ZP2003 и IAU2000.

Let's enumerate the main goals pursued in this work:

1) To determine the departures of the observed nutation angels from those ones predicted by the theories of nutation ZP2003 and IAU2000.

2) To determine, if the linear part of the equations of moments in the theories ZP2003 and IAU2000 has need for corrections in the frequency domain.

3) To perform wavelet-analysis of the discrepancies of the nutation theories ZP2003 and IAU2000 from observations.

4) To determine the empirical corrections to the nutation amplitudes, which could be useful for forecasting of the residual discrepancies of the theories ZP2003 and IAU2000 from observations in the future.

Initial data

We used in the analysis the series of discrepancies between the theories and observations, calculated by processing of VLBI observations since 1984 till 2003 with use of OCCAM 5.0 package [4]. The rough pips lager then 0.01 arcsec and observations not accurate enough with error larger 1 mas were excluded. As a result the series of deviations of $d\varepsilon$ and $d\psi$, containing more then 2000 points were used for analysis. Weighted mean square deviations of discrepancies of the theories from observations were 235 ($d\varepsilon$) and 829 ($d\psi$) μas for ZP2003, 199 and 480 μas for IAU2000.

Structural investigation

Modern nutation models for inelastic Earth belongs to the semianalytic type, it means some parameters of this theories are determined from the best agreement of the theory with observation criteria. That's why there are reasons to believe that the discrepancies of the models from observations cannot be reduced by the improvement of the linear part of these models.

But more accurate consideration shows, that this fact needs to be checked. The fact is that some of the parameters of the models are not varied arbitrarily, but in agreement with special conditions, which restrict the range of their values.

For such a check the corrections to the nutation amplitudes should be estimated and after that the corrections to the parameters of the resonant form of nutation equations should be derived. If the corrections to the parameters of the resonant equation do not improve the agreement with observations, the discrepancies of the theories with observations cannot be improved by addition of corrections to the Earth interior model parameters.

At the first stage the corrections to the amplitudes of the main nutation harmonic oscillations were estimated. In a complex form the model is:

$$\eta = -\sum_{j=1}^{N}\left[(A_{1j} + A_{3j} * T)e^{-\arg_j T} + (A_{2j} + A_{4j} * T)e^{\arg_j T}\right], \tag{1}$$

where each j-oscillation is connected with 4 amplitudes A_{kj} . The corrections to the first 300 harmonics were estimated by the SVD least square method (LSM) algorithm [3]. Fig. 1 illustrates a good approximation of the discrepancies between the theories and observations (dotted curve) by the series calculated with the amplitudes A_{kj} corrections (continues curve). The average level of corrections for A_{1j} and A_{2j} is some tens of μas and one order smaller for A_{3j} and A_{4j}.

