# Panteleev's corrective filtering as the regularizing algorithm for the Earth's Chandler wobble excitation reconstruction

AGU Abstract 362235
G 31B-0658
12.12.2018 8-12 a.m.

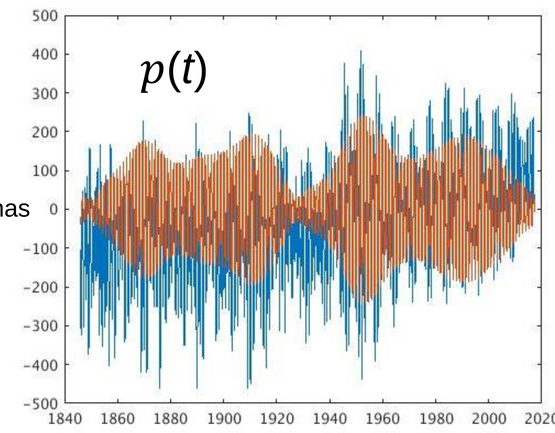
**Leonid Zotov[1,2] (wolftempus@gmail.com), Chaoyang Zhang[3]**

[1]Moscow Institute of Electronics and Mathematics (MIEM) National Research University Higher School of Economics, Russia
[2]Sternberg Astronomical Institute, Lomonosov Moscow State University, Russia, [3]School of Earth sciences, Ohio State University, USA

**Abstract:** We use Panteleev's filtering to extract Chandler wobble (CW) of the Earth's pole. Modelling demonstrates, that it can be extracted from the 50 mas noise with ~1.5% precision. If Euler-Liouville equation gives direct operator, to get the excitation of the resonant CW we need to apply inverse operator. Quite a small excitation maintains resonant Chandler wobble. Thus its recovery is an ill-posed inverse problem. According to Tikhonov [1], regularizing algorithm should converge to the exact solution and give unbiased solution when the observational noise and operator error tend to zero. Panteleev's filtering can be used as a stabilizer, which helps to reject high and low frequencies in spectral domain, where direct operator amplitude response is small. For excitation we obtain the one-parametric family of solutions, with a filter width as parameter. Using the model of the Chandler excitation, modifying operator and noise, we show, that the optimal filter parameter is $f_0$~1/25 year$^{-1}$ [2].

