Astrophysics Introductory Course

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Powerpoint version with the help of Hanna Kotarba

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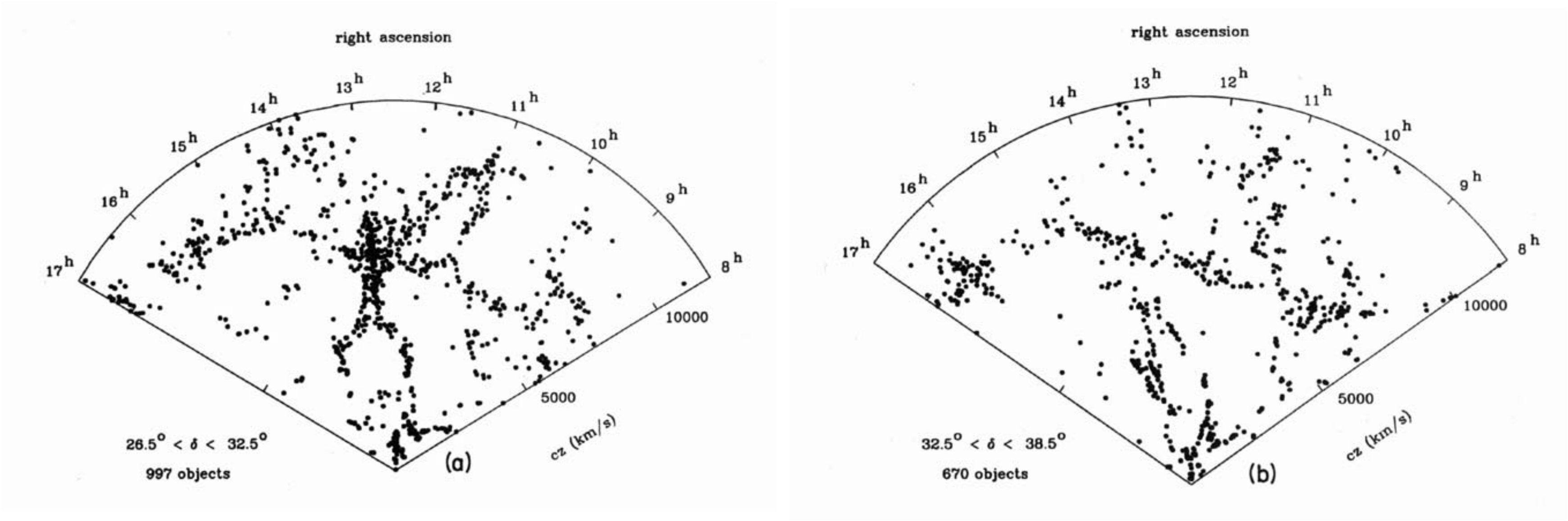
Chapter 13

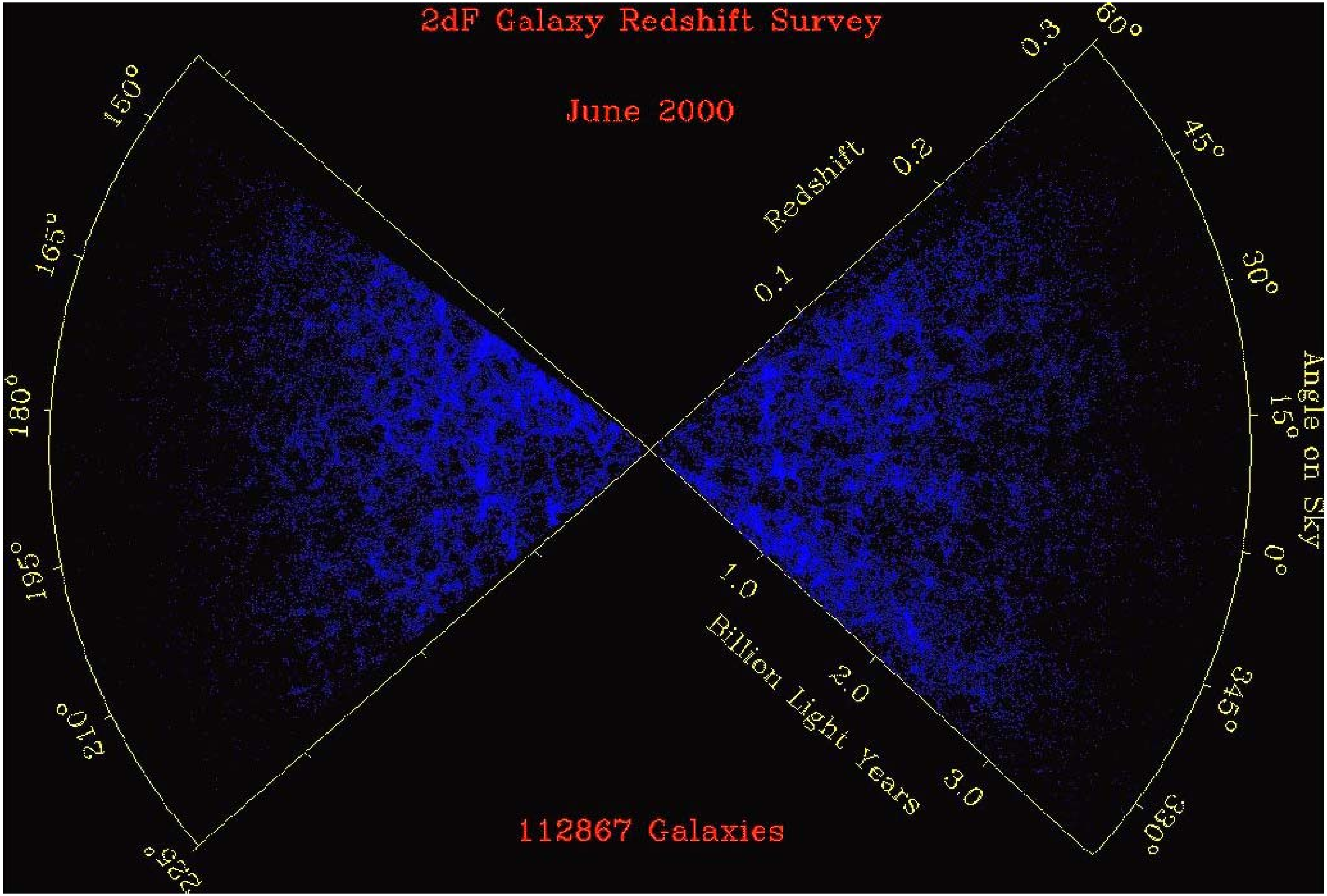
The Large–Scale Distribution of Galaxies

13.1 The local galaxy distribution

Galaxies are not uniformly distributed in space. They rather form large filaments, sheets, and **superclusters** of galaxies, which surround regions with very low galaxy density (**voids**):



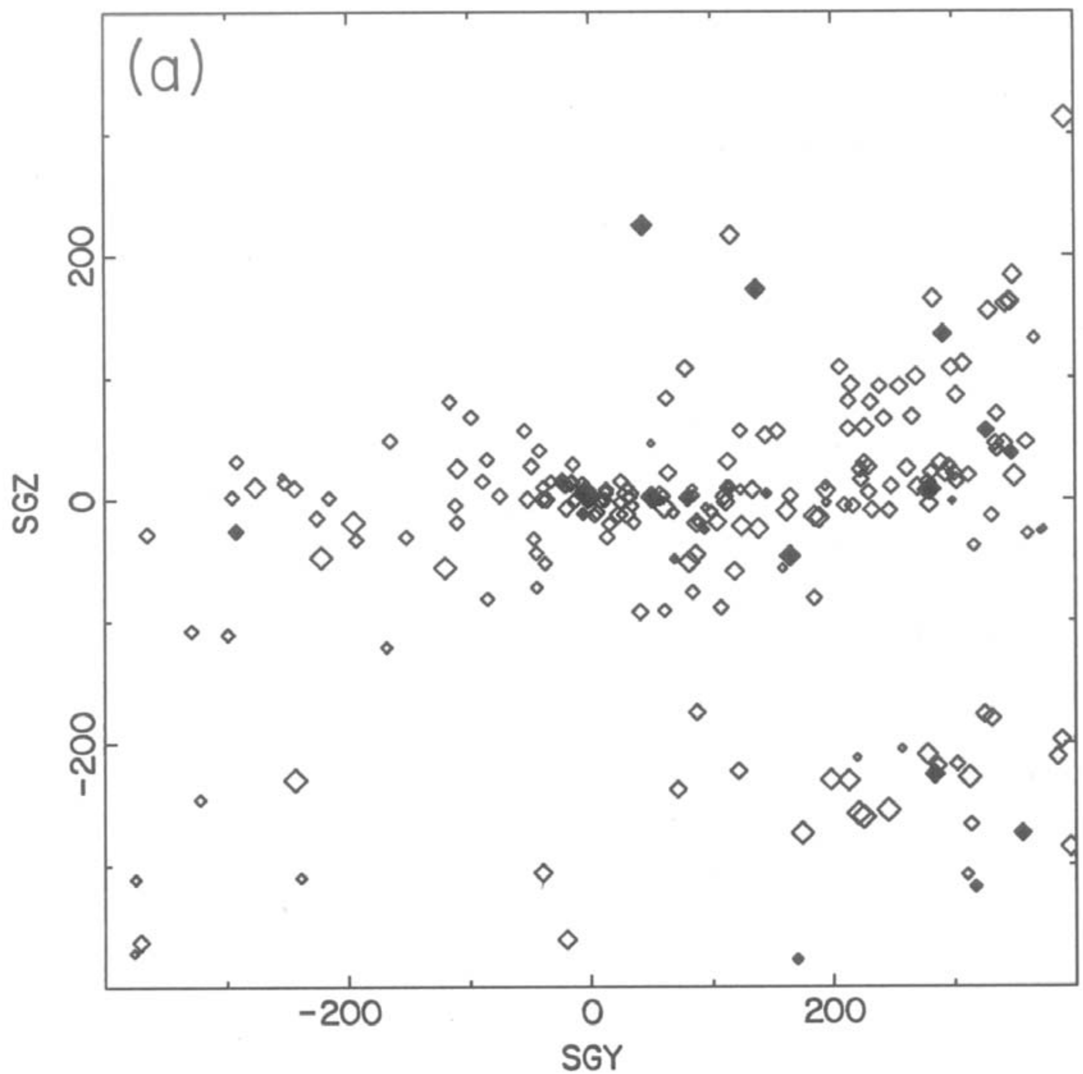
Center for Astrophysics (CFA, Harvard) Survey (from Peebles 1993)