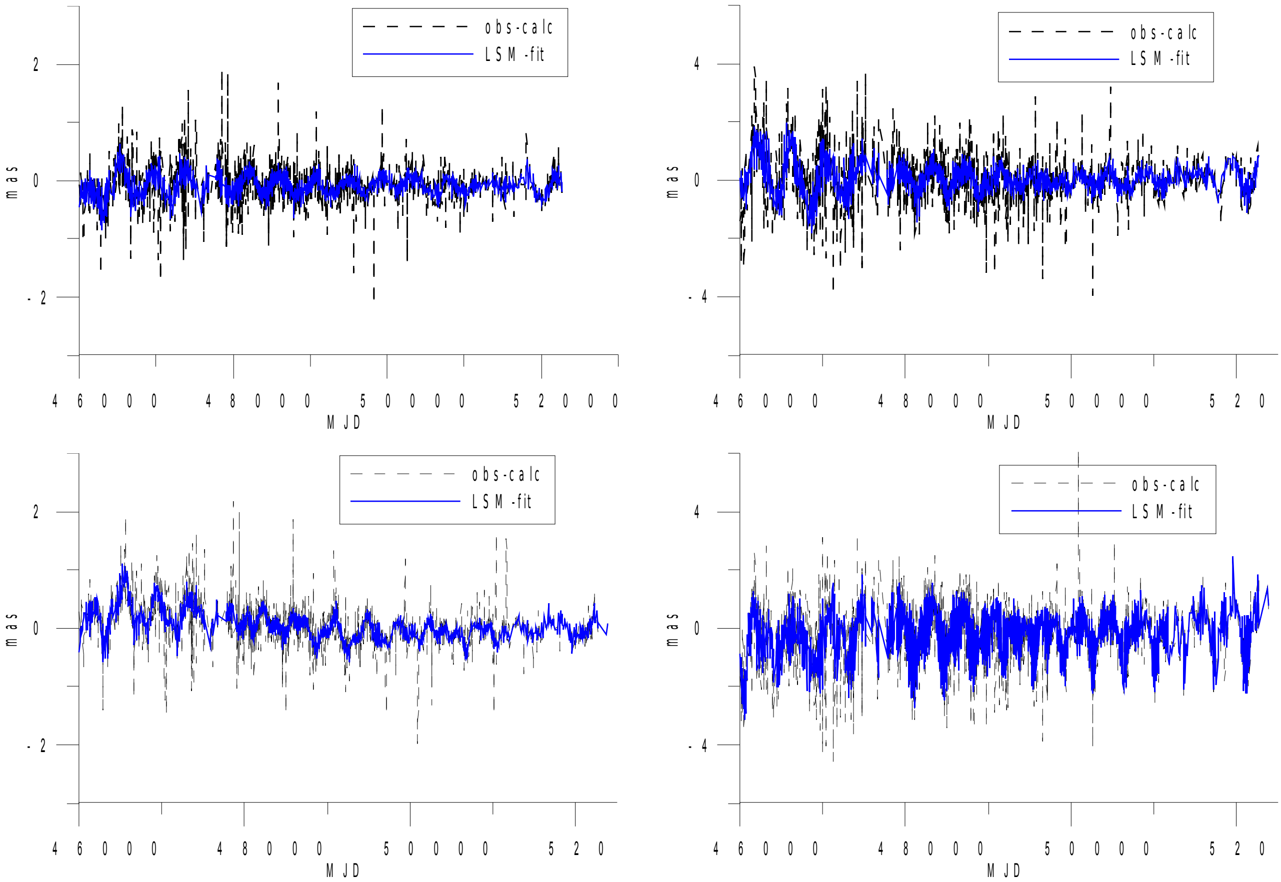

Figure 1. The discrepancies for dε (left) и dψ (right) of the theories IAU2000 (up) and ZP2003 (down) and there LSM fit.

At the second stage the corrections to the transfer function parameters were estimated. Each of the nutation amplitude for the inelastic Earth theory can be calculated by multiplication of corresponding nutation amplitude for the rigid Earth theory by the transfer function with addition of some extra terms

$$\eta(\sigma) = q(\sigma)\eta_R(\sigma) + \sum \delta\eta(\sigma) \tag{2}$$

For the nutation theory ZP2003 the transfer function has a form

$$q(\sigma) = R_1 + R_2(1+\sigma) + \sum_{i=1}^{6} \frac{R_{i+2}}{\sigma - \sigma_i}, \tag{3}$$

where σ is a frequency of oscillation, the values of parameters R_i and σ_i are represented in table 1.

For the nutation theory IAU2000 the transfer function has a form

$$q(\sigma) = \frac{(e_R - \sigma)}{(e_R + 1)} N_0 \left(1 + (1+\sigma) \sum_{i=1}^{4} \frac{N_i}{\sigma - \sigma_i} \right), \tag{4}$$

where e_R =0.003284507, and parameters N_i and σ_i are in table 2.

Table 1. Parameters of the transfer function of ZP2003.

i	R_i		σ_i	
	Real	Imag	Real	Imag
1	1.05142675334988	-3.53932806447103*10^{-6}	0.00253124376931 7792	9.96122648808202*10^{-6}
2	-0.280451410052684	-6.32517757688398*10^{-15}	-1.0023305400605	-1.97141971460492*10^{-5}

3	-0.000579966291835117	7.6698132768425*10^{-6}	0.000329920033796224	-7.30700160251175*10^{-5}
4	-0.00012076676113611	-9.78449258085784*10^{-7}	-0.9964961311824	8.53007679536603*10^{-5}
5	-2.13155865226023*10^{-8}	1.5217615523954*10^{-8}	-0.01175492098217527	0.01313465610040798
6	6.45018411946077*10^{-7}	4.14268265555711*10^{-8}	-0.973722013017465	-0.01313466230866994
7	-5.94852097573844*10^{-8}	1.53555349866495*10^{-8}		
8	5.20787299579148*10^{-8}	-2.80143023629512*10^{-8}		

