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Statistical Parallaxes and Kinematical Parameters of Classical Cepheids and Young Star Clusters

A. S. Rastorguev, E. V. Glushkova, A. K. Dambis, and M. V. Zabolotskikh

Sternberg Astronomical Institute, Universitetskii pr. 13, Moscow, 119899 Russia

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Abstract—The statistical-parallax method is applied for the first time to space velocities of 270 classical Cepheids with proper motions adopted from HIPPARCOS (1997) and TRC (Hog *et al.* 1998) catalogs and distances based on the period-luminosity relation by Berdnikov *et al.* (1996). The distance scale of short-period Cepheids (with periods less than 9^d) is shown to require an average correction of 15–20%, whereas statistical parallaxes of Cepheids with periods > 9^d are found to agree well with photometric distances. It is shown that the luminosities of short-period Cepheids must have been underestimated partly due to the contamination of this subsample by a substantial (20 to 40%) fraction of first-overtone pulsators. The statistical-parallax technique is also applied for the first time to 117 open clusters younger than 100 million years and with proper motions reduced to the HIPPARCOS reference system. It is concluded that a $0.12–0\overset{m}{.}15$ increase of the distance scales of open clusters and Cepheids would be sufficient to reconcile the statistical-parallax results inferred for these two types of objects. Such approach leads to an LMC distance modulus of less than $18\overset{m}{.}40$, which agrees, within the errors, with the short distance scale for RR Lyrae variables and is at variance with the conclusions by Feast and Catchpole (1998) and Feast *et al.* (1998), who argue that the LMC distance modulus should be increased to $18\overset{m}{.}70$. The distance scale based on the Cepheid period-luminosity relation by Berdnikov and Efremov (1985) seems to be a good compromise. Extragalactic distances, which rely on long-period Cepheids, seem to require no substantial correction. In addition to statistical parallaxes, kinematical parameters have been inferred for the combined sample consisting of Cepheids and open-clusters: solar-motion components $(U_0, V_0, W_0) \approx (9, 12, 7)$ km s^{-1} (± 1km s^{-1}); velocity-ellipsoid axes $(\sigma_U; \sigma_V; \sigma_W) \approx (15.0, 10.3, 8.5)$ km s^{-1} (± 1 km s^{-1}); the angular velocity of rotation of the subsystem, $\omega_0 \approx 28.7 \pm 1$ km s^{-1} kpc^{-1}, the Oort constant $A \approx 17.4 \pm 1.5$ km s^{-1}, and the second derivative of angular velocity, $\omega_0'' \approx 1.15 \pm 0.2$ km s^{-1} kpc^{-3}.

INTRODUCTION

The distance-scale problem is among those of prime importance in modern astronomy. In this paper we do not consider the obvious aspects associated with measurement of distances. Instead we point out the main reasons why the problem has remained a focus of astronomers' interest over recent decades. As is well known, open clusters and classical Cepheids are the primary distance indicators among young (galactic-disk) population objects, whereas RR Lyrae variables serve the same function among the objects of old (halo) populations. The halo and disk distance scales are set by different techniques and, as a rule, are not entirely consistent with each other. The obvious requirement of matching these distance scales means that, if applied to the same object (say, to the LMC), they should yield similar distance values. Moreover, the inferred distances should agree with other data and theoretical results. The age of the Universe, as inferred from the Hubble constant, and the ages of globular clusters estimated from the main-sequence turnoff luminosity (Chaboyer *et al.* 1998) are especially sensitive to the adopted distance scale. The distance-scale problem thus touches upon the most fundamental underlying concepts of modern astrophysics.

The distance scale to RR Lyrae variables has been set by applying the statistical parallax analysis to these stars (Pavlovskaya 1953; van Herk 1965). The most reliable recent studies based on the same technique (Hawley *et al.* 1986; Layden *et al.* 1996; Heck and Fernley 1998; Fernley *et al.* 1998) and utilizing proper-motion data adopted from HIPPARCOS and other catalogs, as well as new radial-velocity measurements, imply that the mean absolute magnitude of RR Lyrae-type stars at [Fe/H] = –1.5 should lie within the $\langle M_V \rangle_{RR} = 0.73–0\overset{m}{.}78$ interval. These results agree well with the luminosity-metallicity relation derived using the Baade-Wesselink technique (Carney *et al.* 1992; Cacciari *et al.* 1992). Of all RR Lyrae variables in the HIPPARCOS catalog, the prototype—RR Lyrae itself—has the most confident trigonometric parallax. The absolute magnitude of this star, as inferred from the trigonometric parallax, $\langle M_V \rangle_{RR} = 0.78 \pm 0\overset{m}{.}29$, and its metallicity,