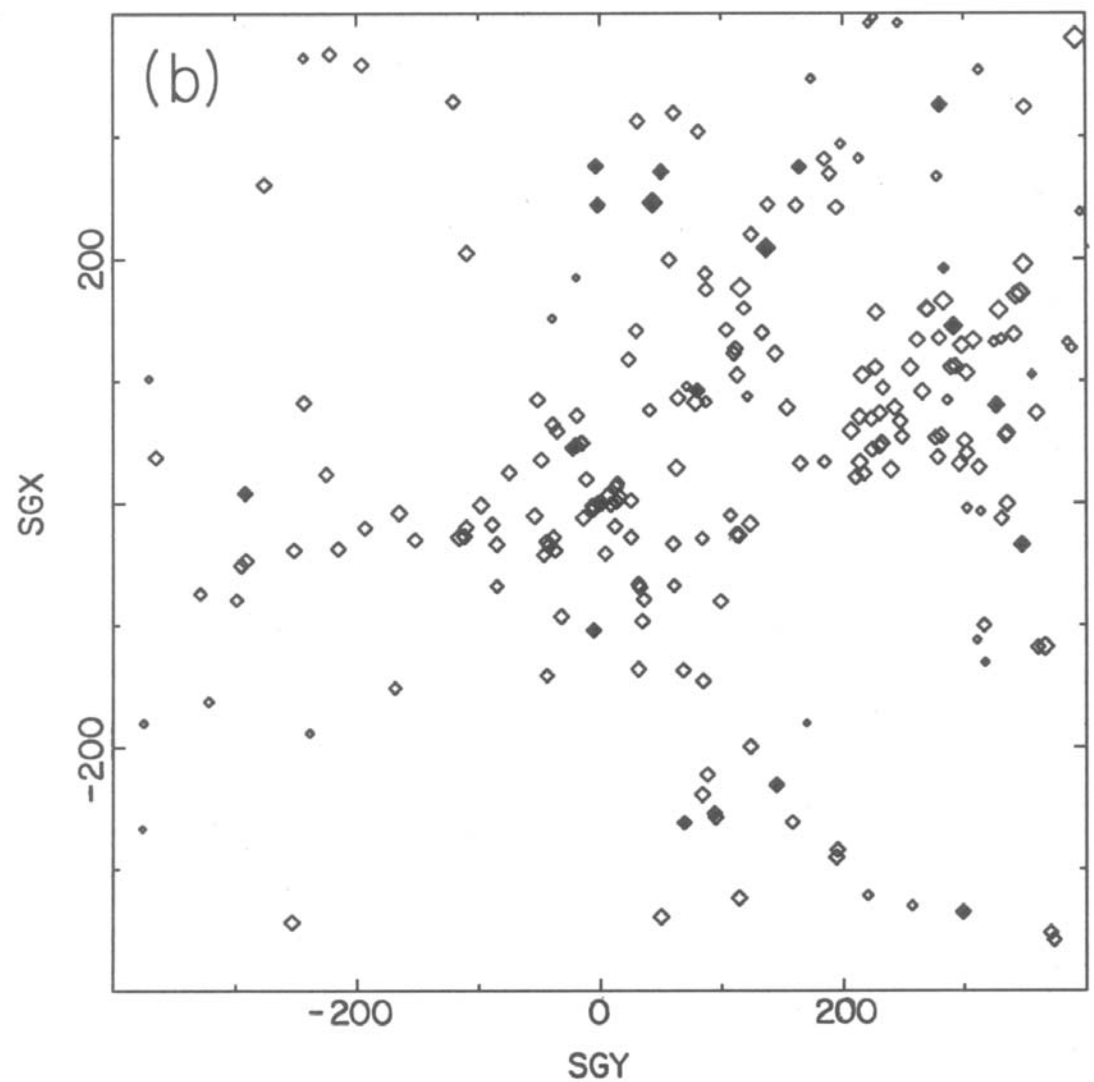
2dF Galaxy Redshift Survey

Also in the vicinity of the Milky Way, galaxies are mostly found concentrated in a plane, the so-called **Supergalactic Plane**. The **Supergalactic Coordinate System** is a coordinate system with the Milky Way at its centre. The (X, Y) plane is chosen to be identical with the Supergalactic Plane, the Y axis roughly points in the direction of the Virgo cluster.

In the following plots (taken from Peebles 1993, *Principles of Physical Cosmology*) we show the Supergalactic Plane edge-on and face-on. The Milky Way is at the centre of the figures. The scale of the axes is given in cz (in units of km/s), making the width of each box 8 h^{-1} Mpc.



Supergalactic Plane
(edge-on)



Supergalactic Plane
(face-on)

Typical Scales of Large–Scale Structure

Galaxies	~ 10 kpc
Groups & Clusters	~ (0.3 . . . 5) Mpc
Superclusters	~ 50 Mpc

Superclusters of galaxies are the largest known structures in the universe.

Distribution of different galaxy types

Ellipticals and S0-galaxies prefer regions of high galaxy density, spirals and irregulars are found in lower denisty environment. Nevertheless all galaxy types cluster along filaments and in groups and galaxy clusters.

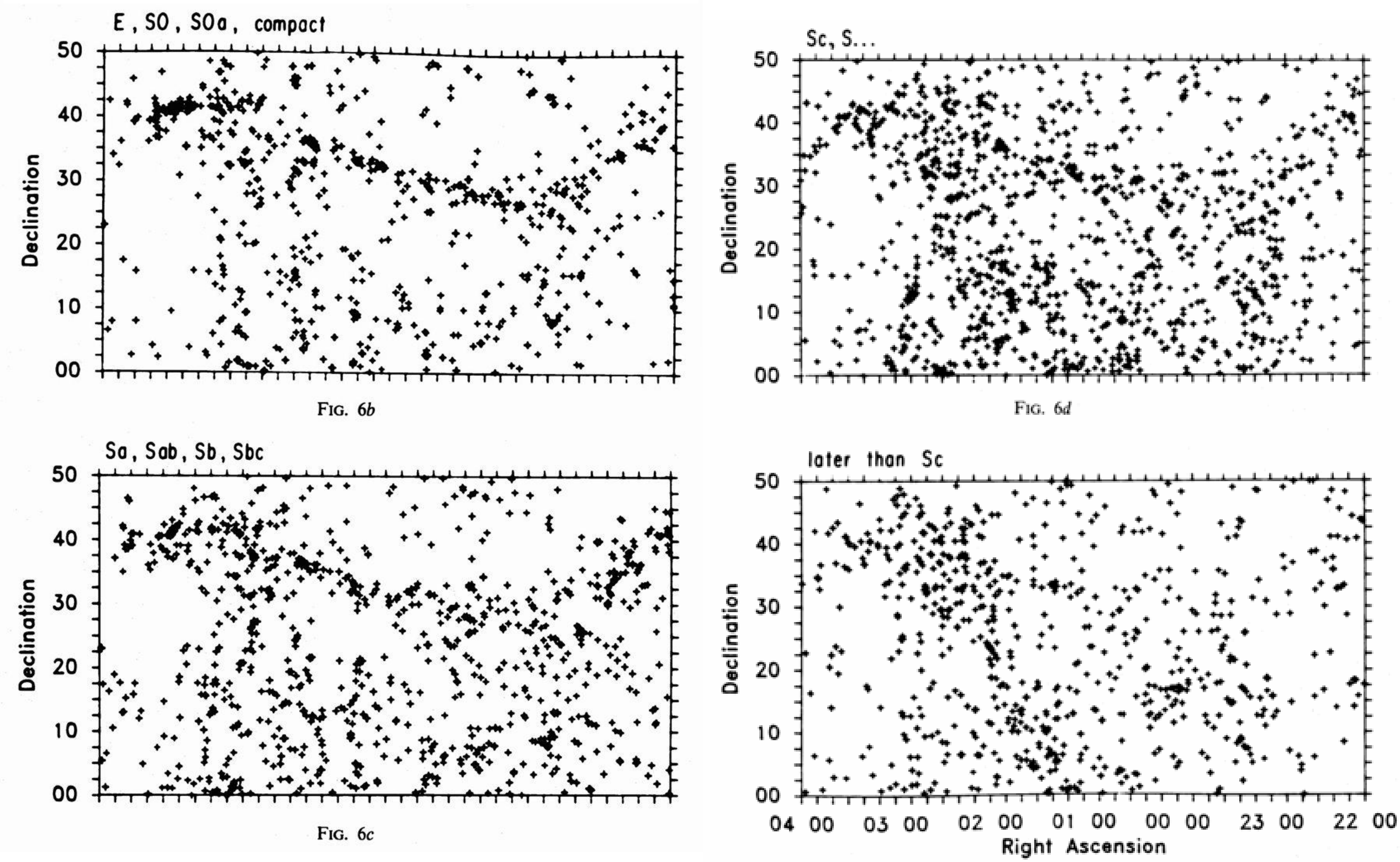


FIG. 6b

FIG. 6c

FIG. 6d

Giovanelli et al. (1986) *ApJ*, **300**, 77.

13.2 The two-point correlation function of galaxies

The two-point correlation function ξ(r) is a quantitative measure of galaxy clustering and is defined via the probability to find pairs of galaxies at a distance r:

$$dN_{pair} = N_o^2 (1 + \xi(r)) dV_1 dV_2$$

where N_o is the mean background density and dV_1 and dV_2 are volume elements around the two positions under consideration.

The two-point correlation function is related to the relative overdensity Δ(x) = $\delta\rho/\rho_o$ because we also have:

$$dN_{pair} = \rho(\vec{x}) dV_1 \rho(\vec{x} + \vec{r}) dV_2 = \rho_o^2 (1 + \Delta(\vec{x}))(1 + \Delta(\vec{x} + \vec{r})) dV_1 dV_2$$

Averaging over a large volume removes the linear terms in $\Delta(\vec{x})$ and we obtain:

$$< dN_{pair} >= \rho_o^2 (1 + < \Delta(\vec{x}) \Delta(\vec{x} + \vec{r}) >) dV_1 dV_2$$

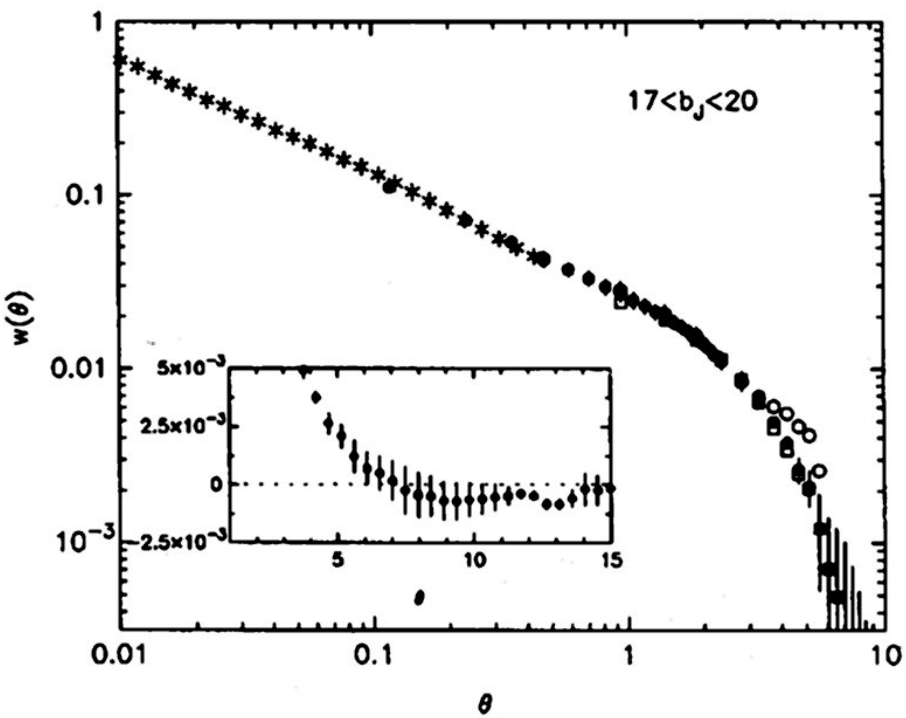
and therefore:

$$\xi(r) = < \Delta(\vec{x}) \Delta(\vec{x} + \vec{r}) >$$

Observationally one obtains averaged over all galaxy types:

$$\xi(r) = \left(\frac{r}{r_o}\right)^{-\gamma}$$

with: γ = 1.8 and r_o = 5/h Mpc (h = H_o/100km/s/Mpc) which is valid for scales from 100kpc to 10Mpc. Beyond 10Mpc the correlation function falls more rapidly.