Table 2. Parameters of the transfer function of IAU2000.

i	N_i		σ_i	
	Real	Imag	Real	Imag
0	1.0000099	-3.7652854*10^{-9}		
1	-0.79952969	0.043796154	0.002594	-0.0001438546
2	0.048964919	0.0016332679	-1.00231861	2.578*10^{-5}
3	0.00029445472	-8.2328898*10^{-5}	-0.998957	0.000687
4	-1.5139223*10^{-5}	-1.1248592*10^{-6}	-0.000413499	2.8*10^{-7}

The transfer function (3) can be transformed into form (2), with four terms under the summation sign with use of the follow equations

$$R_{i+2} = \frac{(e_R - \sigma_i)}{(1 + e_R)}(1 + \sigma_i) N_0 N_i$$

$$R_1 = N_0 + \sum_i \frac{R_{i+2}}{(1 + \sigma_i)} \tag{5}$$

$$R_2(1 + e_r) = -R_1 - \sum_i \frac{R_{i+2}}{(e - \sigma_i)}$$

where e=0.0032845479 is the dynamical Earth's flattening. Such a transformation was performed to make the calculation for ZP2003 and IAU2000 similar.

With the assumption that the corrections to the main harmonics evaluated at the first stage could be determined by the uncertainties of the transfer function parameters R_i and σ_i, we tried to estimate the corrections to them. Differentiation leads to the linear model

$$dq(\sigma) = dR_1 + dR_2(1+\sigma) + \sum_{i=1}^{N} \frac{dR_{i+2}}{\sigma - \sigma_i} + \sum_{i=1}^{N} \frac{d\sigma_i R_{i+2}}{(\sigma - \sigma_i)^2} \tag{6}$$

$$d\eta(\sigma) = \eta_R(\sigma) dq(\sigma), \tag{7}$$

where N is equal to 6 for ZP2003 and 4 for IAU2000. The corrections were estimated with use of SVD LSM.

The results of modelling shows that the attempts to approximate the amplitudes corrections A_{1j}, A_{2j}, A_{3j}, A_{4j} by varying of transfer function parameters do not lead to a success. The amplitudes of the rigid Earth nutation $\eta_R(\sigma)$ decrease rapidly when their number increases, and it cannot be compensated by any selection of the transfer function parameters corrections.

This result testifies that the linear parts parts of the equations of moments in the theories IAU2000 and ZP2003 are modeled well enough and the improvement in the future is possible only by the perfection of the models of the nonlinear effects.

Spectral investigation

The series of discrepancies dε and dψ were smoothed with a one-day sampling by Panteleev filter with parameter ω_0=0.03 [4]:

$$w(t) = \frac{\omega_0}{2\sqrt{2}} e^{-\frac{\omega_0 |t|}{\sqrt{2}}} \left(\cos \frac{\omega_0 t}{\sqrt{2}} + \sin \frac{\omega_0 |t|}{\sqrt{2}} \right) \tag{8}$$

As a result smoothed series of discrepancies between the theories and observations were derived and represented by fig 2. The periodograms of these smoothed series were calculated by Fourier transformation of 4096 points and represented by fig. 3.