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[Fe/H] = –1.39 (Fernley *et al.* 1998), agree well with the estimates mentioned above. Note that high-velocity dispersion of RR Lyrae variables allowed their statistical parallaxes to be estimated rather accurately in earlier works, in spite of appreciable errors in the proper motions used.

The distance scale to classical Cepheids relies on the period-luminosity relation and, ultimately, on the distances to young open clusters. Berdnikov *et al.* (1996) derived a multicolor period-luminosity relation for fundamental-mode Cepheids using the data for nine Cepheid members of open clusters with distances based on Kholopov's (1980) ZAMS. The *V*-band relation is

$$\langle M_V \rangle_I = -1.01 - 2.87^m \log P_{\rm pls}, \qquad (1)$$

where $\langle M_V \rangle_I$ is the intensity-mean absolute magnitude. The distance scales of RR Lyrae variables and classical Cepheids given above agree well with each other, yielding an LMC distance modulus of $(m - M)_0 = 18.25 \pm 0\overset{m}{.}12$ and thereby favoring the case of the **short distance scale**. In particular, earlier analyses based on the period-luminosity relation by Berdnikov and Efremov (1985) [which is similar to relation (1)] yielded solar galactocentric distances of 7–7.5 kpc (Rastorguev *et al.* 1994; Dambis *et al.* 1995; Glushkova *et al.* 1998).

In view of the discussion above, it is now clear why measurement of the trigonometric parallaxes of classical Cepheids was one of the most important projects in the framework of the HIPPARCOS mission. However, contrary to the expectations, the completion of the project failed to unambiguously resolve the distance-scale problem. Thus, an analysis of HIPPARCOS trigonometric parallaxes of classical Cepheids (involving samples consisting of 20 to 200 stars) led Feast and Catchpole (1998) to conclude that the adopted Cepheid distance scale should be increased substantially, bringing the LMC distance modulus to $18.70 \pm 0\overset{m}{.}10$. However, a careful inspection of initial data shows that the overwhelming majority of HIPPARCOS Cepheids used by the authors mentioned above have their parallaxes measured with large fractional errors, a fact which casts strong doubts on the final results. Disregard of the fact that many Cepheids are components of binary systems (Szabados 1997)—at least 20% of the stars of the entire sample belong to this category, judging by the most modest estimates (Rastorguev *et al.* 1997)—adds up to the uncertainty. Berdnikov and Dambis (1998) showed that successive exclusion of Cepheids with the highest fractional parallax errors leads asymptotically to an LMC distance estimate that is close to the one implied by short distance scale (1). Therefore, further analysis is required to investigate whether it is possible to correctly use Cepheid trigonometric parallaxes measured with large errors.

Feast *et al.* (1998) adduced kinematical arguments in favor of higher Cepheid luminosities and a longer distance scale. Their conclusions are based on the fact that the Oort constant *A* inferred from radial-velocity data for the Cepheid sample exceeds significantly the value inferred from HIPPARCOS proper motions for the same stars. One of the versions of the statistical-parallax method consists in imposing the condition that both data types should yield the same constant. Dambis *et al.* (1995) applied this technique to a set of 218 Cepheid radial velocities and 194 proper motions adopted from the Four-Million star catalog (Kuimov *et al.* 1992) to find that the distance scale relying on the period-luminosity relation by Berdnikov and Efremov (1985), which is $0.15–0\overset{m}{.}18$ brighter than relation (1), requires no significant correction.

The so far open status of the Cepheid distance scale problem forces researchers to seek other solutions involving no trigonometric parallaxes, and statistical-parallax technique is one of the possible options. Until very recently, the use of the statistical parallax analysis has been limited to high-velocity stars because of its low accuracy and, most importantly, large and virtually unavoidable systematic errors of available proper motions. The point is that, only for samples with high dispersion of space velocities and high heliocentric velocities (e.g., RR Lyrae and other halo stars), low-accuracy proper motions still bear information about kinematical parameters and luminosities of the objects involved. Cepheids have small residual velocities (10–12 km s^{-1}) and therefore high-precision proper motions are required for the statistical-parallax technique to apply.

