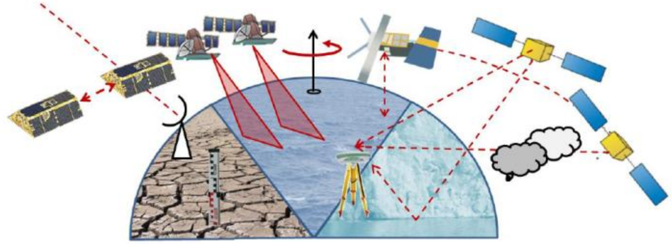

Regularized Kalman Filter in application to the Chandler wobble of the pole

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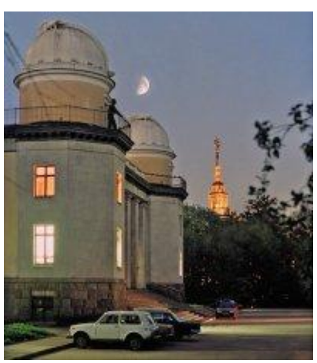


ICCC workshop
“Geodesy for climate research”
29 March 2023

Kalman filter as leas squares

Let's start from observation equation

$$y = Cx + r$$

where y is N-dimensional, x is n-dimensional vectors and recurrent state equation

$$x_{k+1} = Ax_k + Bu_k$$

with A – $[n \times n]$, B – $[n \times m]$ matrices, u_k is m-dimensional input white noise. We would requite

$$||y - Cx|| \to min \tag{5}$$

and

$$||\hat{x}_{k+1} - A\hat{x}_k|| \to min \tag{6}$$

Kalman filter as leas squares

allows to write the matrix system for this optimisation problems in the form of least squares

$$\begin{bmatrix} A\hat{x}_k \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} I \\ C_{k+1} \end{bmatrix} x_{k+1} + \begin{bmatrix} -A\epsilon_k - Bu_k \\ r_{k+1} \end{bmatrix}$$

Let's rewrite above equation using new h, H, s notations for vectors and matrices

$$h = Hx + s$$

Least squares solution is

$$\hat{x}_{k+1} = (H^T S H)^{-1} H^T S h$$

with weight matrix

$$S = \begin{Bmatrix} D^{*-1}_{k+1} & 0 \\ 0 & Q^{-1} \end{Bmatrix}$$

where inverted covariance matrices were used

$$D^*_{k+1} = AD_\epsilon A^T + BD_u B^T$$

$$Q = D_r = \sigma_0^2 I$$

$$H^T S H = D^{-1}_{k+1} = D^{*-1} + \sigma_0^{-2} C^T C$$

$$H^T S h = D^{-1}_{k+1} = D^{*-1}\hat{x}^*_{k+1} + \sigma_0^{-2} C^T y_{k+1}$$

then

$$\hat{x}_{k+1} = \left(D^{*-1} + \sigma_0^{-2} C^T C\right)^{-1} \left(D^{*-1}\hat{x}^*_{k+1} + \sigma_0^{-2} C^T y_{k+1}\right) \tag{7}$$

Regularization

Imagine that matrix C of observational equation is rank-deficient or close to this, then the normal matrix will be ill-posed and LS solution ill-conditioned.

Tykhonov regularization approach recommends to search for normal pseudo-solution by minimizing

$$L_{Tykhonov} = ||y - Cx|| + \alpha||x|| \to min \qquad (11)$$

For least squares

$$\hat{x} = (C^T PC + \alpha I)^{-1} C^T Py$$

$$D_{\hat{x}} = (C^T PC + \alpha I)^{-1}.$$

For recurrent least squares

$$\hat{x}_{N+1} = (C^T PC + a^T pa + \alpha I)^{-1}(C^T Py + a^T py_{N+1}) \qquad (13)$$

denoting

$$D_N^{-1} = (C^T PC + \alpha I), \qquad D_{N+1}^{-1} = (C^T PC + a^T pa + \alpha I)$$

Sequential estimate does not change

$$\hat{x}_{N+1} = \hat{x}_N + Kp^{-1}(y_N - a\hat{x}_N).$$

For covariance matrix update we obtain (see above)

$$D_{N+1} = D_N - K\frac{a}{\sqrt{p}} D_N$$

Kalman filter regularization

$$\hat{x}_{k+1} = (H^T SH + \alpha I)^{-1} H^T Sh = \left(D^{*-1} + C^T Q^{-1} C + \alpha I\right)^{*-1} \left(D^{*-1}\hat{x}^*_{k+1} + C^T Q^{-1} y_{k+1}\right) \quad (17)$$

Recursive expansion of

$$K = D^{reg}_{k+1} C^T Q^{-1}$$

gives

$$K = (Q + CD^* C^T)^{-1}(D^* C^T - \alpha DD^* C^T). \qquad 18$$

which only under some simplifications can be brought to the wildly-used approach 19

Observational error covariance matrix Q can play a role
of regularization parameter.
So, we finally use this approach.

$$x_k = x_k^* + K_k\left(y_k - C_k x_k^*\right),$$

$$K_k = D_k^* C_k^T \left[C_k D_k^* C_k^T + Q_k + \alpha I\right]^{-1}, \qquad 19$$

$$D_k = D_k^* - K_k C_k D_k^*,$$

$$x_{k+1}^* = A_k x_k.$$

3.1 Tikhonov Regularized Kalman Filter and Its Algorithm

The objective function of TRKF is designed as follows

$$\Psi(\widehat{X}_k^{TRKF}) = V_{\widehat{x}_k^{TRKF}}^T R_k^{-1} V_{\widehat{x}_k^{TRKF}} + V_{\overline{x}_k^{TRKF}}^T P_{k/k-1}^{-1} V_{\overline{x}_k^{TRKF}} + \alpha_k Z_k^T H_k Z_k \tag{14}$$

where $\alpha_k>0$ is the regularization parameter, $H_k>0$ is the regularization matrix and $Z_k = \widehat{X}_k^{TRKF} - \widehat{X}_{k/k-1}$.

Using the weighted least square method, we can get

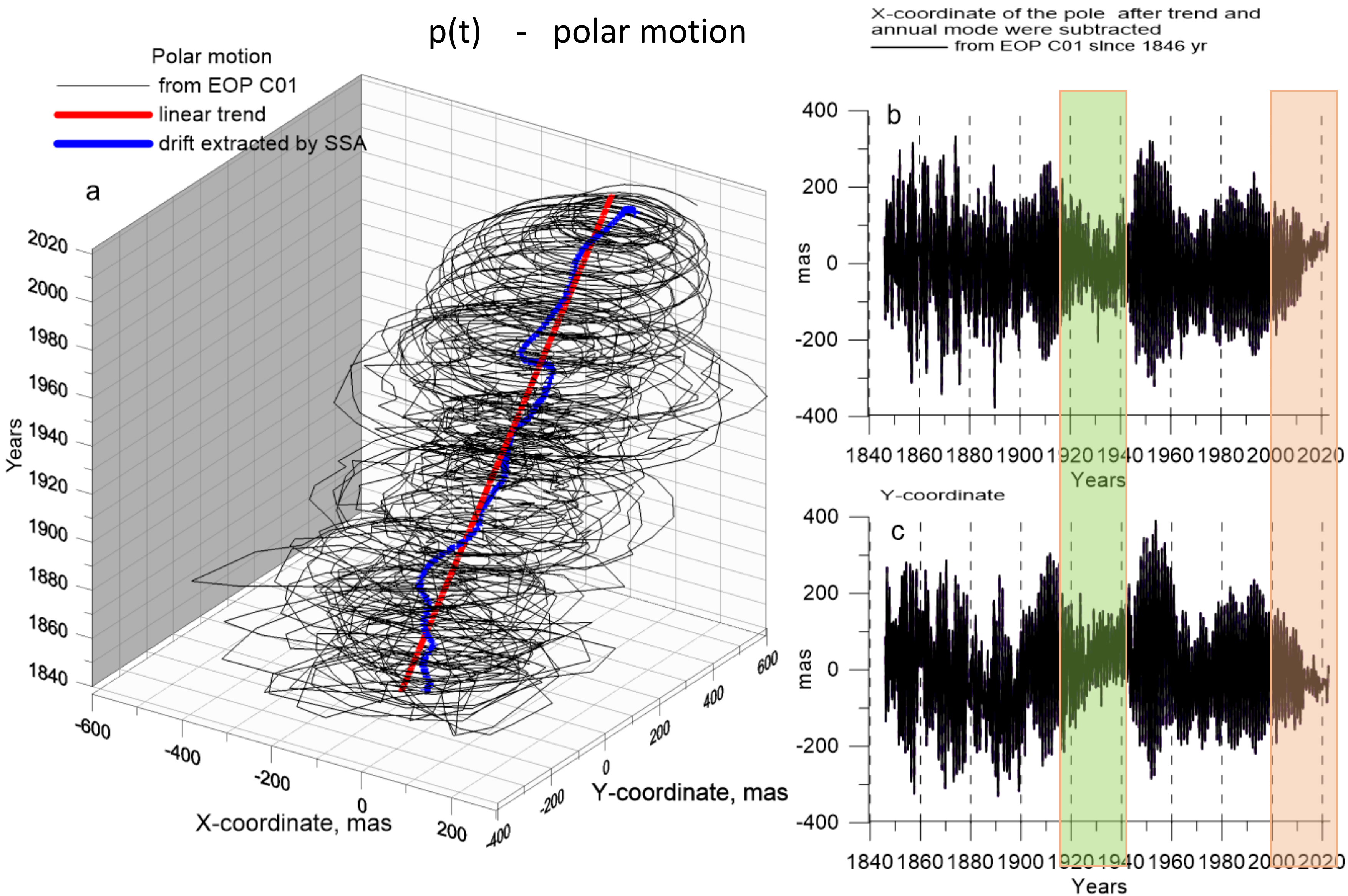
$$K_k^{TRKF} = (A_k^T R_k^{-1} A_k + P_{k/k-1}^{-1} + \alpha_k H_k)^{-1} A_k^T R_k^{-1} \tag{15}$$