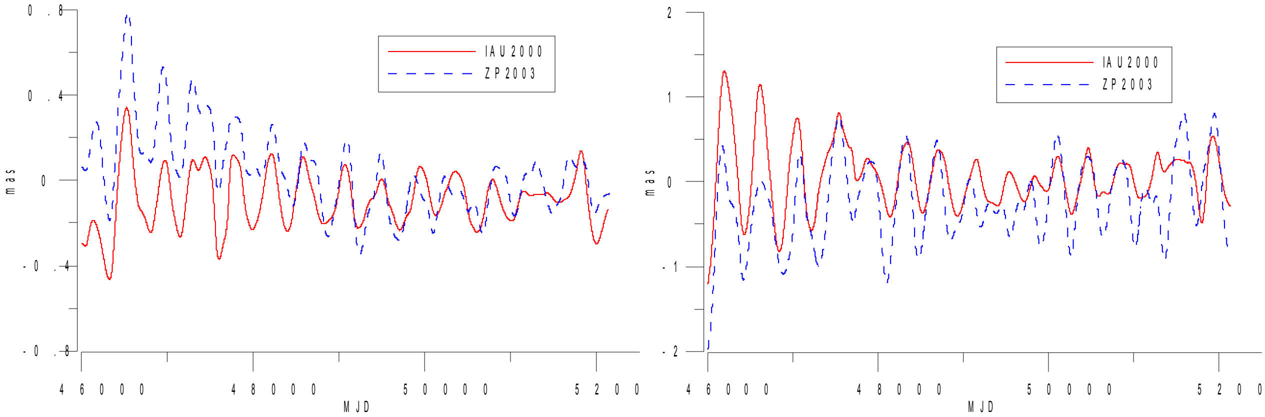

Figure 2. Comparison of smoothed discrepancies dε (left) and dψ (right) of the theories IAU2000 and ZP2003 from VLBI observations.

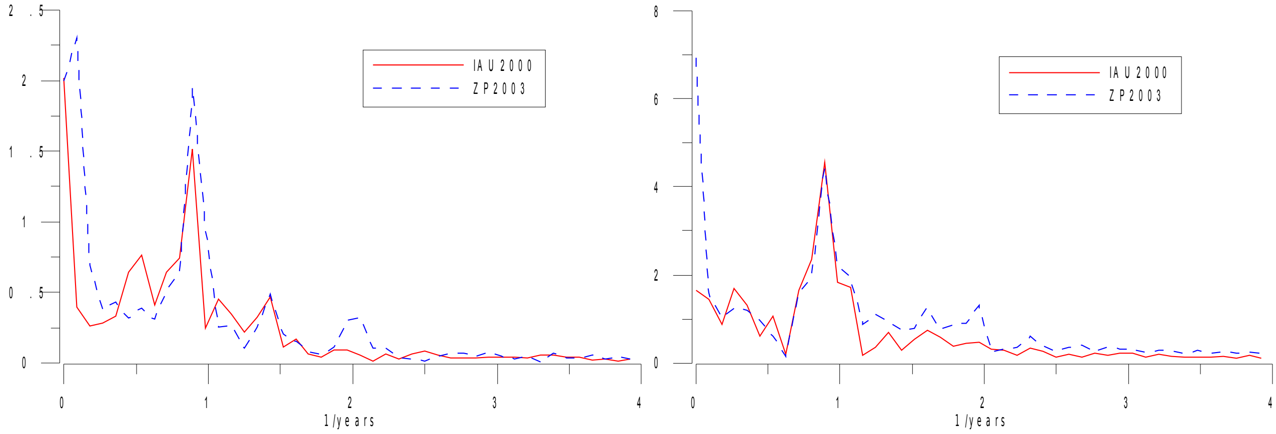

Figure 3. Periodograms of discrepancies dε (left) and dψ (right) of the theories IAU2000 and ZP2003 from VLBI observations. The frequency in years^{-1} in X-direction.

For more detailed investigation of spectral composition of the series of the nutation theories discrepancies from VLBI observations wavelet-analysis was performed. Morlet wavelet [7,8] with parameter α=10 was used. Scalograms, which illustrate the temporal evolution of the periodical components, are represented by fig. 4, their "ridges" found reflection in skeletons (fig. 5).

It can be seen that the discrepancies of the theory ZP2003 from VLBI observations contain more energy at lower frequencies, then the discrepancies of IAU2000. Also the components at semiannual frequencies present in the discrepancies of ZP2003, but absent in the discrepancies of IAU2000. The major part of the discrepancies of both theories is determined by the free core nutation, which needs to be investigated and monitored more in the future.