The publication of mass high-precision proper-motion catalogs HIPPARCOS (1997) and TRC (Hog *et al.* 1998; Kuz'min 1998) with low quoted systematic errors allowed for the first time the statistical parallax technique to be used to refine the luminosities of galactic-disk objects characterized by low dispersion of space velocities and low heliocentric velocities. HIPPARCOS gives absolute proper motions for 246 Cepheids with known photometric distances. The individual errors of the proper-motions components range from 0.0005 to 0.006 arcsec yr^{-1} with a median error equal to 0.0012 arcsec yr^{-1}. The errors do not exceed 0.002 and 0.003 arcsec yr^{-1}, for 83 and 96% of the stars, respectively. It can be easily shown that at a distance of 1 kpc the median error mentioned above translates into a linear velocity error of about 5 km s^{-1}, well below the dispersion of residual velocities. A statistical parallax analysis allows not only the luminosities to be refined but also a self-consistent solution to be found for all kinematical parameters of the sample, including those describing the rotation curve and the shape and the size of the velocity ellipsoid.

In this paper we used the statistical parallax technique, with allowance for the circular rotation law, to refine the distance scales and kinematical parameters of a classical Cepheid sample and of a genetically similar sample of young open clusters (Rastorguev *et al.* 1998).

METHOD OF ANALYSIS

The statistical-parallax technique is based on the simple idea of matching tangential and radial velocities for a sample of objects whose distances are inferred from known distance moduli (i.e., based on adopted luminosity values). The tangential components are computed from proper motions and distances, making them explicitly dependent on the adopted distances to objects drawn from a certain homogeneous sample, i.e., on the adopted luminosities of these objects. The optimum distance scale should balance radial and tangential velocities of sample objects and provide for a triaxial ellipsoidal distribution of residual velocities. In the most rigorous and coherent way, the basic underlying ideas of the statistical-parallax analysis have been described by Murray (1986). In this paper we derive basic relations in the form that is convenient for use in a computational algorithm.

In the first place, we assume that the residual space velocities of objects studied are distributed in accordance with (Schwartzschild's) ellipsoidal law. We further assume that the sizes of velocity-ellipsoid axes remain constant within the region considered, and that the axes themselves are pointed along the principal directions of the galactic coordinate system connected with the centroid under study (Fig. 1). Our sample occupies a wide solar neighborhood (up to heliocentric distances of 6 kpc) and therefore the model used should allow not only for differential rotation of the subsystem studied but also for the change of the velocity-ellipsoid orientation relative to the line of sight within the region in question. The observed space velocity of each object includes the following components: (a) heliocentric motion of the local centroid of the sample studied; (b) pure differential rotation of centroids (we disregard the radial centroid motions because the real expansion or contraction of the sample cannot be distinguished from effects due to spiral-arm induced perturbations); (c) dispersion of residual object velocities (relative to the centroid velocity), and (d) observational errors in radial velocities and proper motions. Our aim is to derive the distribution function for observed velocities and to determine its parameters.

The principal result obtained by applying the statistical-parallax technique is the refinement of a previously adopted distance scale for objects studied. Below we denote the computed (for the Cepheid sample, based on the period-luminosity relation) heliocentric distances as r_e (expected) and the true, or refined, distances as r_0.

Allowance for the solar motion and differential rotation of the sample. Figure 1 shows schematically the triangle Sun–Galactic center–S, where S is the centroid which we are considering, and the velocity ellipsoid which is connected with this centroid. Let the minor axis of this ellipsoid be parallel to the rotation axis of the Galaxy and the major axis point to the Galactic center. Here $\rho = r_0 \cos b$ is the true distance to