$$\widehat{X}_k^{TRKF} = \widehat{X}_{k/k-1} + K_k^{TRKF}(L_k - A_k \widehat{X}_{k/k-1}) \tag{16}$$

$$P_k^{TRKF} = (I - K_k^{TRKF} A_k) P_{k/k-1} \tag{17}$$

Polar motion since 1846 from EOP C01 bulletin

p(t) - polar motion



Zotov L.V. et al., Moscow University Physics Bulletin, N3, 2022

DYNAMICAL MODEL OF THE POLAR MOTION

$$\frac{i}{\sigma_c}\frac{dp(t)}{dt}+p(t)=\chi(t)$$

$$p=p_1+ip_2$$

$$\chi=\chi_{mass}+\chi_{motion}$$

$$\sigma_c=2\pi f_c(1+i/2Q)$$

$$f_c=\frac{1}{433}\ \text{days}^{-1}\qquad Q=100$$

In time domain

$$\frac{-\omega\hat{p}(\omega)}{\sigma_c}+\hat{p}(\omega)=\hat{\chi}(\omega)$$

$$\frac{\sigma_c-\omega}{\sigma_c}\hat{p}(\omega)=\hat{\chi}(\omega)$$

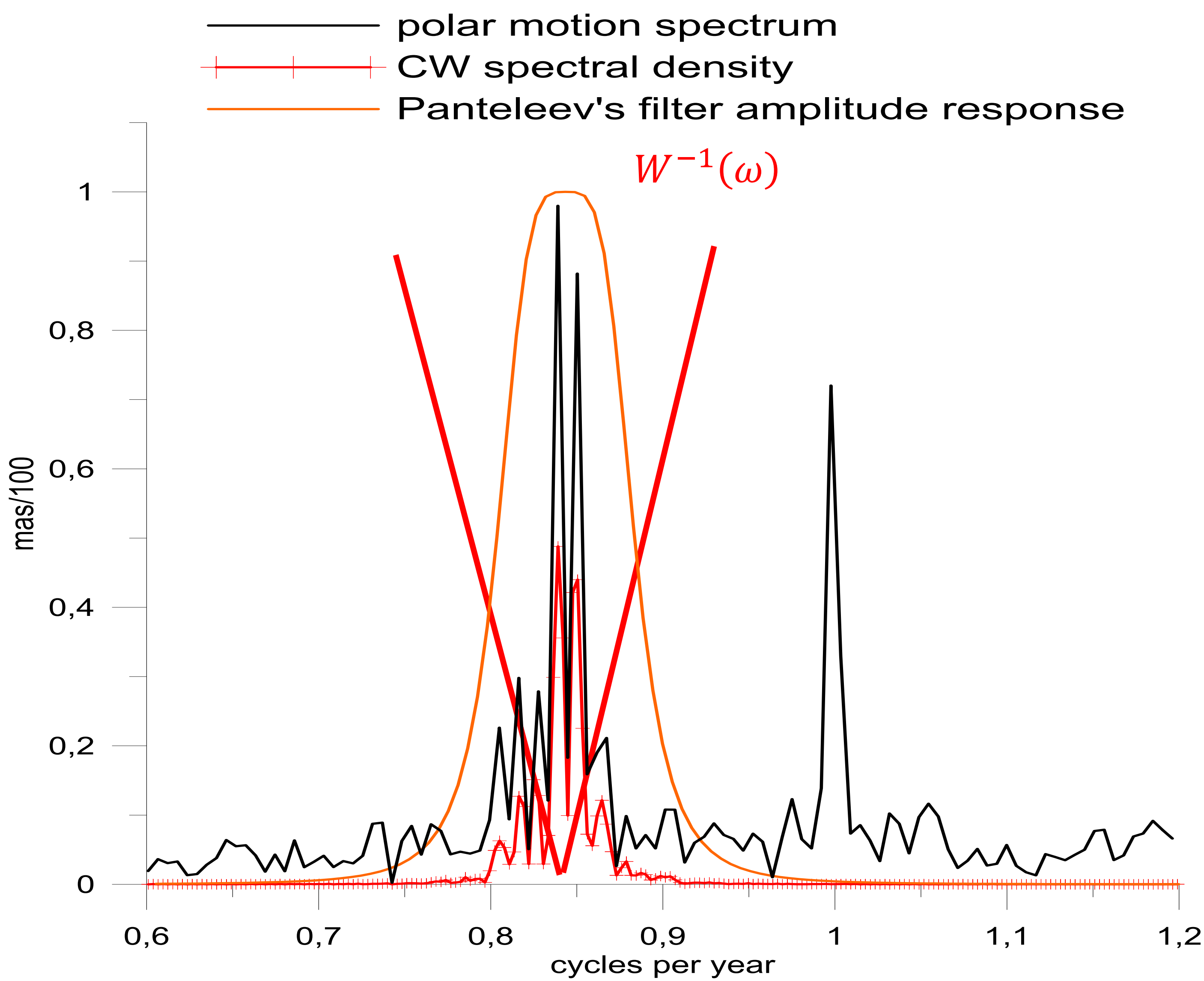
$$\hat{p}(\omega)=W(\omega)\,\hat{\chi}(\omega)$$