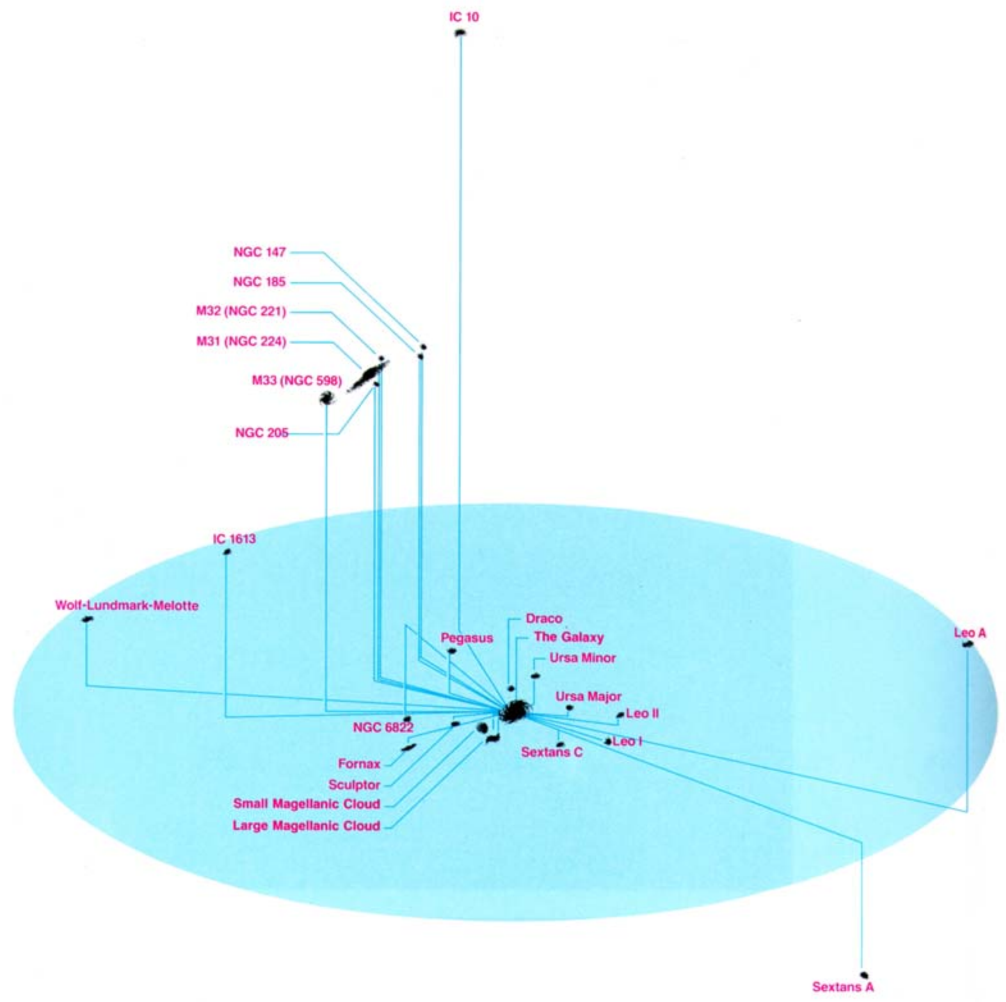
angular correlation function of galaxies from APM survey Maddox et al. 1990, MN 242, 43

13.3 The Local Group

- The Milky Way belongs to a loose collection of galaxies called the **Local Group**.
- The brightest members of the Local Group are the Andromeda Galaxy (M31), the Milky Way, and M33, three spiral galaxies. Apart from M32 (which is not very typical) there are no elliptical galaxies found in the Local Group. The most frequent galaxy types in the Local Group members are the irregulars (like the Large and the Small Magellanic Cloud) and dwarf ellipticals.
- The total number of galaxies known to belong to the Local Group is about 40, but there probably exists a number of dwarf galaxies which may have remained undetected (especially behind the Milky Way plane).
- All Local Group galaxies are gravitationally bound (M31 approaches the Milky Way with 120 km/s).

The distribution of Local Group members in space

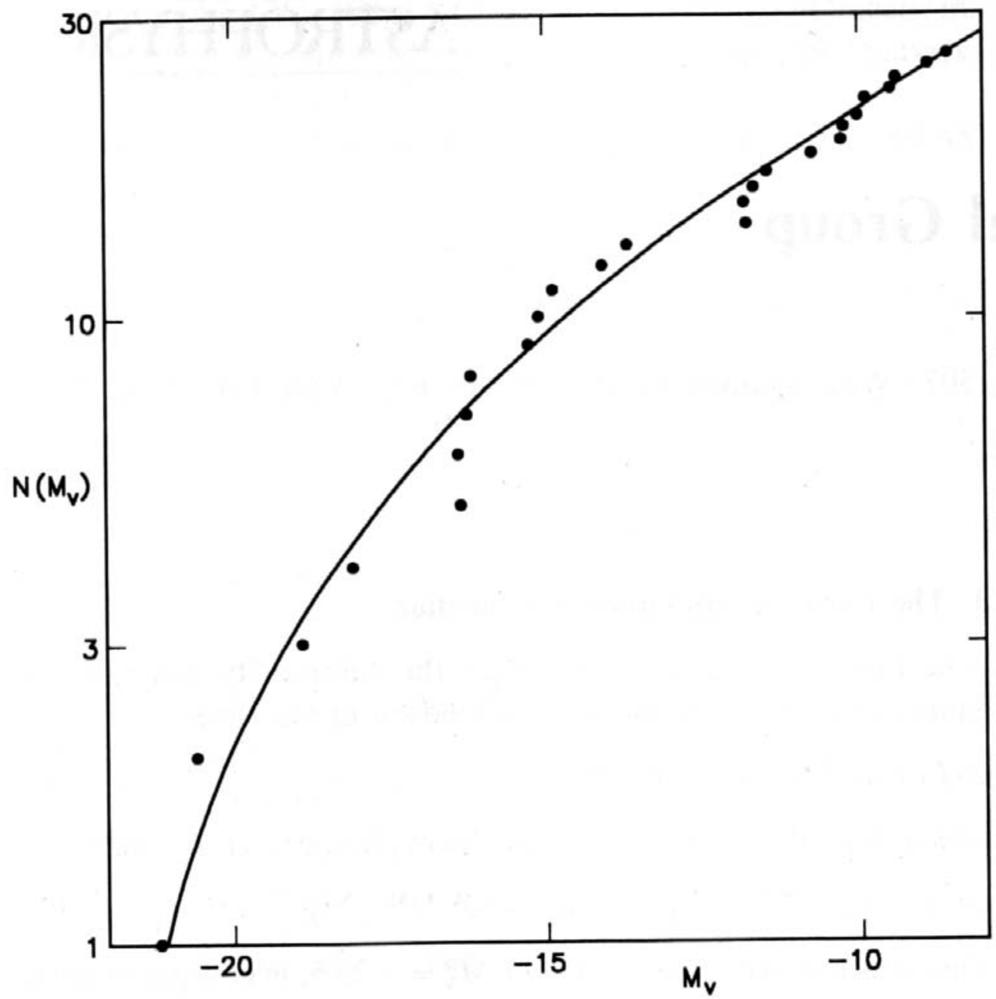
(*Cambridge Atlas of Astronomy* Third Edition, Cambridge 1994)



List of Local Group Members

The following table gives the Names, the celestial coordinates α and δ (for the equinox 2000), the Hubble type, the distance D (in kpc), the absolute V magnitude M_V , and the radial velocity V_0 (in km/s) of Local Group galaxies. The data were taken from M. Irvin's page on the local group (http://www.ast.cam.ac.uk/~mike/local members.html).

Name		Coordinates		Type	D(kpc)	Mv	Vo(km/s)	Name		Coordinates		Type	D(kpc)	Mv	Vo(km/s)
M31	NGC 224	00 40.0	+40 59	Sb	725	-21.1	-299	And VII	Cas Dw	23 24.1	+50 25	dE3	760	-12.0	
Galaxy		17 42.4	-28 55	Sbc		-20.6		Leo I	DDO 74	10 05.8	+12 33	dE3	270	-12.0	285
M33	NGC 598	01 31.1	+30 24	Sc	795	-18.9	-180	Leo A	DDO 69	09 56.5	+30 59	Irr	692	-11.7	+26
LMC		05 24.0	-69 48	Irr	49	-18.1	270	And II		01 13.5	+33 09	dE3	587	-11.7	
IC 10		00 17.7	+59 01	Irr	820	-17.6	-343	And I		00 43.0	+37 44	dE0	790	-11.7	
NGC 6822	DDO 209	19 42.1	-14 56	Irr	540	-16.4	-49	And VI	Peg Dw	23 49.2	+24 18	dE3	775	-11.3	
M32	NGC 221	00 40.0	+40 36	E2	725	-16.4	-190	SagDIG		19 27.9	-17 47	Irr	1150	-11.0	-79
NGC 205		00 37.6	+41 25	E5	725	-16.3	-239	Antlia		10 01.8	-27 05	dE3	1150	-10.7	361
SMC		00 51.0	-73 06	Irr	58	-16.2	163	Sculptor		00 57.6	-33 58	dE	78	-10.7	107
NGC 3109	DDO 236	10 00.8	-25 55	Irr	1260	-15.8	403	And III		00 32.6	+36 12	dE6	790	-10.2	
NGC 185		00 36.2	+48 04	E3	620	-15.3	-208	Leo II	DDO 93	11 10.8	+22 26	dE0	230	-10.2	76
IC 1613	DDO 8	01 02.2	+01 51	Irr	765	-14.9	-236	Cetus		00 23.6	-11 19	dE4	775	-10.1	
NGC 147	DDO 3	00 30.5	+48 14	E4	589	-14.8	-157	Sextans		10 10.6	-01 24	dE4	90	-10.0	224
Sextans A	DDO 75	10 08.6	-04 28	Irr	1450	-14.4	325	Phoenix		01 49.0	-44 42	Irr	390	-9.9	56
Sextans B	DDO 70	09 57.4	+05 34	Irr	1300	-14.3	301	LGS 3		01 01.2	+21 37	Irr/dE	760	-9.7	-277
WLM	DDO 221	23 59.4	-15 45	Irr	940	-14.0	-116	Tucana		22 38.5	-64 41	dE5	900	-9.6	
Sagittarius		18 51.9	-30 30	dE7	24	-14.0	140	Carina		06 40.4	-50 55	dE4	87	-9.2	223
Fornax		02 37.8	-34 44	dE3	131	-13.0	53	And V		01 07.3	+47 22	dE	810	-9.1	
Pegasus	DDO 216	23 26.1	+14 28	Irr	759	-12.7	-181	Ursa Minor	DDO 199	15 08.2	+67 23	dE5	69	-8.9	-250
								Draco	DDO 228	17 19.2	+57 58	dE3	76	-8.6	-289



<u>The cumulative luminosity function of Local Group galaxies</u>

is consistent with a Schechter function.