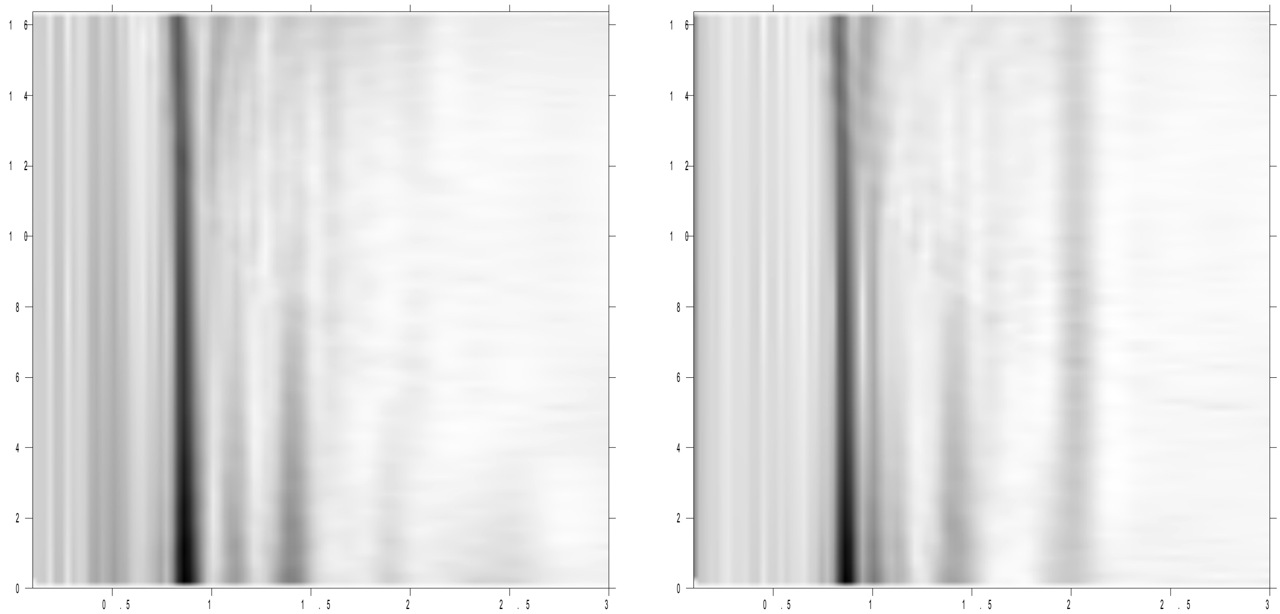

Figure 4a. Scalograms of the discrepancies of dε for IAU2000 (left) and ZP2003 (right) theories. The frequency in years^{-1} in X-direction, years since 1984 in Y-direction.