$$W(\omega)=\frac{\sigma_c}{\sigma_c-\omega}$$

In frequency domain

Munk W.H., MacDonald G.J.F., The rotation of the Earth, 1960

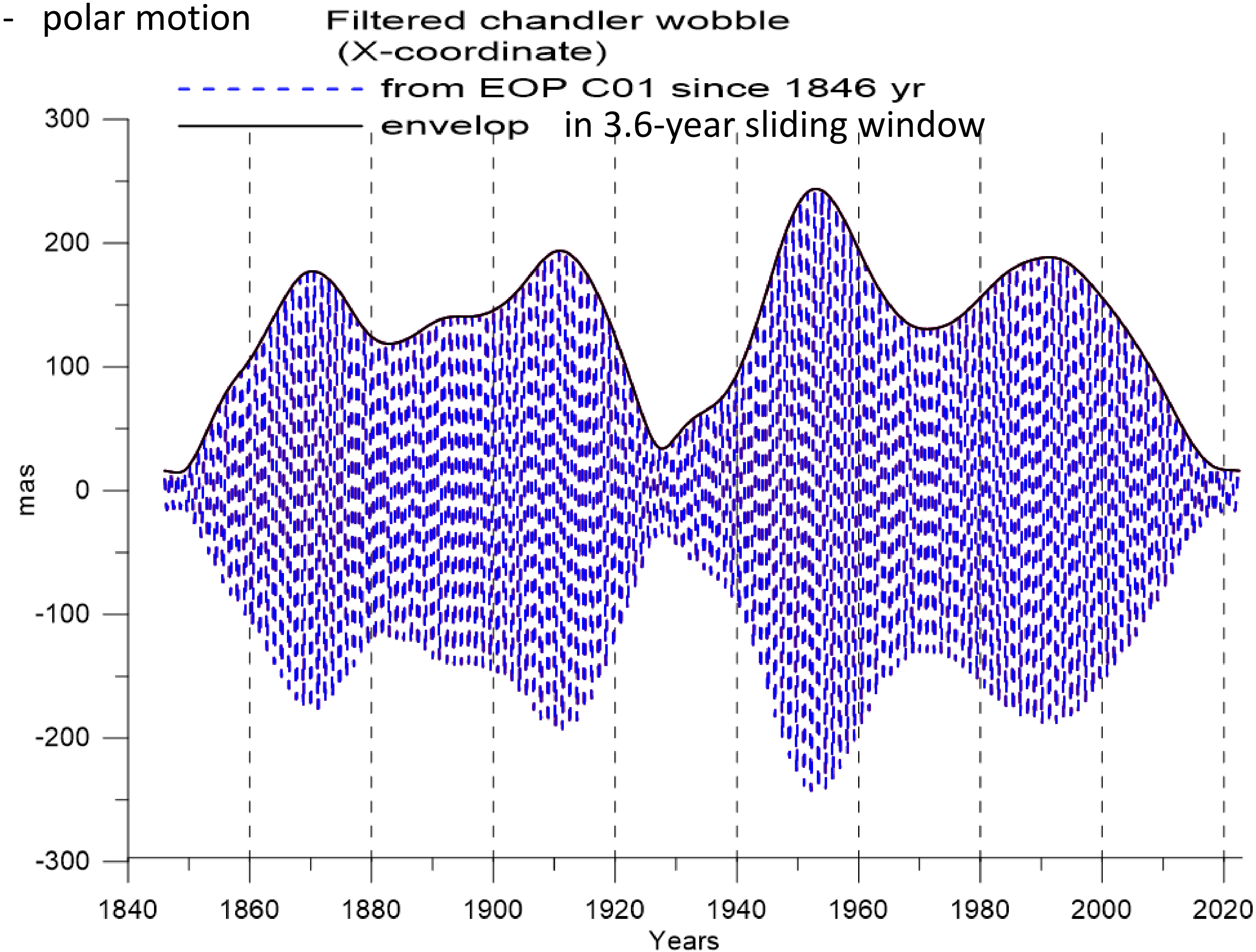
Polar motion spectrum and Panteleev's corrective filtering



Zotov L.V. et al., Moscow University Physics Bulletin, N3, 2022

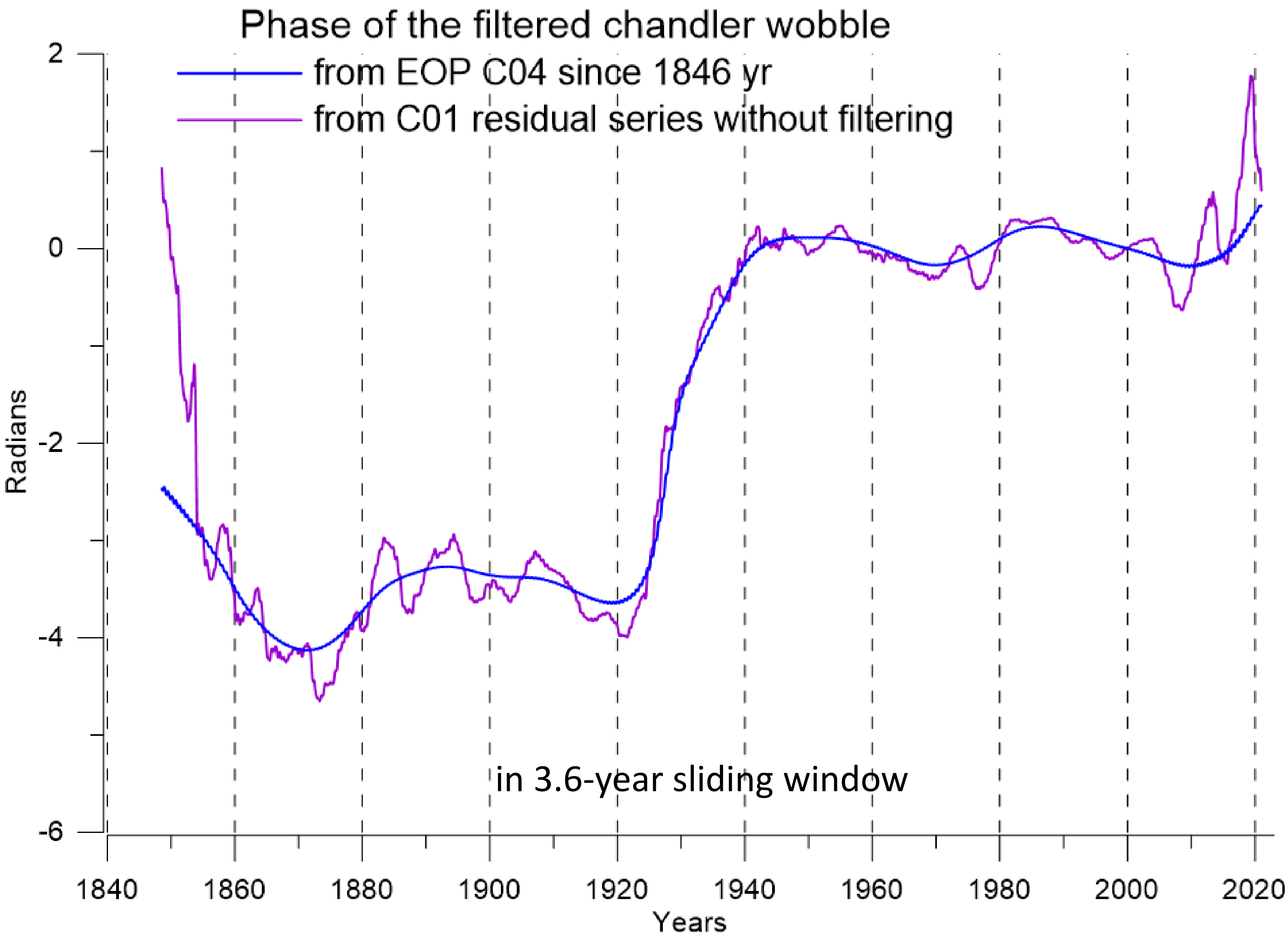
Changes of the Chandler wobble amplitude

$p_1(t)$ - polar motion



Zotov L.V. et al., Moscow University Physics Bulletin, N3, 2022

Changes of the Chandler wobble phase



Zotov L.V. et al., Moscow University Physics Bulletin, N3, 2022

Matrix form of Euler-Liouville equations for the polar motion

State equation with coordinates of the pole $p_1 = x$ (along Greenwich) and $p_2 = -y$ (meridian 90° East) can be given in the form

$$D\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} -\beta & -\alpha \\ \alpha & -\beta \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} \beta & \alpha \\ -\alpha & \beta \end{bmatrix}\begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix},$$

Where α, β – are real and imaginary parts of complex-valued chandler frequency $\sigma_c = \alpha + i\beta = 2\pi f_c + i\frac{\pi f_c}{Q}$, Q –quality factor (≈100), $f_c = 1/433$ day^{-1} – real Chandler frequency, $D =$

$D = \frac{d}{dt}$ – operator of differentiation,

χ_1, χ_2 – components of the input excitation, *i* – imaginary unit.

$x = [p_1, p_2, \chi_1, \chi_2]^T$ - ?