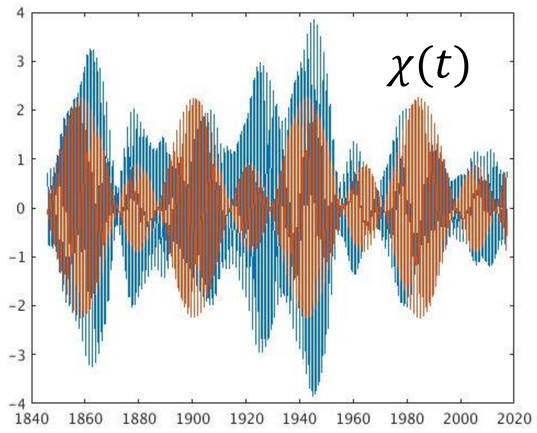
Filtered Chandler Wobble

Polar motion X

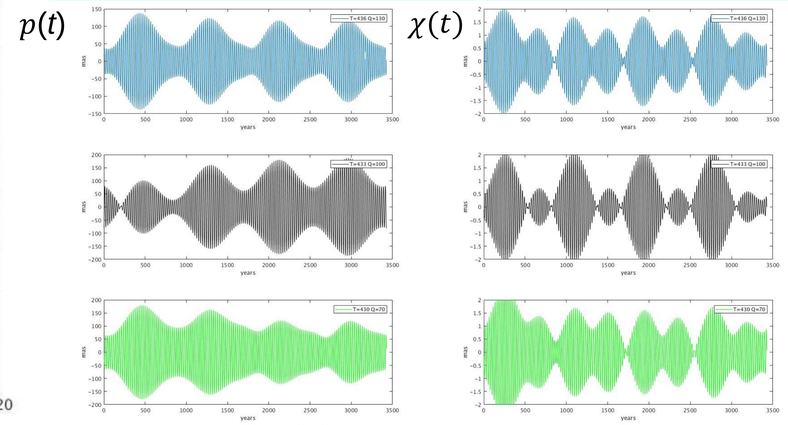



Modelled Excitation

Reconstructed excitation



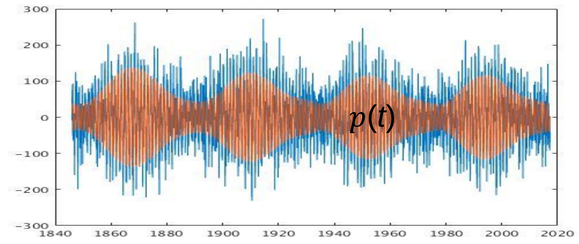

Left panel: Chandler wobble obtained for the modeled excitation with error-in-operator (changes in parameters $f_c$=1/T and Q)
right: reconstructed excitations for that cases, but with T=433, Q=100



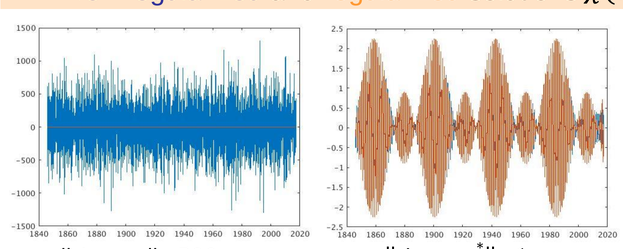

$h\|\chi\|$~130 mas, $h$ – error in operator

Observational noise added to modeled CW

$\delta = \sigma_{noise} = 50\ mas$




Non-regularized and regularized solutions $\chi(t)$



$\|A\chi - p\|$~500 $mas$ $\|A\chi - p^*\|$~1 $mas$

## Euler-Liouville equation:

Time domain

$$\frac{i}{\sigma_c}\frac{dp(t)}{dt} + p(t) = \chi(t)$$

$$p = p_1 + ip_2$$

$$\chi = \chi_{mass} + \chi_{motion}$$

$$\sigma_c = 2\pi f_c(1 + i/2Q)$$

$$f_c = \frac{1}{433}\ \text{days}^{-1} \qquad Q = 100$$

Frequency domain

$$\frac{-\omega\hat{p}(\omega)}{\sigma_c} + \hat{p}(\omega) = \hat{\chi}(\omega)$$

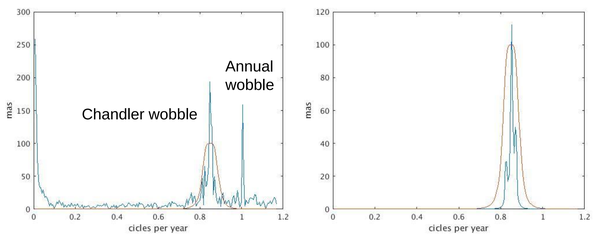
$$\frac{\sigma_c - \omega}{\sigma_c}\hat{p}(\omega) = \hat{\chi}(\omega)$$

direct operator:

$$\hat{p}(\omega) = W(\omega)\,\hat{\chi}(\omega)$$

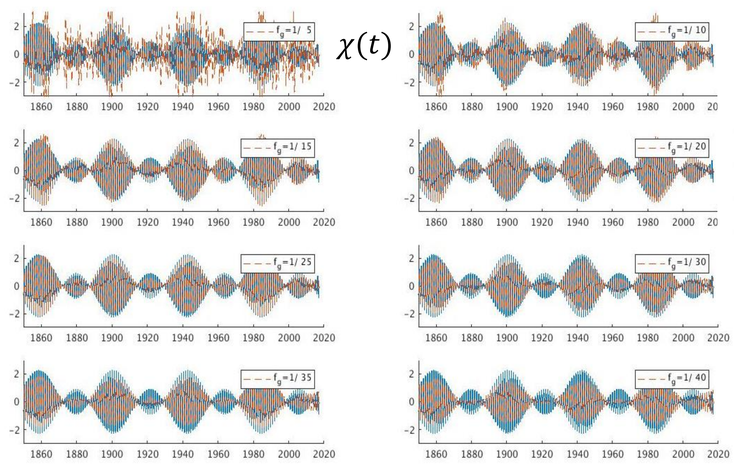
$$W(\omega) = \frac{\sigma_c}{\sigma_c - \omega}$$