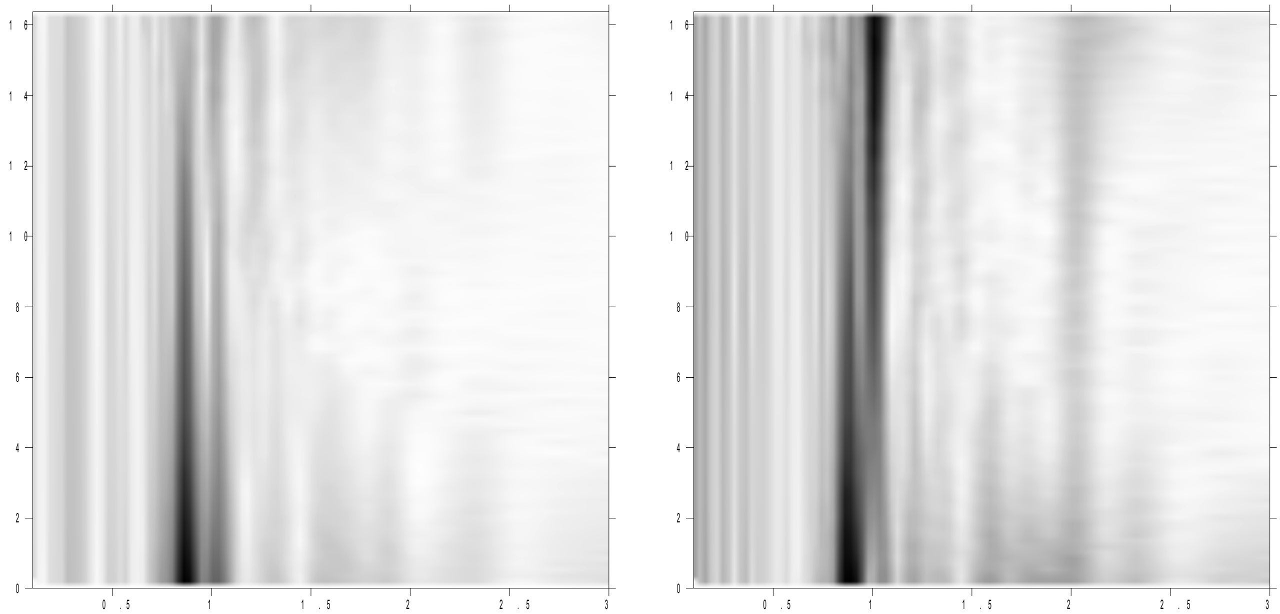

Figure 4b. Scalograms of the discrepancies of dψ for IAU2000 (left) and ZP2003 (right) theories. The frequency in years^{-1} in X-direction, years since 1984 in Y-direction.

Table 3. Corrections to the amplitudes of ZP2003 main harmonic oscillations.

j	f, years^{-1}	$\lvert A_{1j} \rvert$, mas	$\lvert A_{2j} \rvert$, mas	$\lvert A_{3j} \rvert$, mas	$\lvert A_{4j} \rvert$, mas
1	0.053726	0.150329	0.061415	0.007118	0.003692
2	2.000043	0.157420	0.081102	0.007450	0.003811

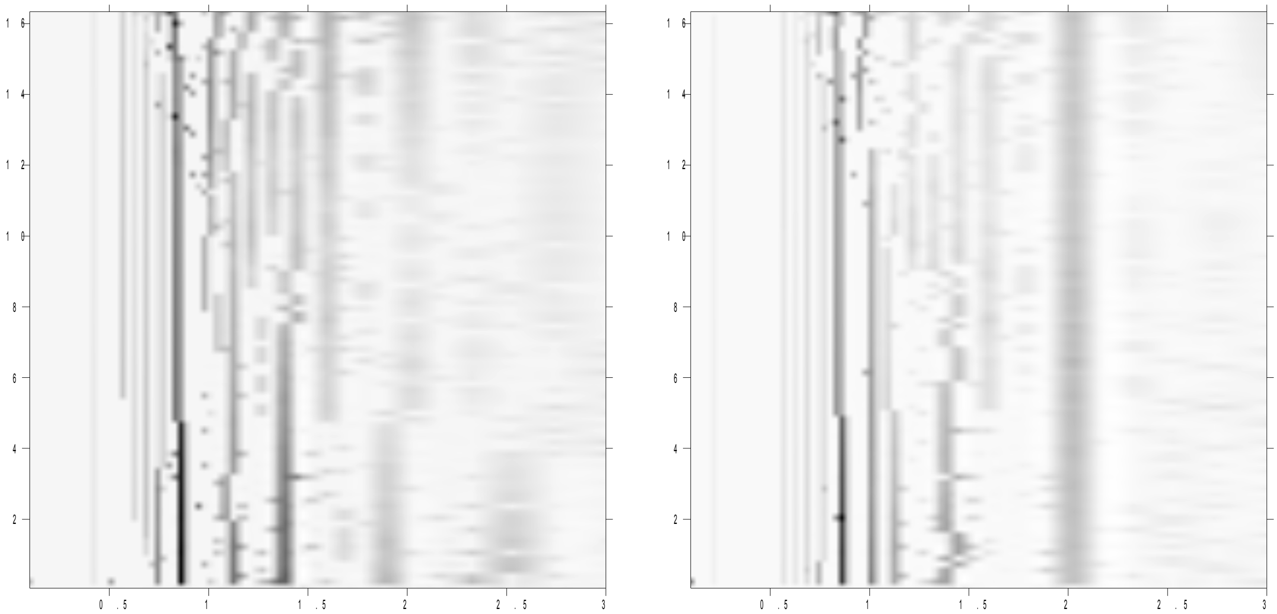

Figure 5a. Sceletons of discrepancies of dε for IAU2000 (left) and ZP2003 (right). The frequency in X-direction, years since 1984 in Y-direction.