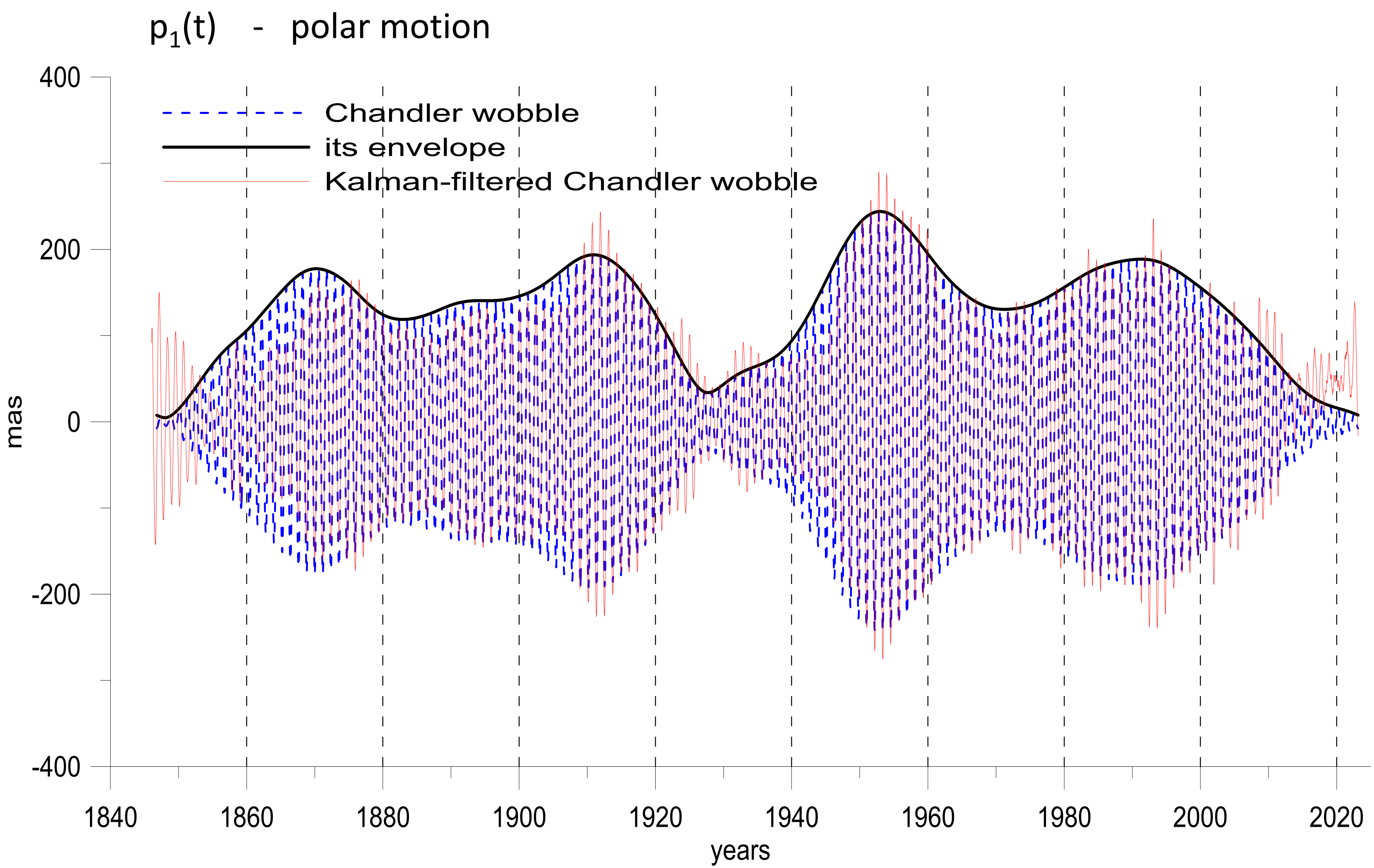
The problem of input excitation reconstruction is inverse

$$\begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} = \begin{bmatrix} \beta & \alpha \\ -\alpha & \beta \end{bmatrix}^{-1}\left(DI - \begin{bmatrix} -\beta & -\alpha \\ \alpha & -\beta \end{bmatrix}\right)\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = W^{-1}(D)\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

It can be corrected through regularization

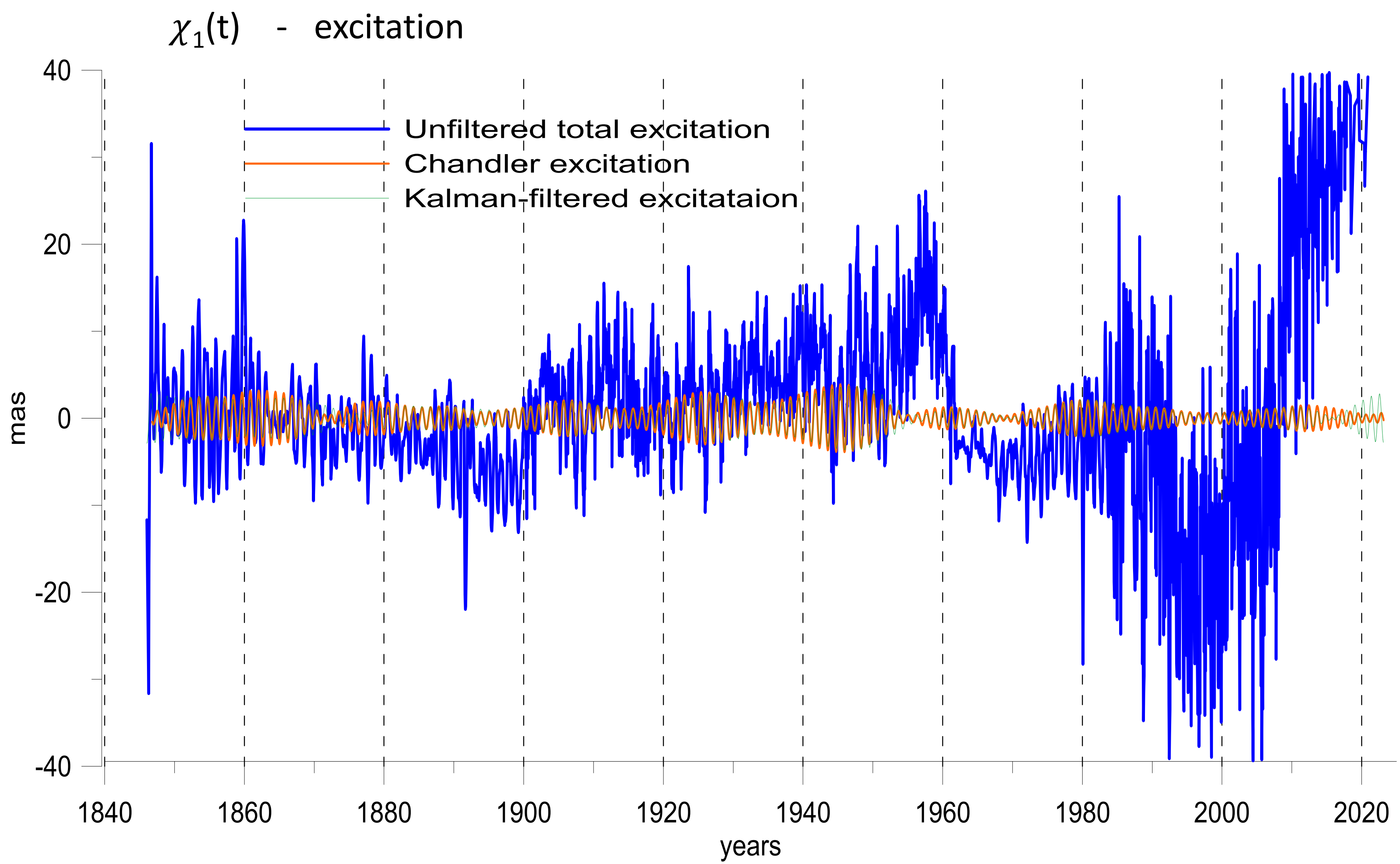
$$W_{corr}(f)W^{-1}(f) = \frac{W^*(f)}{W^*(f)W(f) + \alpha I},$$