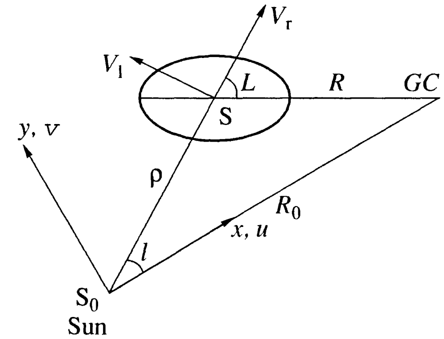

Fig. 1. Schematic sketch showing the mutual arrangement of the centroid S, of the local centroid S_0, and of the velocity ellipsoid projected onto the galactic plane. GC is the galactic center; R_0, the galactocentric distance of the Sun, and ρ, the distance of the object from the rotation axis of the galaxy. Angle L determines the orientation of the velocity ellipsoid relative to the line of sight and l is galactic longitude.

centroid S projected on the Galactic plane; V_r and V_l are the radial and tangential (along the galactic longitude) velocity components, respectively, and R_0 is the distance from the Sun to the Galactic center. The proper motions are in arcsec yr^{-1}; linear velocities, in km s^{-1}, and angular velocities, in km s^{-1} kpc^{-1}. For convenience, we introduce the factor k = 4738 km s^{-1} kpc^{-1} (arcsec yr^{-1})$^{-1}$ that enters formula $V_t = k\mu r_e$, where V_t is the tangential velocity component.

We now denote the local (circumsolar) centroid as S_0. Let us assume that the heliocentric velocity of the local sample is

$$\mathbf{V}_0 = \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix}, \qquad (2)$$

where the velocity components are in the Cartesian galactic coordinate system (the x-axis points to the galactic center, the y-axis points in the direction of Galactic rotation, and the z-axis points to the North Galactic Pole). The true galactocentric distance of the object, R, is computed from the true heliocentric distance r_0 and from the galactic coordinates as follows:

$$R^2 = R_0^2 + r_0^2 \cos^2 b - 2 r_0 R_0 \cos b \cos l.$$

Auxiliary angle L shown in Fig. 1, which determines the orientation of the velocity ellipsoid in the vicinity of centroid S relative to the line of sight, is given by the formula

$$\log L = \frac{R_0 \sin l}{R_0 \cos l - r_0 \cos b}.$$

The residual velocity distribution is most easily analyzed in the *local reference frame*, which is connected with the direction to the object and galactic coordinates l and b. The velocity components in this reference frame are either known from observations (radial velocity) or can be easily computed from the distance and the proper-motion components. The tangential velocities are computed using the expected distance r_e and therefore the velocity vector in the local reference frame can be written as follows:

$$\mathbf{V}_{\mathrm{loc,e}} = \begin{pmatrix} V_r \\ V_l \\ V_b \end{pmatrix} = \begin{pmatrix} V_r \\ k r_e \mu_l \\ k r_e \mu_b \end{pmatrix}. \quad (3)$$

The true contribution of galactic differential rotation is given by Bottlinger formulas (Kulikovskii 1985), which can be written in terms of the true distance r_0 as follows:

$$\mathbf{V}_{\mathrm{rot,0}} = \begin{pmatrix} R_0(\omega - \omega_0)\sin l \cos b \\ (R_0 \cos l - r_0 \cos b)(\omega - \omega_0) - r_0 \omega_0 \cos b \\ -R_0(\omega - \omega_0)\sin l \sin b \end{pmatrix}, \quad (4)$$

where $\omega(R)$ and $\omega_0 = \omega(R_0)$ are the angular velocity of the centroid studied at a galactocentric distance R and at a solar galactocentric distance, respectively. In our case we can expand the difference in angular velocities into a Taylor series leaving only the second- and lower-order terms. Such an expansion yields good results even at heliocentric distances as high as 5–6 kpc:

$$(\omega - \omega_0) \approx \omega_0'(R - R_0) + \frac{1}{2}\omega_0''(R - R_0)^2 + \dots. \quad (5)$$

We finally introduce the principal unknown term—the distance-scale factor that relates the true and the computed (expected) distances:

$$p = \frac{r_e}{r_0}, \quad (6)$$

and the distance-scale matrix:

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix},$$

which transforms the true velocity components into expected velocity components in the local reference frame:

$$\mathbf{V}_{\mathrm{loc,e}} = P \times \mathbf{V}_{\mathrm{loc,0}}.$$

The radial velocity is independent of the adopted distance and therefore the first diagonal element in matrix P is equal to unity.