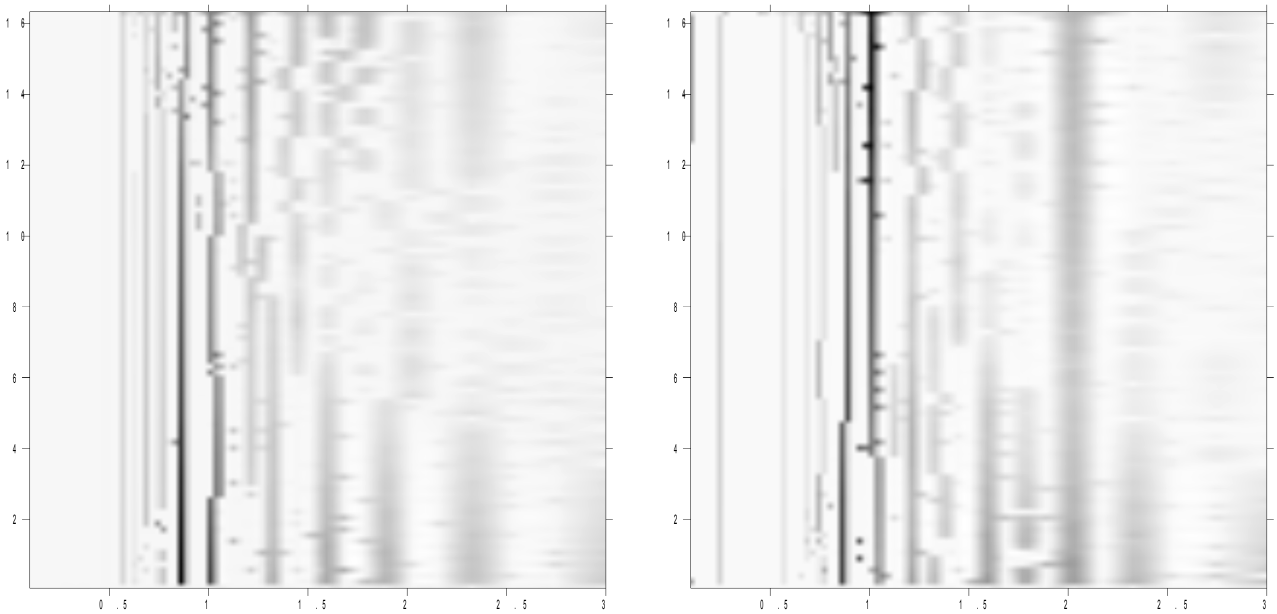

Figure 5b. Scalograms of discrepancies of dψ for IAU2000 (left) and ZP2003 (right). The frequency in X-direction, years since 1984 in Y-direction.

Discussion, ZP2003 model with corrections

The correction at the main nutation frequency 18,6-year can be connected with the problem of separation of harmonic oscillations, which have close frequencies.

The existence of correction at semiannual frequency has to be discussed in details. The IAU2000 theory doesn't include the atmosphere correction at this frequency, it was not estimated in it [1]. It was suggested by authors of IAU2000, that an unknown process at this frequency compensates the influence of atmosphere. From fig 4 and 5 it can be seen, that the residual energy at this frequency absents in the discrepancies of IAU2000 and presents in the discrepancies of ZP2003, where the atmosphere was taken in account in the equations of momentum, but no other unknown processes was. If such an unknown process exists, the fact that it was not modeled in ZP2000 should bring to the appearance of a peak at semiannual frequency in fig. 4 and 5. So the spectral investigation proves the existence of an unknown process, which compensates the atmosphere influence at semiannual frequency.

The agreement of ZP2003 nutation theory with observations can be improved by introduction the corrections at semiannual and 18.6-year periods. These corrections were estimated with use of SVD LSM (table. 3). After they were taken in account the agreement of ZP2003 became better and weighted mean square deviations of discrepancies reach the level of 207 ($d\varepsilon$) and 808 ($d\psi$) μas. Figure 6 represents for comparison the periodograms of the discrepancies before and after the introduction of corrections to 2 main harmonic oscillations.

The estimation of corrections to 300 first harmonics was also performed. After these corrections were taken in account weighted mean square deviations of discrepancies became 146 ($d\varepsilon$) and 345 ($d\psi$) μas. The periodograms are represented by fig 7 for comparison, rhombs represent the frequencies corrections were estimated at.