(van den Bergh 1992, *A& A*, **264**, 75)

13.4 Galaxy Clusters: overview

Rich clusters of galaxies are the most massive virialized, high-overdensity systems known. In the optical light galaxy clusters have the following ranges of properties:

- **Richness** (number of cluster galaxies with luminosities 2 magnitudes dimmer than the third brightest cluster galaxies): 30-300 galaxies
- **Radius** (where the surface density of galaxies drops to 1% of the core density): 1-2 Mpc
- **Radial velocity dispersion:** 400-1400 km/s
- **Mass (r<1.5 Mpc):** $10^{14}\text{-}10^{15} M_\odot$
- **Optical B-Band luminosity: (r<1.5 Mpc):** $10^{11}\text{-}10^{13} L_\odot$
- **Mass-to-light ratio:** $\approx 300 M_\odot / L_\odot$
- **Cluster number density:** $10^{-5}\text{-}10^{-6}\,\mathrm{Mpc}^{-3}$
- **Cluster correlation scale:** $22 \pm 4\mathrm{Mpc}$
- **Fraction of galaxies with L>L* in clusters:** ~5%

Some important optical cluster catalogues are:

By visual inspection of photographic plates:

Abell (1958, ApJS 3, 211); Abell et al. (1989, ApJS, 70, 1); Zwicky et al. (1961-68)

From (deep) CCD (multicolor) images:

Postman et al. (1996, AJ, 111, 615), Gladders & Yee (2000, AJ, 120, 2148), Goto et al. (2002, AJ, 123, 1808, SLOAN)

Two nearby clusters

(Part of) the Virgo Cluster

central part of Coma cluster with two cDs.

Two nearby clusters

Virgo Cluster:

- Nearest large galaxy cluster with more than 2000 galaxies
- brighter than MB ≈ −14 ($L_B \sim 10^{7.8} L_\Theta$)
- Distance ~ 17Mpc (dependent on H_0)
- Extend ~ 10° ^= 3Mpc × 3Mpc
- Irregular cluster, densest regions dominated by ellipticals
- Velocity dispersion of galaxies about 600km/s

Coma Cluster:

- One of the most luminous clusters known
- Distance ~ 100Mpc (dependent on H_0)
- Regular cluster with probably subcluster merging from SW
- Dominated by ellipticals and S0s, two central cDs and one in subcluster
- Velocity dispersion of galaxies about 1000km/s
- Strong X-ray source

Spatial distribution and galaxy content

The King profile used for globular clusters or ellipticals describes well the cluster galay number density:

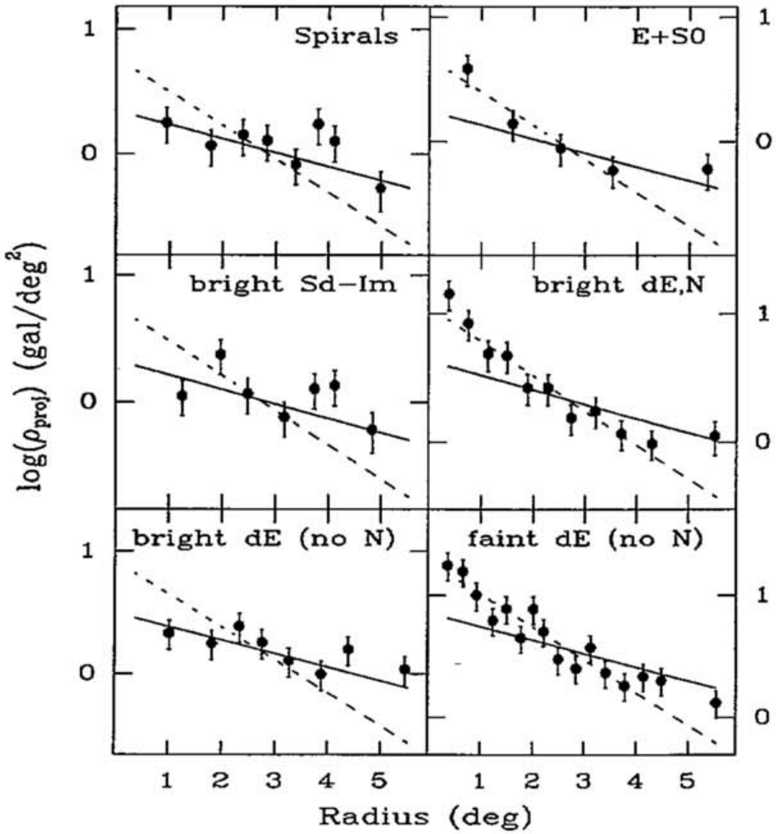
$$n_g(r) = n_0 \left[1 + \frac{r^2}{R_{core}^2} \right]^{-3/2}$$

where r is the radial distance from the center, R_{core} the core radius (0.1-0.25 Mpc) and n_0 the central galaxy number density ($\approx 10^3 Mpc^{-3}$)

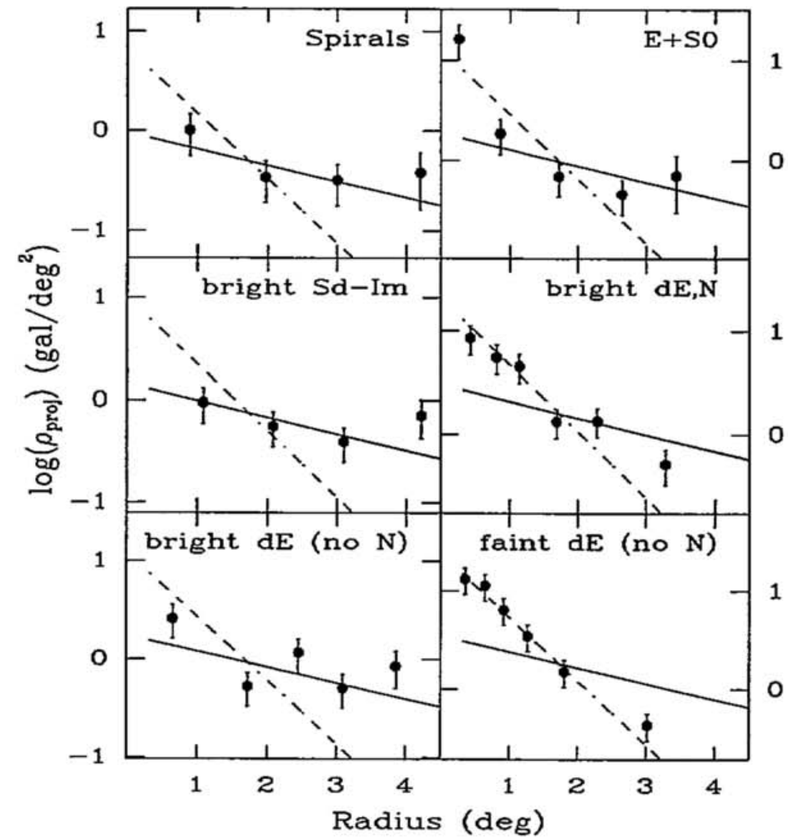
Cluster type	E	S0	Sp
Regular clusters	35%	45%	20%
Intermediate clusters	20%	50%	30%
Irregular clusters	15%	35%	50%
Field	10%	20%	70%

Morphology-density relation
Evolution with redshift: Butcher-Oemler effect

Morphology-Radius Relation:



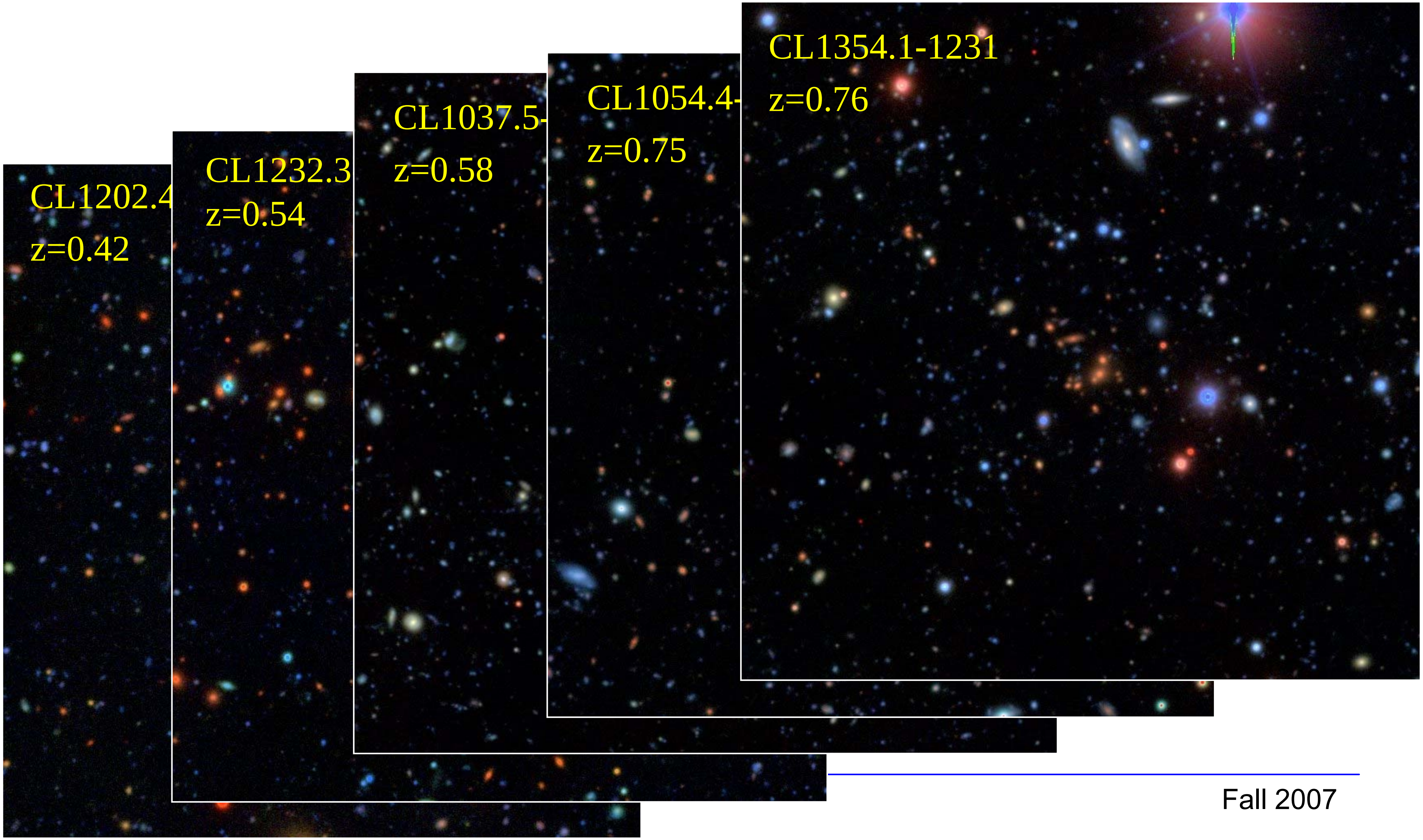
Virgo



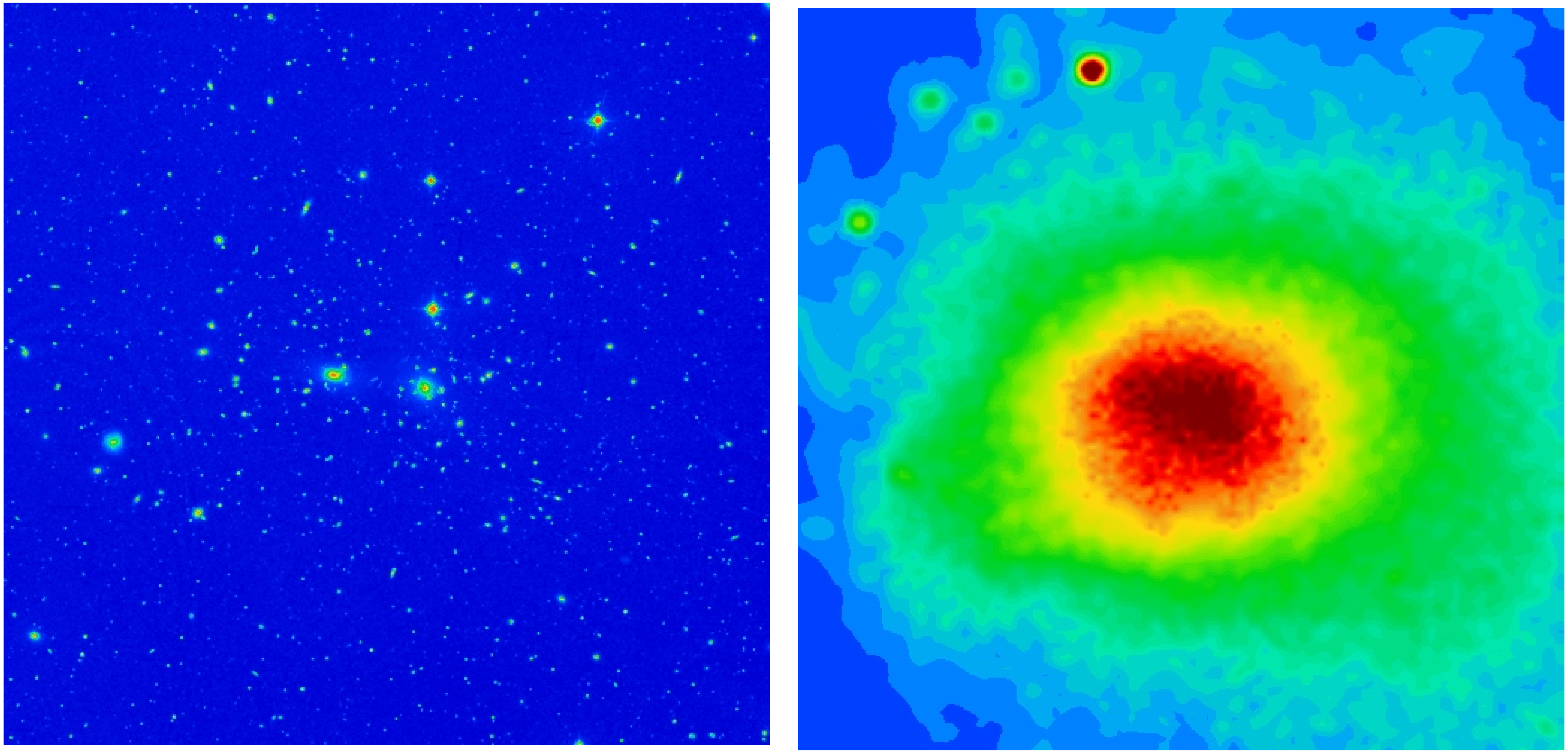
Fornax

Ferguson, Sandage (1989) *ApJ*, **346**, L53. Ellipticals and S0's are more concentrated than spirals and irregulars.

Distant clusters: EDisCS



13.5 X-Ray Gas in Galaxy Clusters



Coma cluster (left: optical image, right: X-ray image)

The baryonic gas is compressed in the deep cluster potential wells and shock-heated up to X-ray emitting temperatures T. The X-ray spectra show the characteristics of Bremsstrahlung of a $\sim 10^8$K hot gas. Therefore the emissivity at frequency ν is:

$$\varepsilon(\nu)=\frac{32\pi Z^2 n_e n_i}{3m_e c^3}\sqrt{\frac{2\pi}{3km_e T}}\exp\left(\frac{-h\nu}{kT}\right)g_{ff}(T,\nu)$$

where g_{ff} is the Gaunt factor. Integrating over frequency one gets the volume emissivity:

$$\varepsilon = 2.4\ 10^{-27}\mathrm{T}^{-1/2}n_{\mathrm{e}}^2\left[\frac{erg}{cm^3 s}\right]$$

The cooling time of the plasma is: $t_{cool} = \frac{3n_e kT}{\varepsilon} \simeq \frac{10^{11}}{n_e}T^{1/2}$ [s]

X-ray surface brightness distribution (β model): $S(R)=S(0)\left(1+R^2/r_{core}^2\right)^{-3\beta+1/2}$

X-ray cluster properties for rich clusters:

Temperature: $2-14 keV$ or $2\times10^6-10^8 K$

Luminosity: $10^{42.5}-10^{45} erg/s$

Core radius: 0.1-0.2 Mpc

Central electron density: $n_e \simeq 10^{-3} cm^{-3}$

Gas mass: $M_{gas} \simeq 10^{13}-10^{14} M_\odot$

Fe abundance: $\approx$ 1/3 solar

For the center of the Coma cluster:

$L \simeq 10^{44} erg/s$ $\quad \overline{\tau_{cool}} \simeq 10^{10} yrs$

$\overline{n}_e \simeq 10^{-3} cm^{-3}$ $\quad M_{gas} \simeq 10^{13} M_\odot$

Important X-ray catalogues:

Böhringer et al. (2001, 2004): clusters with z<0.45 from the ROSAT all-sky survey
Rosati et al. (1998): clusters with z<1.2 from ROSAT pointed observations

Most distant X-ray cluster: z=1.39 (Mullis et al. 2005, ApJ 623, 185)

Masses of galaxy clusters

From dynamics:

Virial Theorem:

$$M = \frac{r_G < v^2 >}{G}$$

$$\text{where } \mathrm{r_G} = \frac{\sum_i m_i}{\sum_{i \neq` j} \frac{m_i m_j}{r_{ij}}} \text{ and } < v^2 >= \frac{\sum_i m_i (v_i - \overline{v})^2}{\sum_i m_i}$$

From X-rays:

Hydrostatic equilibrium:

$$\frac{1}{\rho_g} \frac{dP}{dr} = -\frac{GM(< r)}{r^2}$$

As for elliptical galaxies:

$$\mathrm{M(<r)=-}\frac{\mathrm{kT(r)r^2}}{G \mu m} \left[\frac{d \ln \rho_g(r)}{dr} + \frac{d \ln T(r)}{dr} \right]$$

The following correlations exist between the different components of galaxy clusters:

• The central galaxy density is higher for higher L_X.

• The fraction of spirals is lower for higher L_X.

• The temperature T is proportional to L_X and typically 10^8K.

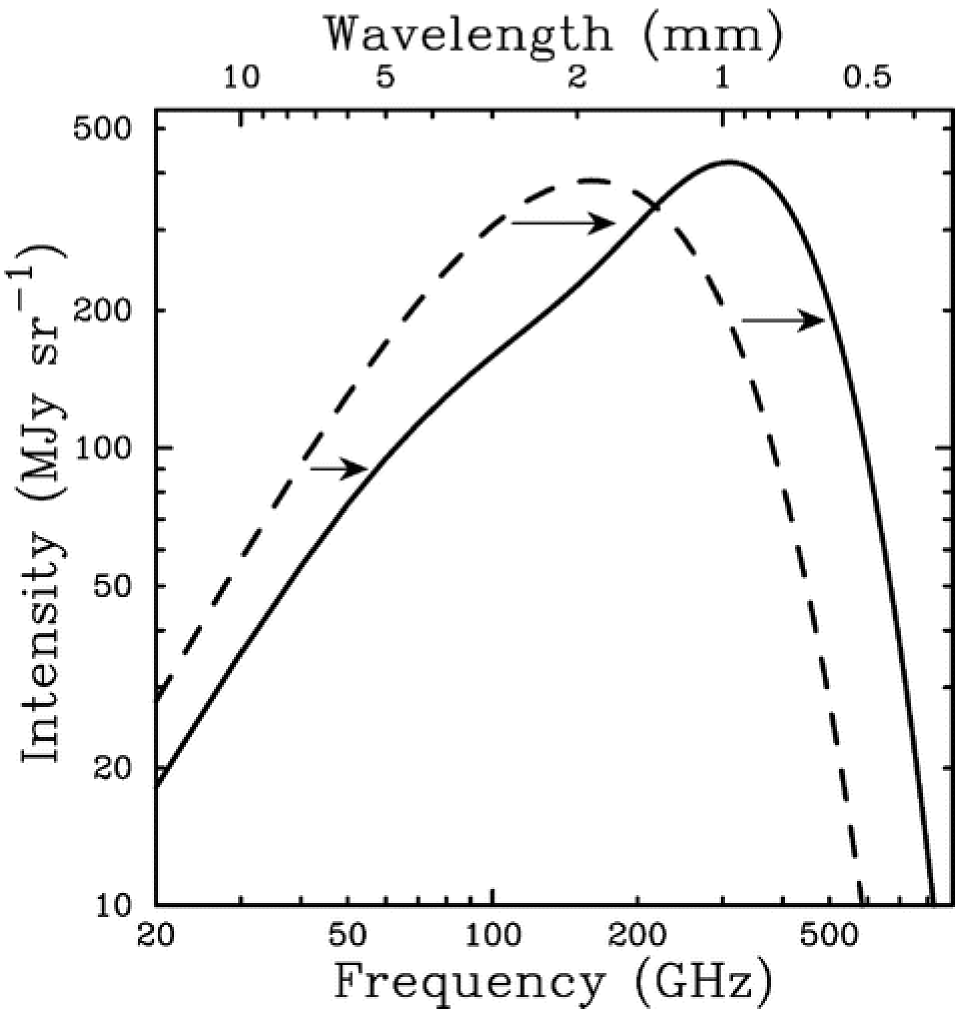
• The gas metallicity is lower for higher T and typically 1/3 of solar.

• The ratio of gas-mass to galaxy-mass increases with T up to 5 or more.