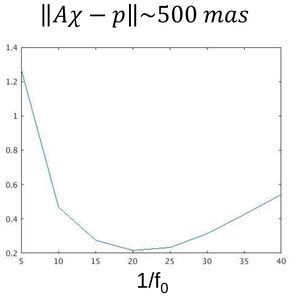
Munk W.H., MacDonald G.J.F., The rotation of the Earth, 1960




Real PM spectrum CW model spectrum

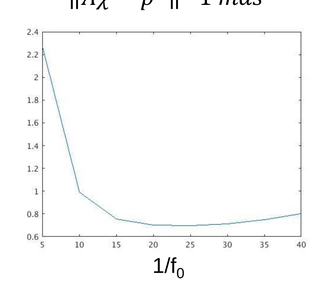
Excitation reconstructed from modeled CW with noises $\sigma_{noise}$ for different filter half-widths $f_0$, compared with modelled excitation.



1/$f_0$

Reconstruction error (mas) depending on the filter parameter 1/$f_0$ (observational noise only)



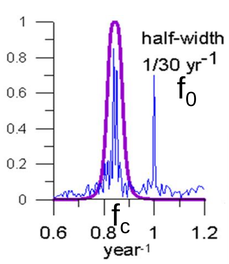
1/$f_0$

Reconstruction error depending on the filter parameter 1/$f_0$ (error-in-operator + observational noise)

Panteleev's band-pass filter

$$h(t) = \frac{\omega_0}{2\sqrt{2}} e^{-\left(\frac{\omega_0|t|}{\sqrt{2}} - i2\pi f_c t\right)} \left(\cos\frac{\omega_0 t}{\sqrt{2}} + \sin\frac{\omega_0|t|}{\sqrt{2}}\right),$$

$$L_h(f) = \frac{f_0^4}{(f - f_c)^4 + f_0^4}.$$




We generated annual signal with amplitude 100 mas and CW of 150 mas amplitude, added them together and superimposed white noise of amplitude *a*.
CW filtering error is given in the table below:

| Dependence of CW extraction error from the noise level | | | |
|---|---|---|---|
| **a - noise amplitude, mas** | **10** | **30** | **60** |
| **σ, mas** | **2.4 (1%)** | **2.6 (1%)** | **4.5 (1.5%)** |

generalized uncertainty:

$$\mu = \|A\chi - p^*\| + \delta + h\|\chi\|$$

Panteleev's corrective filter

$$W_{corr}(\omega) = \frac{L_h(2\pi f)}{W(\omega)}$$

**Reference:**

[1] Tikhonov A.N., Goncharsky A.V., Stepanov V.V., Yagola A.G., Numerical Methods for the Solution of Ill-Posed Problems, Springer Science+Business Media Dordrecht, ISBN 978-94-015-8480-7, 253 p.

[2] Zotov L., C. Bizourad On modulations of the Chandler wobble excitation, Journal of Geodynamics, special issue "Earth rotation", N 62 (2012) p. 30-34, DOI: 10.1016/j.jog.2012.03.010

**Conclusions:** Panteleev's corrective filtering proposed in [2] for Chandler excitation reconstruction is a regularizing algorithm, converging to exact solution when the noise tends to zero, giving the unbiased solution, since the filter is centered at the CW frequency. It implements the generalized selection principle of Tikhonov [1] and the optimal parameter (half-width of the filter) value $f_0$~1/25 year$^{-1}$ is selected in accordance with the generalized uncertainty principle [1].

**Acknowledgements:** the work is supported by 111 plan (grant No. B17033), and RFBR grant No 16-05-00753.