Kalman-Filtered Chandler wobble



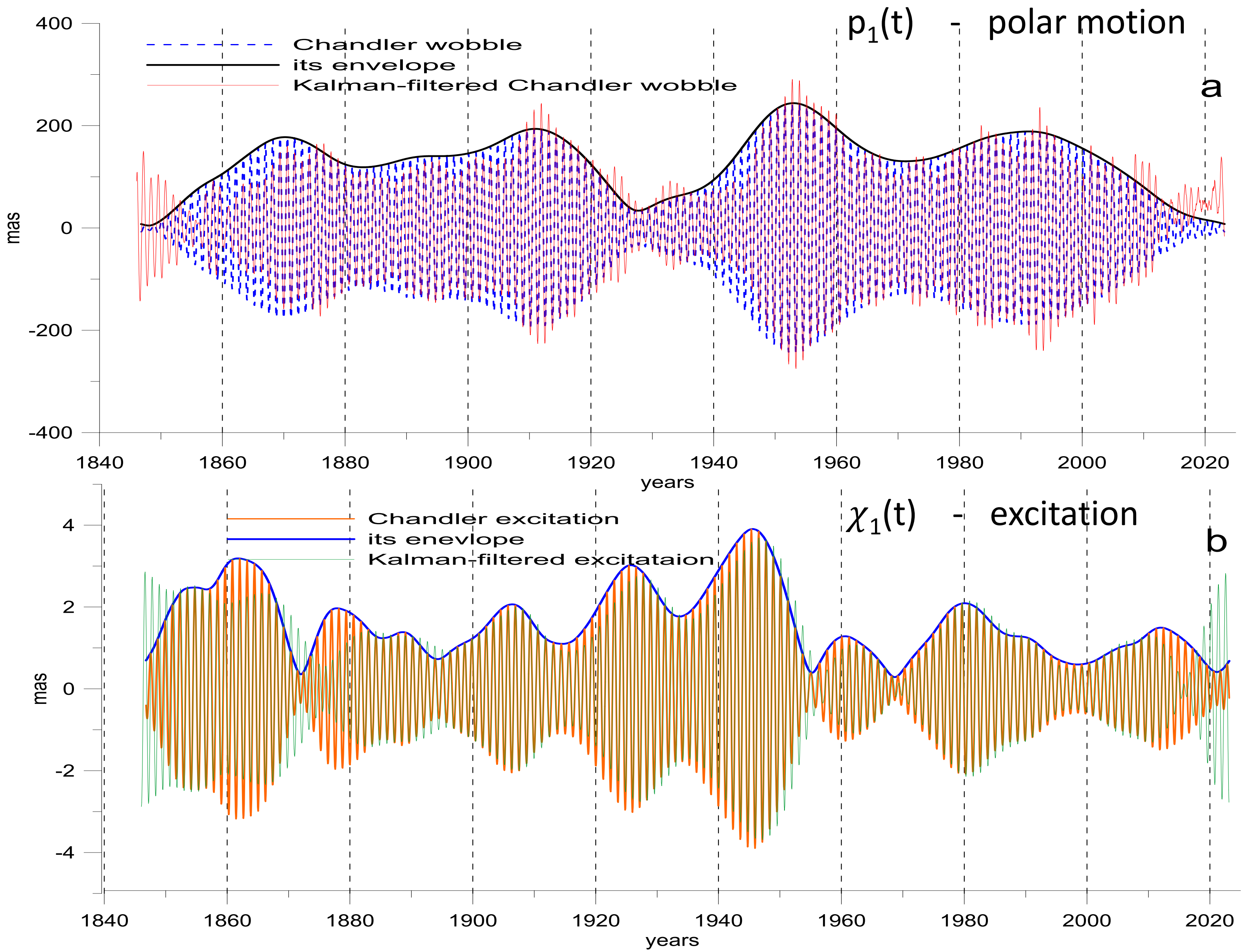
Zotov L.V. et al., 2023

Unfiltered total excitation



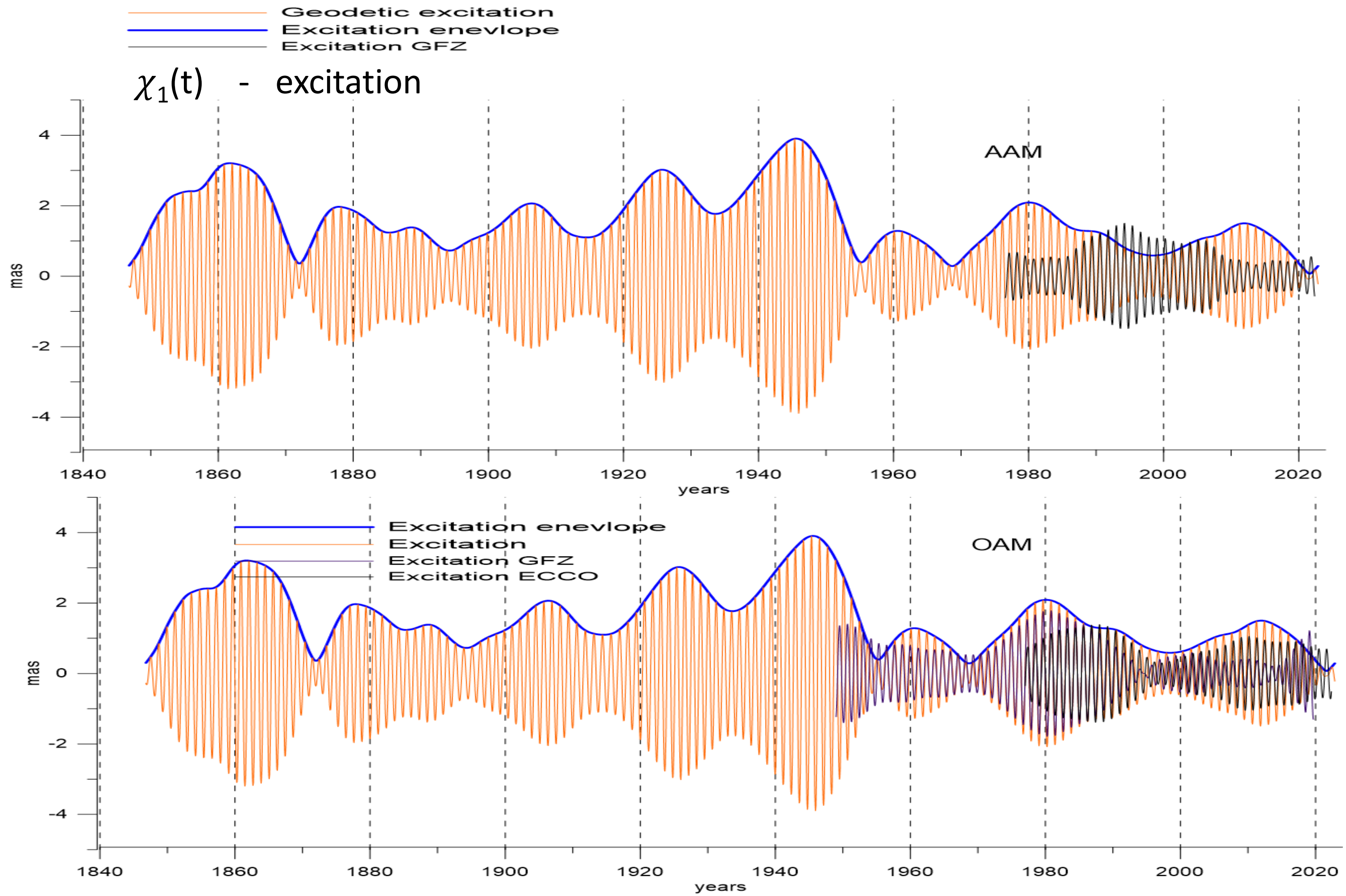
Zotov L.V. et al., 2023

KF - Chandler wobble and its filtered excitation



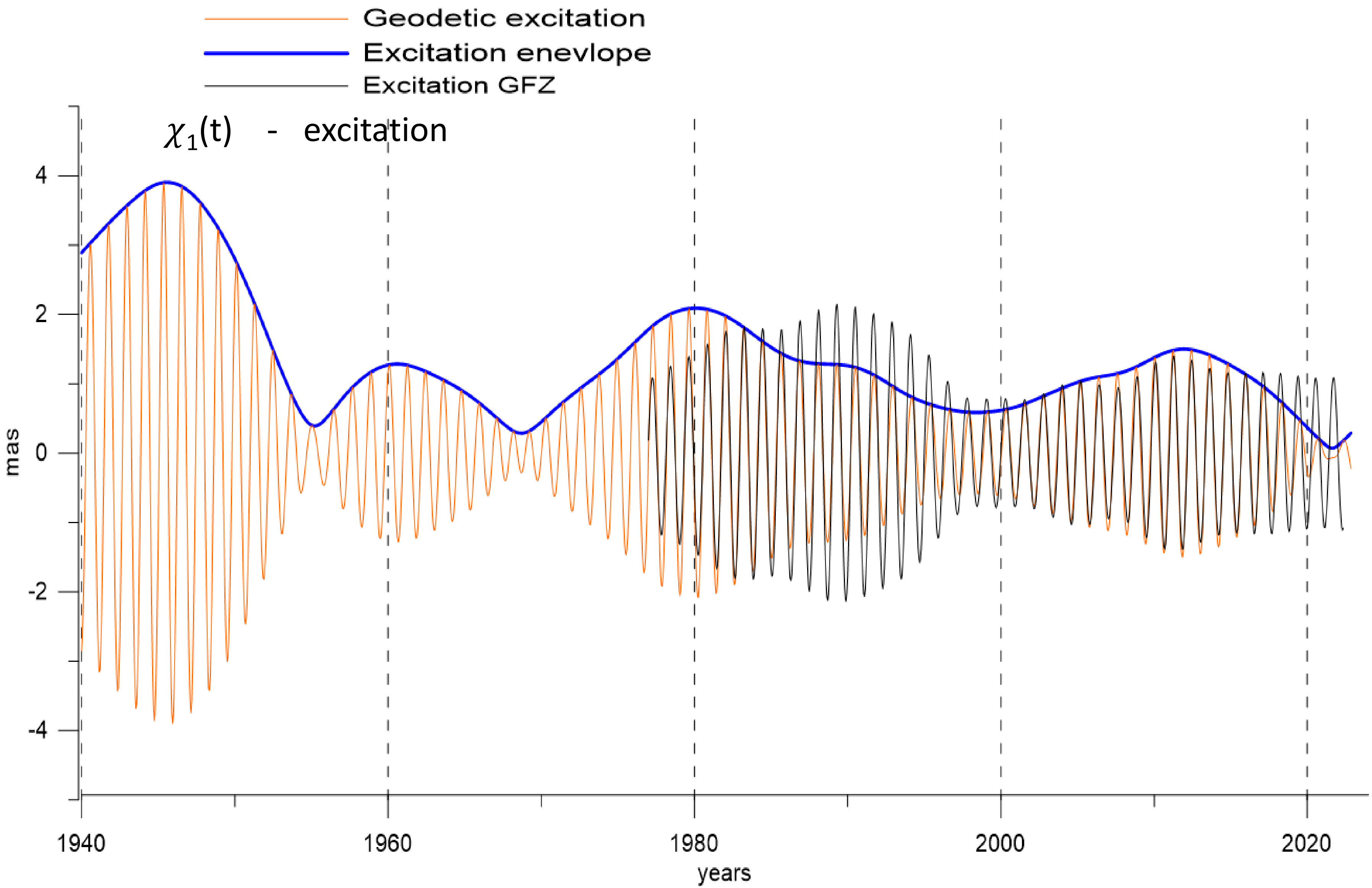
Zotov L.V. et al., 2023

Chandler excitation compared with atmospheric and oceanic OAM and AAM (GFZ)



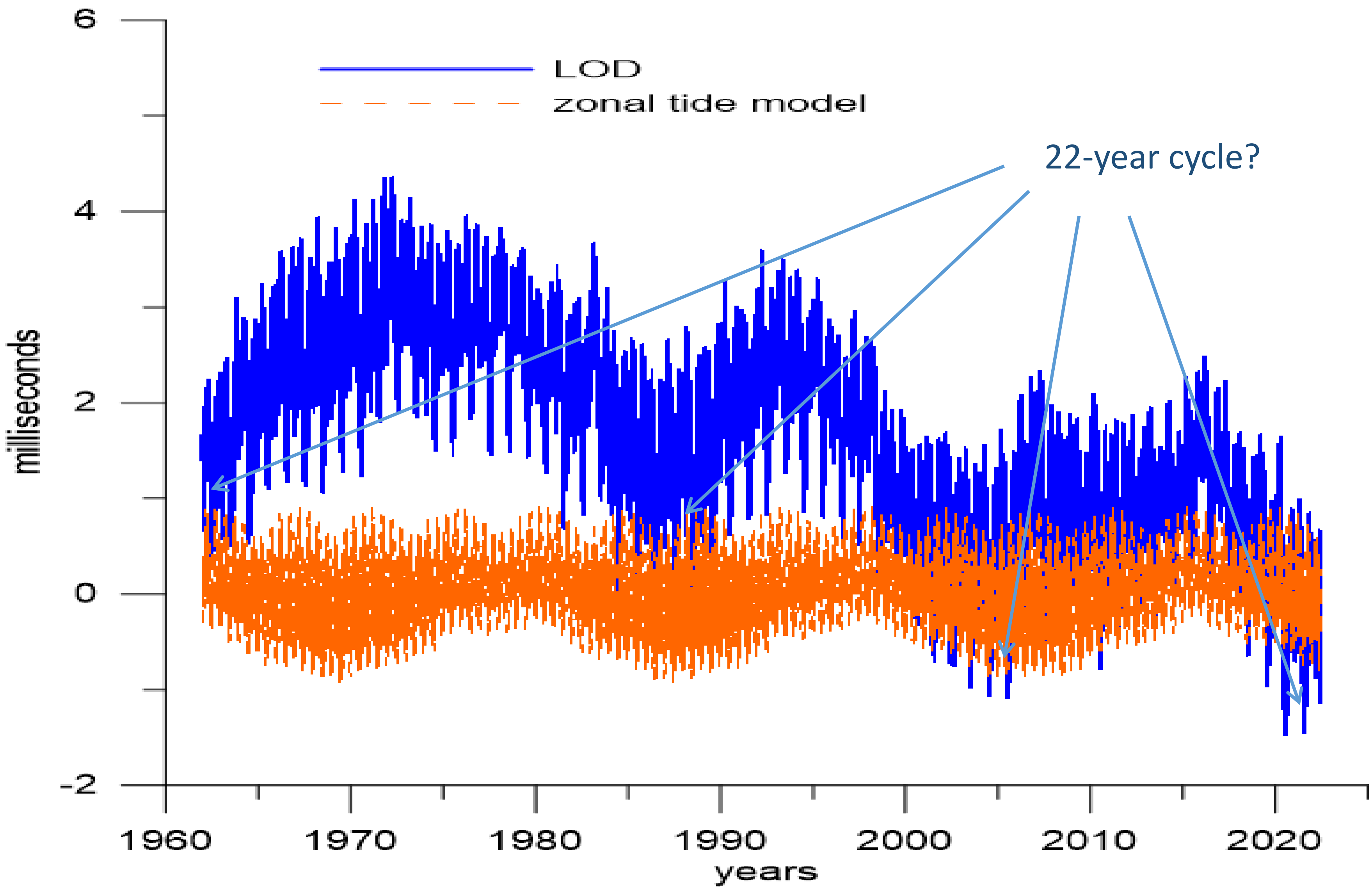
Zotov L.V. et al., 2023

Comparison of geodetic and geophysical AAM+OAM



Zotov L.V. et al., 2023

Length of day (LOD) from EOP C04 and zonal tides



Zotov L.V. et al., 2023

GEODESY AND GEODYNAMICS 2016, VOL X NO X, 1–7

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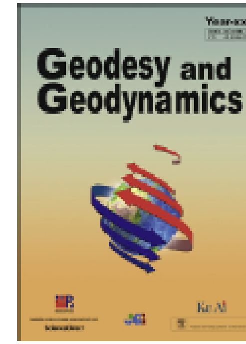

A possible interrelation between Earth rotation and climatic variability at decadal time-scale