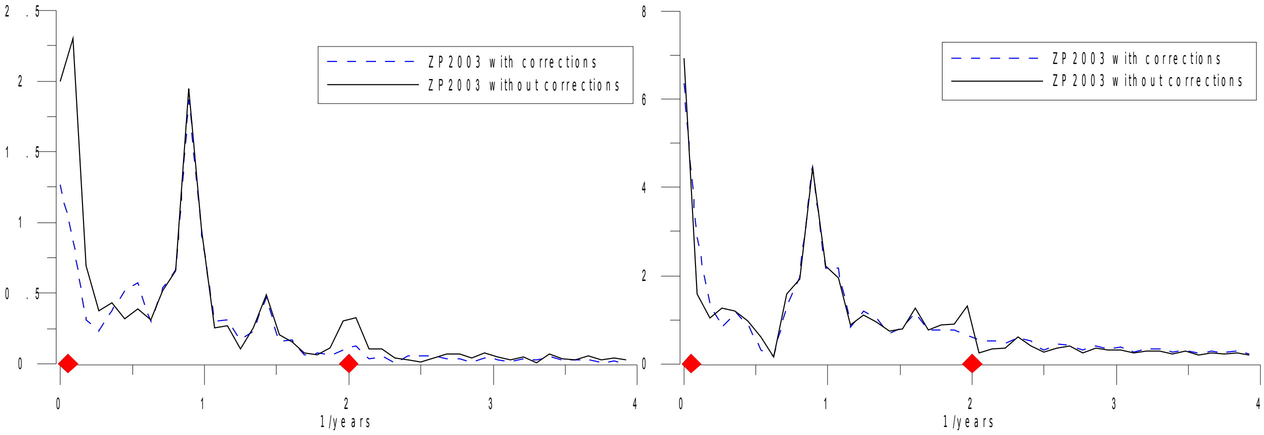

Figure 6. Periodograms of discrepancies $d\varepsilon$ (left) and $d\psi$ (right) of the theory ZP2003 from VLBI observations before and after the corrections to 2 main frequencies were taken in account.

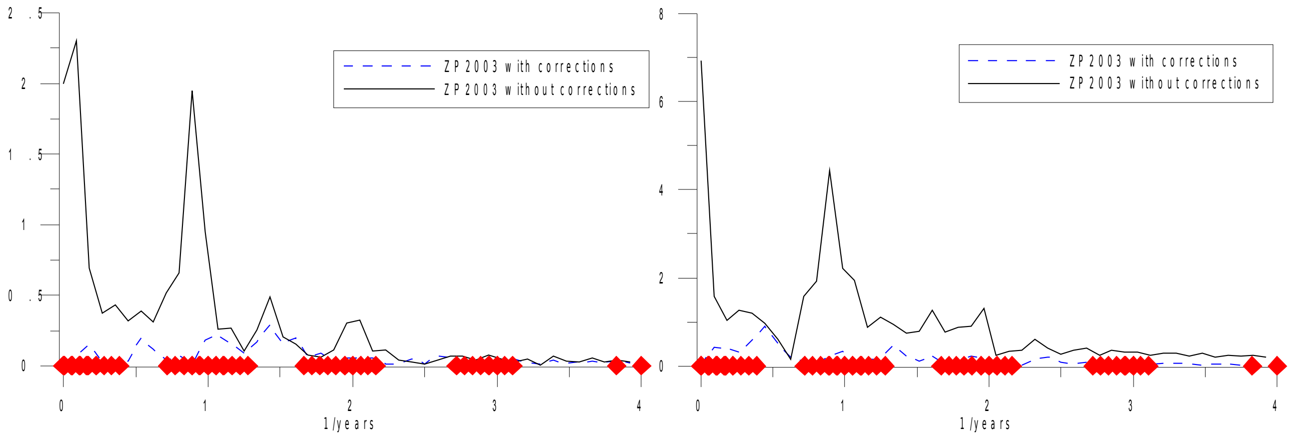

Figure 7. Periodograms of discrepancies $d\varepsilon$ (left) and $d\psi$ (right) of the theory ZP2003 from VLBI observations before and after the corrections to 300 first (by energy) frequencies were taken in account.

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