• The dominant component in all clusters is dark matter. This follows consistently from the dynamics of galaxies, the hydrostatic equilibrium of the X-ray gas and from gravitational lensing. The typical mass ratios are:

galaxies : X-ray-gas : dark-matter ≈ 1 : 5 : 25

Clusters in microwaves and the Sunyaev & Zel‘dovic effect

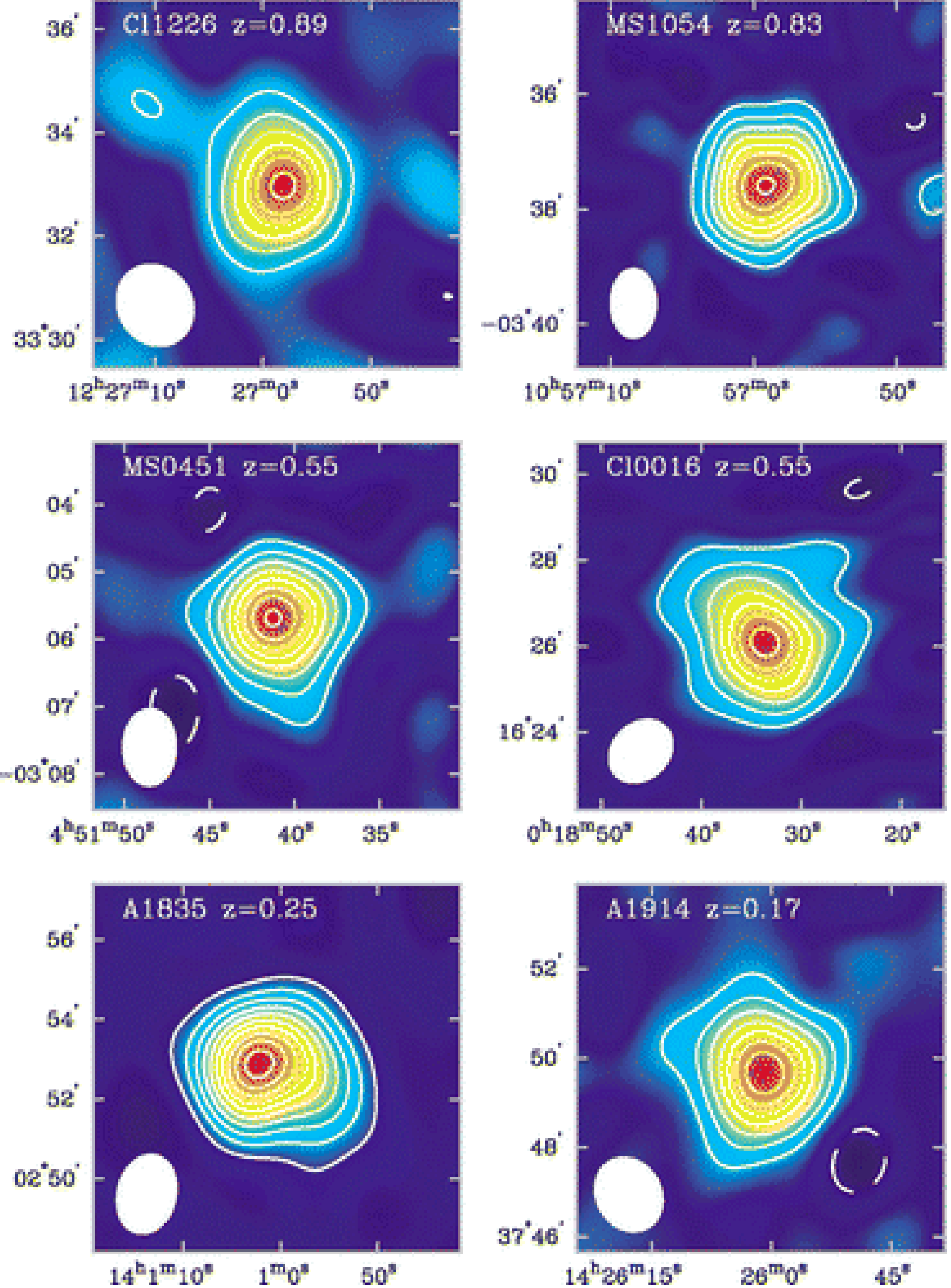


The photons of the CBR suffer Inverse Compton scattering against the hot electrons of the intracluster medium, preferentially gaining energy. The CMB spectrum gets shifted to higher frequencies: at wavelengths <1.4 mm the clusters appear as bright pacthes in the CMB.

To first order, the CMB distortion is proportional to the integral along the line of sight of the electron density times its thermal energy:

$$\frac{\Delta I}{I} = 2y, y = \int n_e \sigma_T \frac{kT}{m_e c^2} dl$$

Carlstrom, Holder & Reese, 2002, ARAA, 40, 643



The SZ effect is independent of Redshift, unlike the optical or X-ray surface brightness that suffer from cosmological dimming (see cosmology lessons) $(1+z)^4$

Therefore it is a powerful method to detect clusters at high redshifts.

Cluster surveys are starting with e.g. APEX (Atacama Pathfinder Experiment)

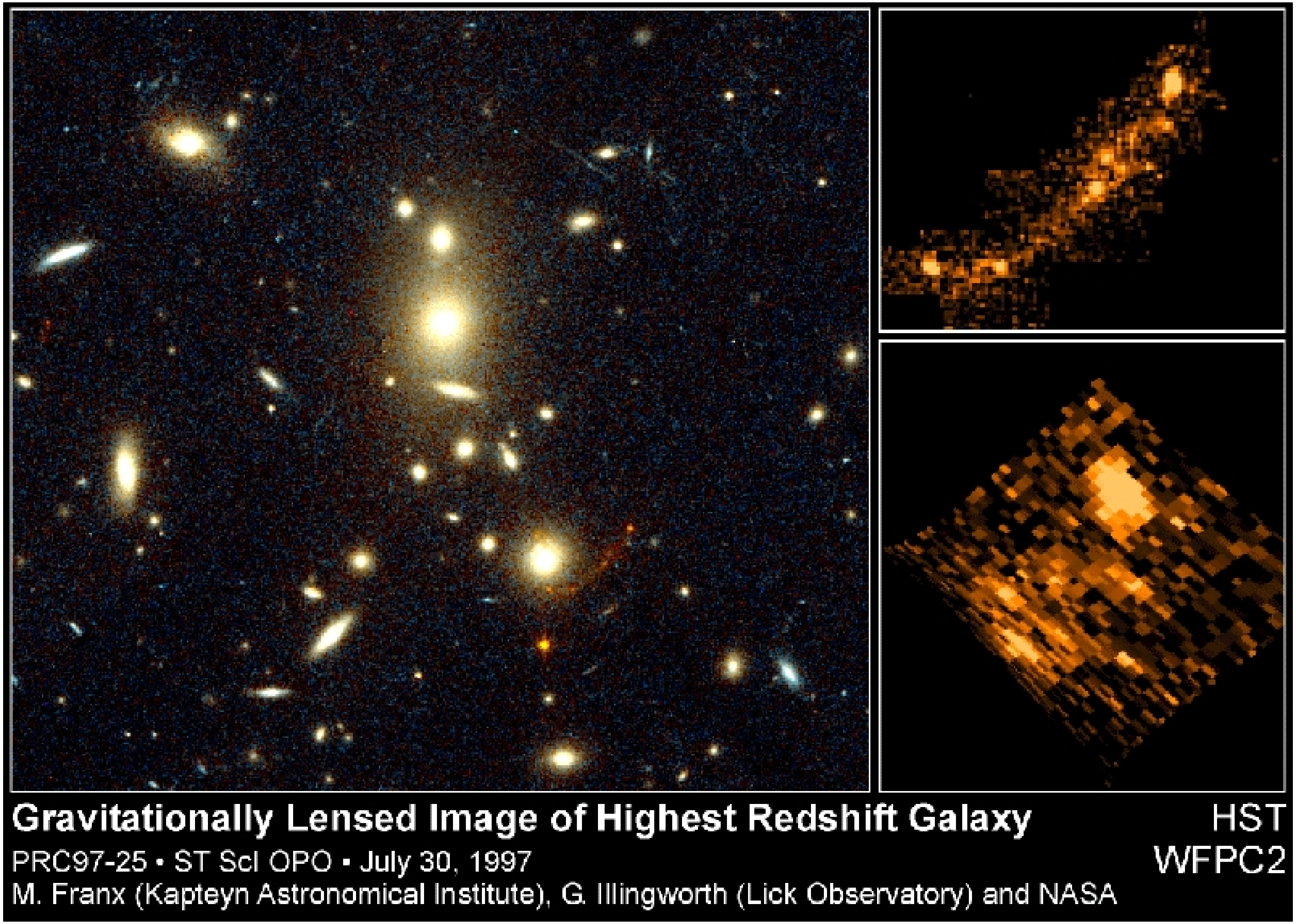
If the cluster is not resolved, the SZ signal measures the thermal energy of the electrons:

$$Y = \int y dA \approx \int n_e T dV \propto M_{gas} T$$

13.6 Masses of galaxy clusters from gravitational lensing

Galaxy Cluster Abell 2218 HST • WFPC2

NASA, A. Fruchter and the ERO Team (STScI, ST-ECF) • STScI-PRC00-08


Gravitationally Lensed Image of Highest Redshift Galaxy
HST
WFPC2
PRC97-25 • ST ScI OPO • July 30, 1997
M. Franx (Kapteyn Astronomical Institute), G. Illingworth (Lick Observatory) and NASA

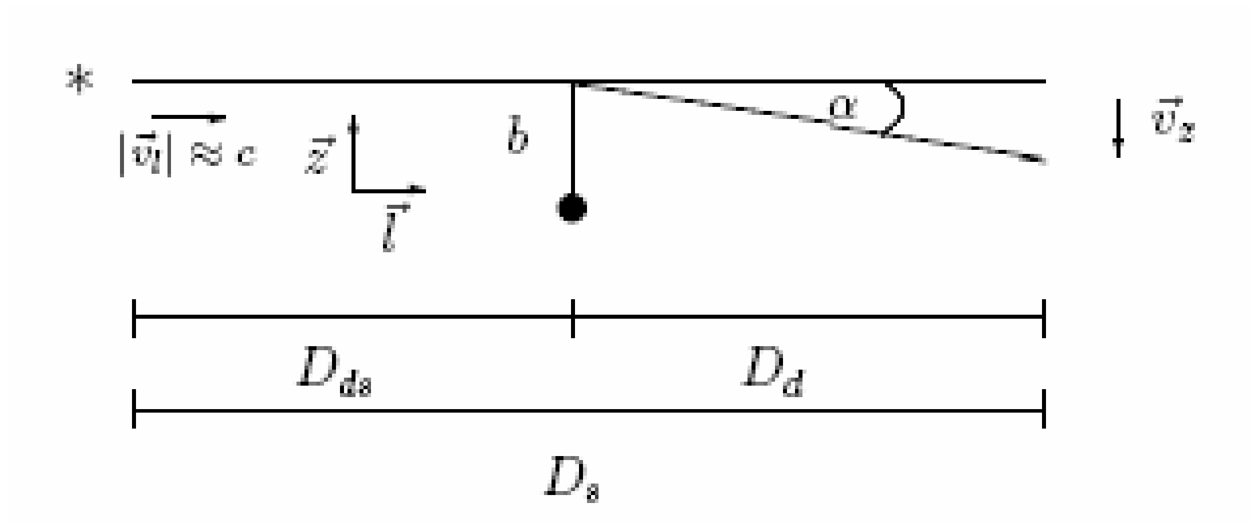
13.6.1 Basics of Gravitational Lensing

One consequence of Einstein's Theory of Relativity is that light rays are deflected by gravity. Einstein calculated the magnitude of the deflection that is caused by the sun. Since the potential and the velocity of the deflecting mass are small ($v \ll c$ and $\Phi \ll c^2$) the deviation angle is expected to be small as well. According to Einstein's formula, a light ray passing the surface of the Sun tangentially is deflected by 1.7". This deflection angle has in the mean time been confirmed with a very high accuracy (0.1 %).