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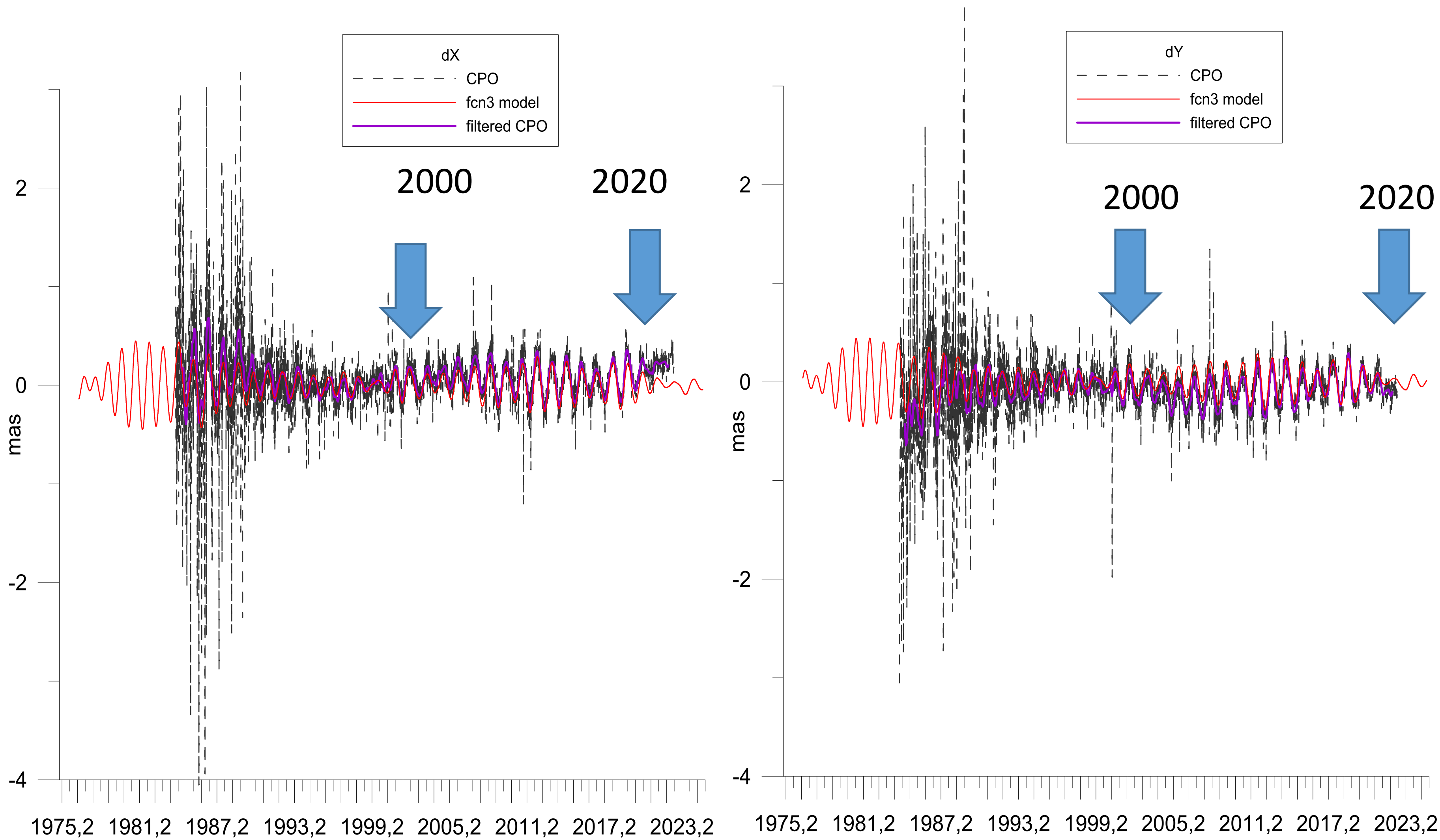
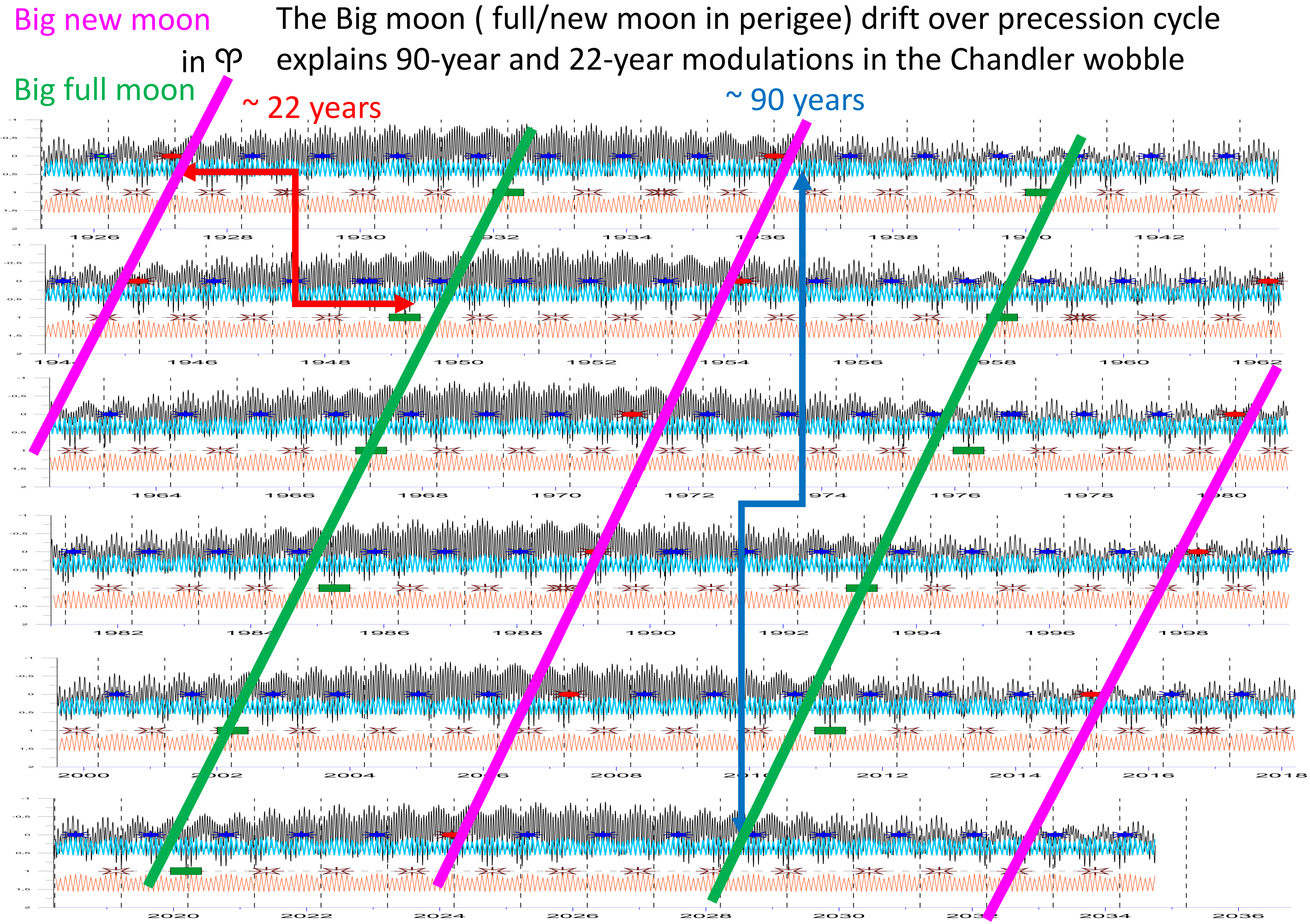
Free core nutation disappeared?