For further information see also:

- R. Narayan, M. Bartelman: Lectures on Gravitational Lensing; in:
  Formation of Structure in the Universe
  Edited by Avishai Dekel and Jeremiah P. Ostriker. Cambridge:
  Cambridge University Press, 1999., p.360

- Schneider, Ehlers, Falco: Gravitational Lenses
  Springer Verlag



The light path from the source to the observer can then be broken up into three distinct zones:

1. Light travels from the source to a point close to the lens through unperturbed spacetime, since b « D_d.

2. Near the lens the light is deflected.

3. Light travels to the observer through unperturbed spacetime, since b « D_{ds}.

In a naive Newtonian approximation one would derive:

$$\alpha = \frac{v_z}{c} = \frac{1}{c}\int \underbrace{\frac{d\Phi}{dz}}_{*} dt = \frac{1}{c^2}\int \frac{d\Phi}{dz} dl$$

*: acceleration in z direction; because the acceleration doesn't depend on the energy of the photons, gravitational lenses are achromatic.

This result differs only by a factor of two from the correct general relativistic result:

$$\vec{\alpha} = \frac{2}{c^2}\int \vec{\nabla}_{\perp}\Phi dl \qquad \text{G.R.}$$

where the deflection angle α, written as vector $\vec{\alpha}$ perpendicular to the light propagation $\vec{l}$, is the integral of the potential gradient perpendicular to the light propagation.

For a point mass the potential can be written as:

$$\Phi(l,z) = \frac{-GM}{(l^2+z^2)^{1/2}}$$

Therefore:

$$\frac{d\Phi}{dz} = \frac{+GMz}{(l^2+z^2)^{3/2}} \qquad = (\vec{\nabla}_{\perp}\Phi)$$

After integration:

$$\alpha = \frac{2}{c^2}\int_{-\infty}^{\infty}\frac{GMz}{(l^2+z^2)^{3/2}}dl = \frac{4GMz}{c^2}\int_{0}^{\infty}\frac{dl}{(l^2+z^2)^{3/2}} = \frac{4GMz}{c^2}\left[\frac{l}{z^2(l^2+z^2)^{1/2}}\right]_0^{\infty}$$

Thus the deflection angle α for a light ray with impact parameter b = z near the point mass M becomes:

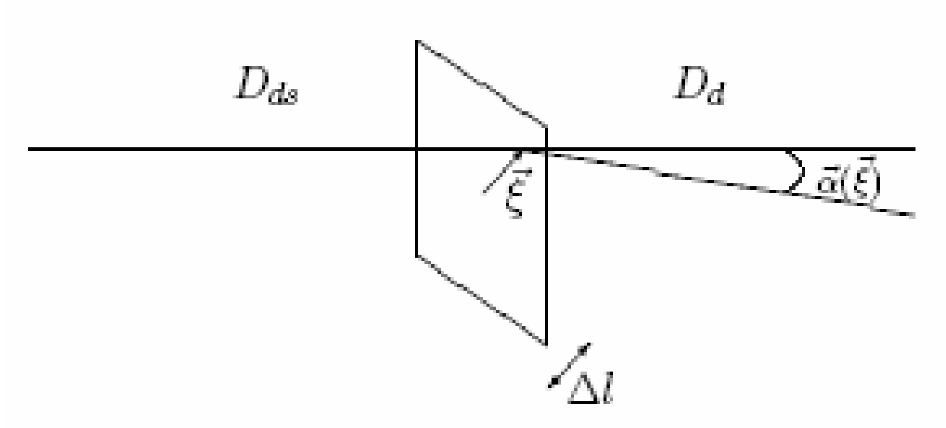
$$\boxed{\alpha = \frac{4GM}{c^2 b} = \frac{2R_S}{b}}$$

where $R_S = 2GM/c^2$ is the **Schwarzschild radius** of the mass M, i.e. the radius of the black hole belonging to the mass M.

Therefore for the sun ($M_\odot \approx 2 \cdot 10^{33}$ g → $R_S \approx 3.0$ km) we get a deflection angle α at the Radius of the sun (≈ 700000km) of:

$$\alpha_{\odot,R_\odot} \simeq 1.7''$$

In order to calculate the deflection angle α caused by an arbitrary mass distribution (e.g. a galaxy cluster) we use the fact that the extent of the mass distribution is very small compared to the distances between source, lens and observer:
$\Delta l \ll D_{ds}$ and $\Delta l \ll D_d$



Therefore, the mass distribution of the lens can be treated as if it were an infinitely thin mass sheet perpendicular to the line-of-sight. The surface mass density is simply obtained by projection.

The plane of the mass sheet is called the **lens plane**. The mass sheet is characterized by its surface mass density

$$\sum(\vec{\xi}) = \int_{\Delta l} \rho(\vec{\xi}, \vec{l})dl$$

The deflection of a light ray passing the lens plane at $\vec{\xi}$ by a mass element $dm = \sum(\vec{\xi}')d^2\xi'$ at $\vec{\xi}'$ is:

$$d\alpha = \frac{4Gdm}{c^2 |\vec{\xi} - \vec{\xi}\,'|}$$

To get the deflection caused by all mass elements, we have to integrate over the whole surface. Doing this we must take into account that, e.g., the deflection caused by mass elements lying on opposite sides of the light ray may cancel out. Therefore we must add the deflection angles as vectors:

$$\boxed{\vec{\alpha}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}\,')\Sigma(\vec{\xi}\,')}{|\vec{\xi} - \vec{\xi}\,'|^2} d^2\xi\,'}$$

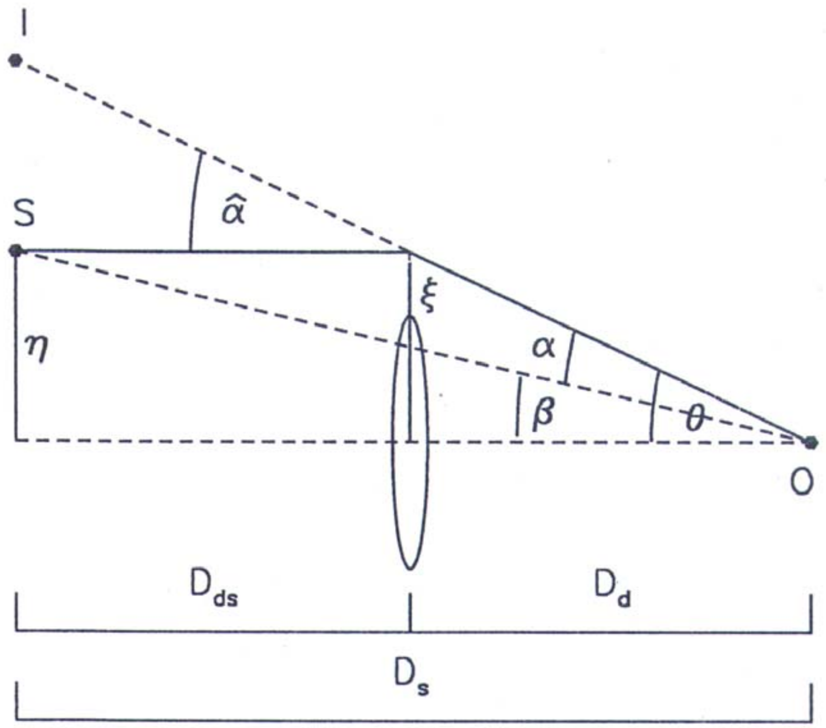
Special case: For a spherical mass distribution the lensing problem can be reduced to one dimension. The deflection angle then points toward the center of symmetry and we get:

$$\boxed{\alpha(\xi) = \frac{4GM(<\xi)}{c^2 \xi}}$$

where ξ is the distance from the lens center and M(< ξ) is the mass enclosed within radius ξ,

$$\boxed{M(<\xi) = 2\pi \int_0^{\xi} \Sigma(\xi\,')\xi\,' d\xi\,'}$$

13.6.2 Lensing Geometry and Lens Equation



Important relations:

$$\hat{\alpha} \cdot D_{ds} = \alpha \cdot D_s \qquad (13.1)$$

$$\theta \cdot D_s = \beta \cdot D_s + \hat{\alpha} \cdot D_{ds} \qquad (13.2)$$

Note: The distances D are angular diameter distances.

Using the previous two equations one obtains the so called lens equation:

$$\boxed{\beta = \theta - \alpha = \theta - \frac{D_{ds}}{D_s}\hat{\alpha}} \qquad (13.3)$$

The lens equation relates the real position (angle) of the source (without a lens) with the position of the lensed image.

Important note: only angular distances are needed for deriving the lens equation. In general, i.e. over cosmological distances: $D_{ds} \neq D_s - D_d$.

13.6.3 Einstein radius and critical surface density

Consider now a circularly symmetric lens with an arbitrary mass profile. Due to the rotational symmetry of the lens system, a source, which lies exactly on the optical axis ($\theta = \alpha \leftrightarrow \beta = 0$) is imaged as a ring. This ring is the so called **Einstein ring**:

$$\beta = 0 \quad \mapsto \quad \theta = \alpha \tag{13.4}$$

$$= \frac{D_{ds}}{D_s} \cdot \hat{\alpha} \tag{13.5}$$

$$= \frac{D_{ds}}{D_s} \cdot \frac{4G}{c^2} \cdot \frac{M(<\xi)}{\xi} \tag{13.6}$$

$$= \frac{D_{ds}}{D_s} \cdot \frac{4\pi G}{c^2} \cdot \frac{M(<\xi)}{\pi\xi^2}\xi \tag{13.7}$$

$$= \frac{D_{ds}}{D_s} \cdot \frac{4\pi G}{c^2} \cdot \Sigma_{cr} \cdot D_d \cdot \theta \tag{13.8}$$

Therefore the **critical surface density** to observe an Einstein ring is:

$$\boxed{\Sigma_{cr} = \frac{c^2}{4\pi G} \cdot \frac{D_s}{D_{ds} D_d} = 0.3 \frac{g}{cm^2} \frac{D_s \cdot 1Gpc}{D_{ds} \cdot D_d}} \qquad \text{critical surface density}$$

Note: The critical surface density depends only on the angular distances between source, lens and observer.