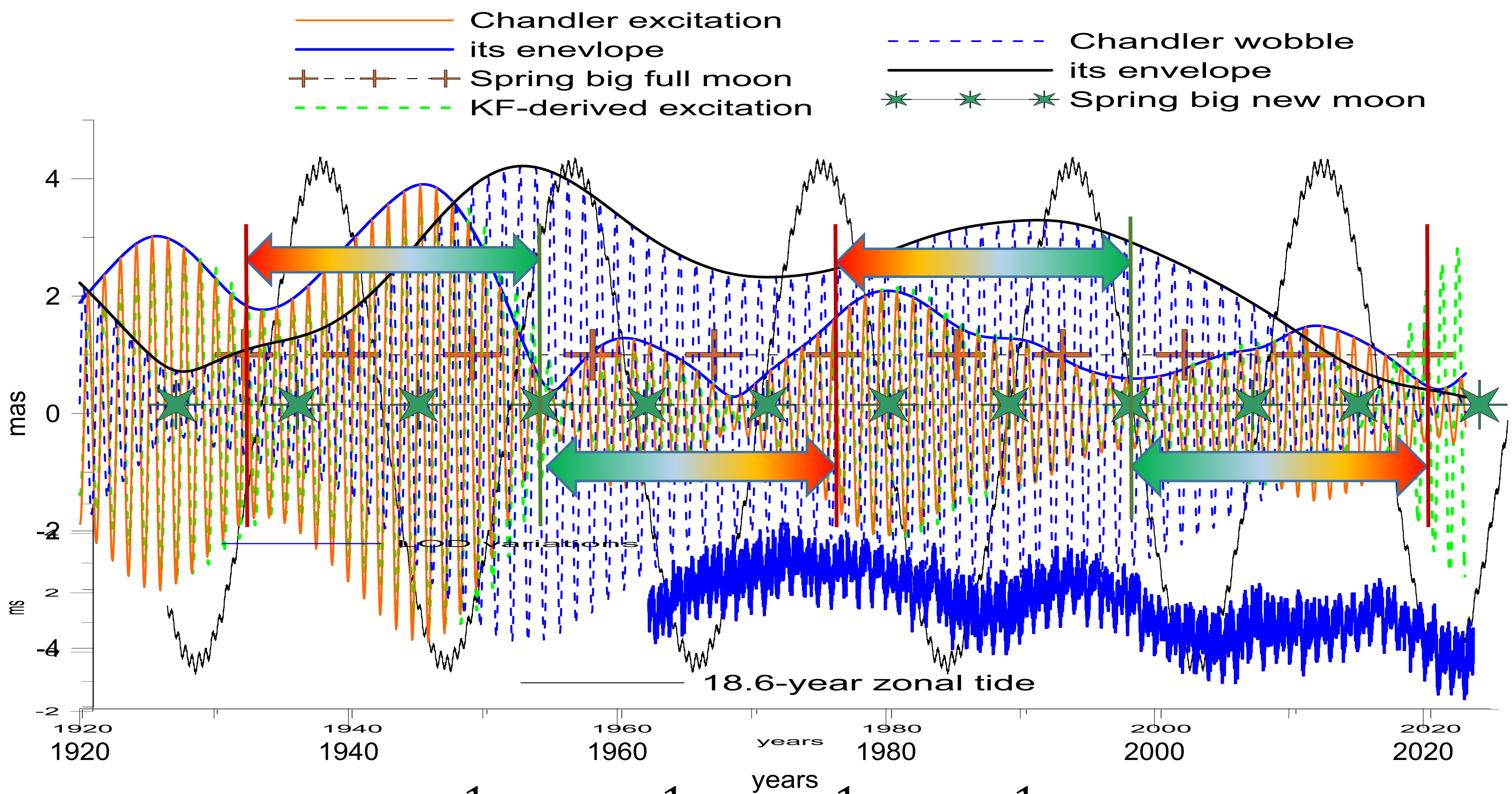
Zotov L.V. et al., 2023

The Big moon (full/new moon in perigee) drift over precession cycle explains 90-year and 22-year modulations in the Chandler wobble



Zotov L.V. et al., ICCC workshop, 29 March 2023

Chandler wobble modulations can be explained by the Moon

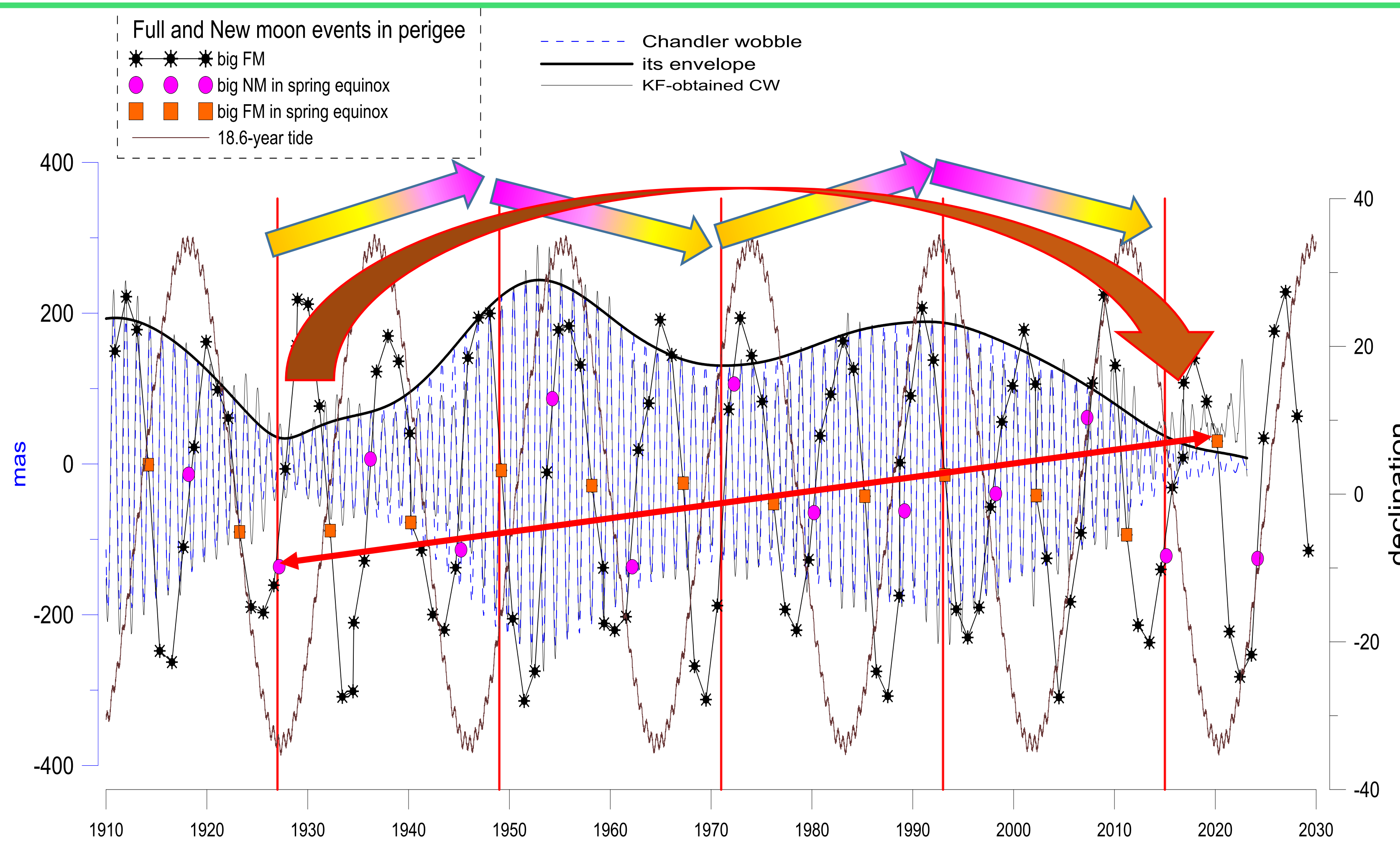


$$\frac{1}{Tchandler} = \frac{1}{412} - \frac{1}{22 * 365} = \frac{1}{433}$$

Zotov L.V. et al., ICCC workshop, 29 March 2023

Ideas about big moon from Yu Avsyuk and tidal cycle from N. Sidorenkov

Chandler wobble is excited by the series of syzygies occurring at different declinations



* - big full moon

Zotov L.V. et al., ICCC workshop, 29 March 2023

Conclusions

- We tried to develop regularized Kalman filter procedure to extract Chandler wobble, we see that is disappeared in 2017-2020 and now appeared again, its phase is now changing like in the 1930-s
- Atmospheric angular momentum AAM, oceanic angular momentum OAM, hydrological angular momentum and Sea Level changes can not quite explain acceleration of the Earth since 2016 and Chandler wobble disappearing in 2020-s
- If it is a mass term, we would see it in the gravity field
- Decadal LOD changes can be related to the oscillations in the core and/or lunar tide influence on geophysical processes
- Free Core Nutation disappeared as well
- Earth rotation velocity and Chandler wobble changes can be produced by the regular mechanism which we believe is related to the drift of the Big Moon events over the precession cycle

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Thank you for attention!