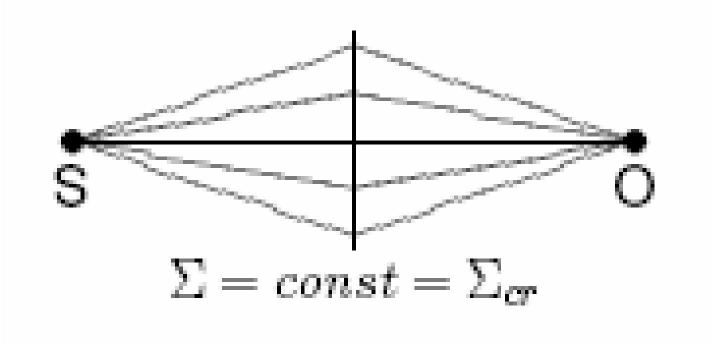
The radius of the Einstein ring can be calculated using the previous equation and $\xi = D_d\theta$:

$$\boxed{\theta_E^2 = \frac{D_{ds}}{D_s D_d} \cdot \frac{4G}{c^2} \cdot M_{<\theta_E}}$$

Einstein radius

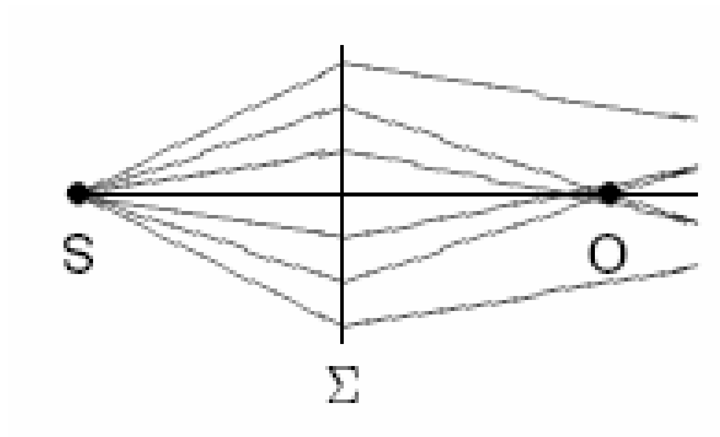
where $M_{<\theta E}$ is the projected mass within θ_E.

If the surface mass density has the value Σ_{cr} and is constant in ξ, we get a ideal convex lens. All light rays would then be focused in the point of observation:



For a typical gravitational lens Σ decreases as a function of the radius.

Therefore only at a certain radius the condition for a circular image is fulfilled:



Furthermore gravitational lenses are hardly ever spherically symmetric. For an elliptical mass distribution one observes only parts of the ring, the so called *arcs*.

Examples of Einstein angles θ_E

1. Galaxy clusters:

typical mass: $M \simeq 10^{14} M_\odot$

typical distances: $\simeq 1\, Gpc$

This leads to:

$$\theta_E \simeq 10'' \left(\frac{M}{10^{13} M_\odot} \right)^{1/2} \left(\frac{D}{Gcp} \right)^{-1/2}$$

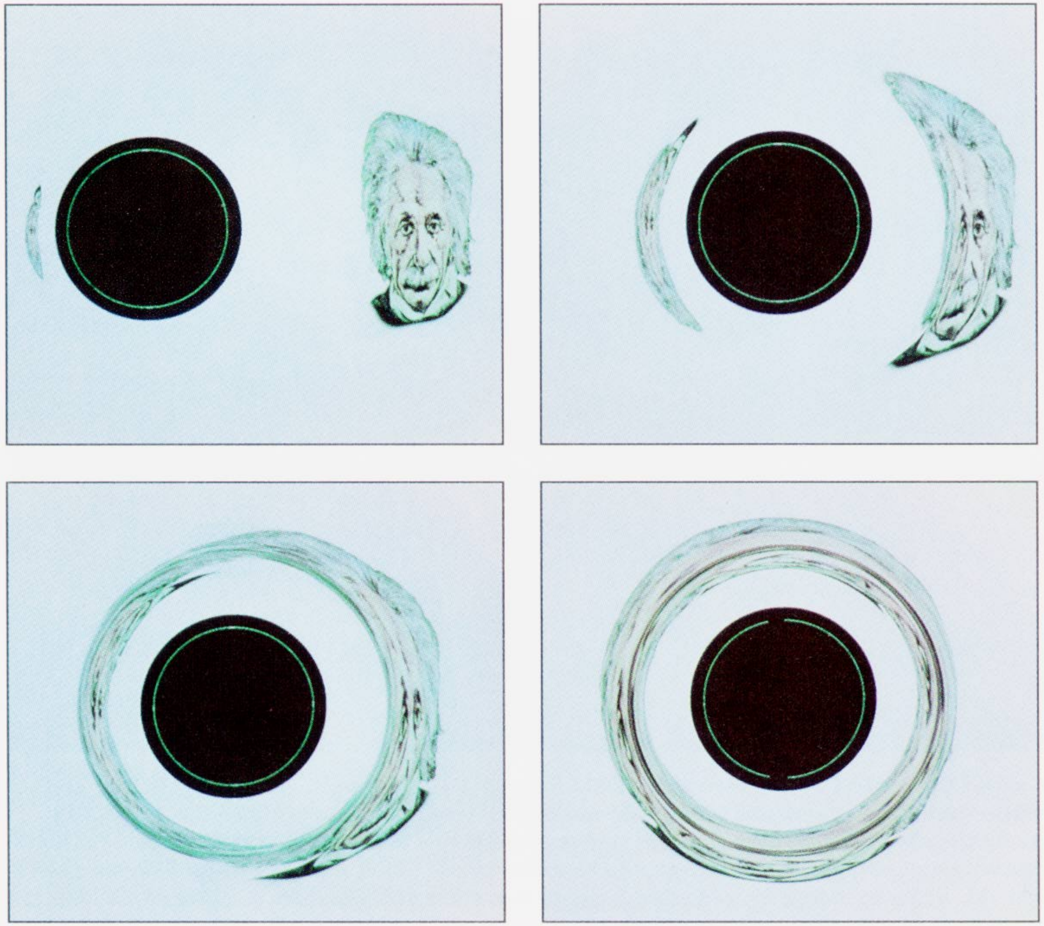
where $D = \dfrac{D_s \cdot D_d}{D_{ds}}$.

Thus for massive galaxy cluster (M > $10^{14}M_\Theta$ within a few hundreds of kpc) we get observable angles in the order of ten arcsecs.

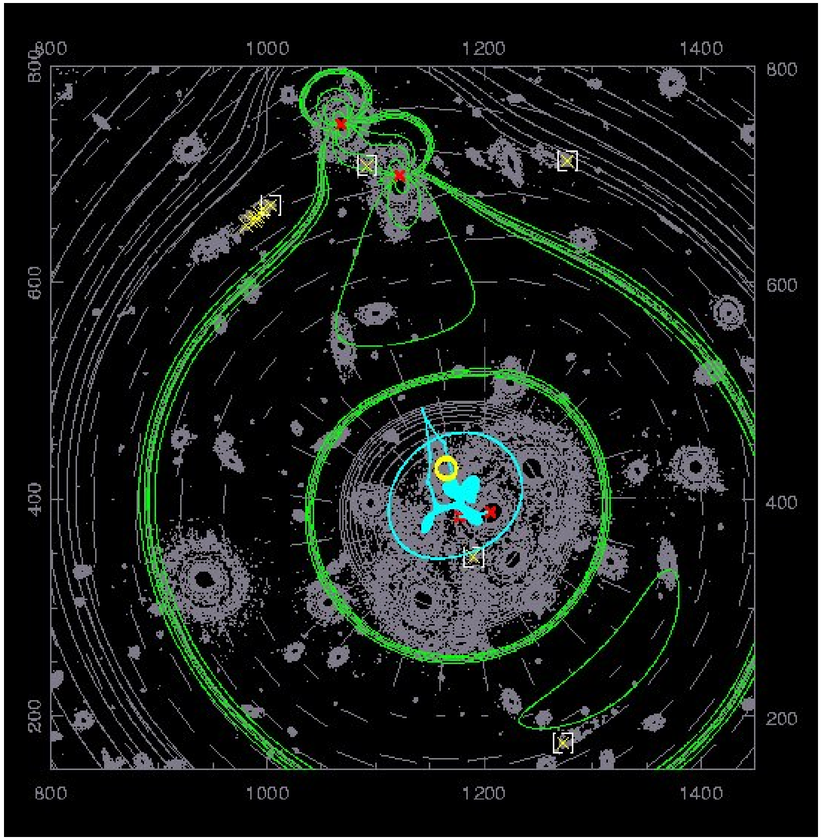
2. Stars (or similar objects) in the Milky Way:

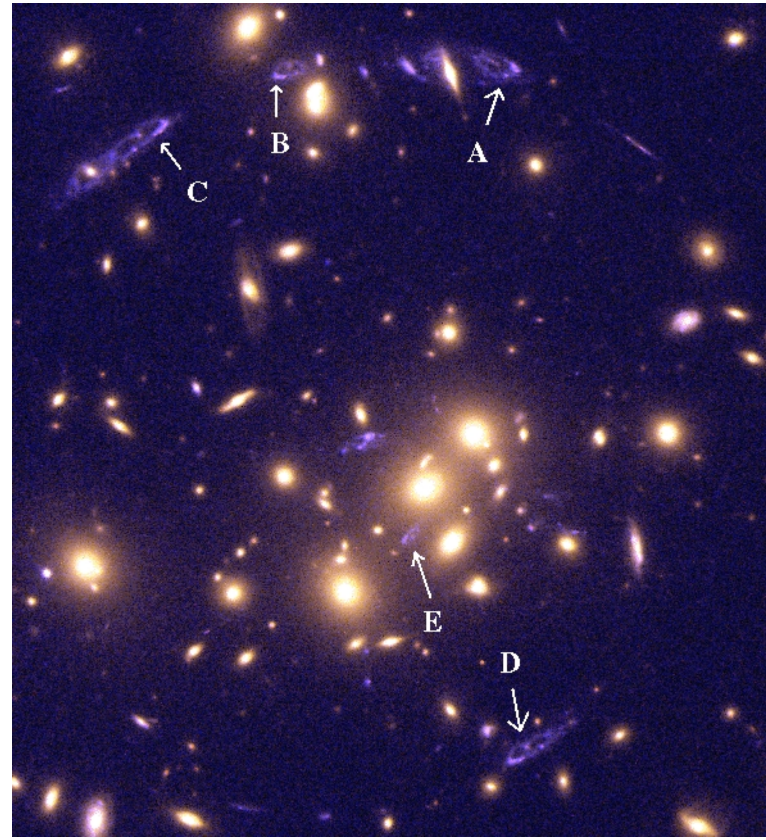
$$\theta_E \simeq 0.001'' \left(\frac{M}{M_\odot} \right)^{1/2} \left(\frac{D}{10kpc} \right)^{-1/2}$$

Such a tiny angle cannot be directly observed, but sometimes it is possible to detect the amplification it causes.



To make an Einstein ring, place a giant portrait of Albert Einstein far behind a black hole. In this computer-generated simulation of gravitational lensing, the central portion of the disk is the black hole itself. The thin white circle is formed by photons from the background wall that orbit the black hole before reaching the observer. The outside dark ring is the image of the dark universe behind the observer. Courtesy C. Zahn and H. Ruder.





Galaxy Cluster Cl0024+1645, strong lensing reconstruction (left, courtesy S. Seitz) of HST image (right, Colless et al.); light blue = caustic structure, bold green = critical lines of ’infinite’ amplification, squares = observed positions of multiple imaged source (A,B,C,D,E in color image), yellow crosses = predicted position of the lensmodel, yellow circle = position of source in source plane, red crosses = mass centers used for the lens model. The caustics are obtained by mapping the critical lines into the source plane.