Transformation of the coordinates, of the velocities, and of the covariance tensor. The relation between velocity-dispersion components in the local reference frame and the axes of velocity ellipsoid S is determined by the orientation of the basis vectors of the local right-hand frame relative to the principal axes of the velocity ellipsoid, i.e., by a pair of angles (b, L).

Let $\mathbf{e}_S$ be a unit column vector in the frame connected with the principal axes of ellipsoid S and let $\mathbf{e}_{\mathrm{loc}}$ be the same unit vector in the local frame. The components of these vectors are related to each other by a rotation that transforms the principal axes of velocity ellipsoid into the local-frame axes $\mathbf{e}_{\mathrm{loc}} = G_S \times \mathbf{e}_S$. The matrix of this transformation is

$$G_S = \begin{pmatrix} \cos b \cos L & \cos b \sin L & \sin b \\ -\sin L & \cos L & 0 \\ -\sin b \cos L & -\sin b \sin L & \cos b \end{pmatrix}. \quad (7)$$

We assume that the shape and the size of the velocity ellipsoid is given by the velocity-dispersion tensor with constant coefficients. In the reference frame connected with principal axes the latter tensor can be written as

$$L_{S,0} = \begin{pmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_w^2 \end{pmatrix}.$$

Then the true covariance tensor in the local reference frame is

$$L_{\mathrm{loc,0}} = G_S \times L_{S,0} \times G_S^{\mathrm{T}},$$

where G_S^{T} is the transposed matrix. After reduction to the distance scale used, the covariance tensor acquires the following form:

$$L_{\mathrm{loc,e}} = P \times G_S \times L_{S,0} \times G_S^{\mathrm{T}} \times P^{\mathrm{T}}.$$

We now allow for the errors in the initial data. The resulting observed velocity distribution in the local frame is described by the following modified covariance tensor:

$$L_{\mathrm{obs}} = L_{\mathrm{loc,e}} + L_{\mathrm{err}}, \quad (8)$$

where the tensor of observational errors can be written as follows:

$$L_{\mathrm{err}} = \begin{pmatrix} \sigma_{Vr}^2 & 0 & 0 \\ 0 & k^2 r_e^2 \sigma_{\mu l}^2 & 0 \\ 0 & 0 & k^2 r_e^2 \sigma_{\mu b}^2 \end{pmatrix},$$

Here $(\sigma_{Vr}, \sigma_{\mu l}, \sigma_{\mu b})$ are standard errors of radial velocities and proper-motion components. The aim of our analysis is to estimate the total correction factor for the adopted distance scale and here we disregard the small cosmic errors in individual distances.

We now write the contribution to the observed velocity due to the heliocentric motion of the local sample. In view of (2) we have

$$\mathbf{V}_{S_0} = G_0 \times \mathbf{V}_0,$$

where, by analogy with (7), l plays the part of L. The rotation matrix for the local centroid is

$$G_0 = \begin{pmatrix} \cos b \cos l & \cos b \sin l & \sin b \\ -\sin l & \cos l & 0 \\ -\sin b \cos l & -\sin b \sin l & \cos b \end{pmatrix}.$$

The distribution of residual velocities and the likelihood function. In view of the discussion above, after subtracting all systematic motions (i.e., the solar motion relative to the local sample and differential rotation), the local-frame residual velocity $\Delta\mathbf{V}$ relative to centroid S located at (r, l, b) becomes

$$\Delta\mathbf{V} = \mathbf{V}_{\mathrm{loc,e}} - P \times G_0 \times \mathbf{V}_0 - P \times \mathbf{V}_{\mathrm{rot},0}, \qquad (9)$$

where the observed velocity $\mathbf{V}_{\mathrm{loc,e}}$ is given by formula (3). The distance scale is adjusted by multiplying the velocity components (2) and (4) based on true distances by distance-scale matrix P, and therefore all velocities in (9) are reduced to the adopted distance scale.

The distribution function of residual velocities can be written in the following general form [see p. 284 in Murray (1986)]:

$$f(\Delta\mathbf{V}) = (2\pi)^{-3/2} |L_{\mathrm{obs}}|^{-1/2} \exp\left\{-\frac{1}{2}\Delta\mathbf{V}^{\mathrm{T}} \times L_{\mathrm{obs}}^{-1} \times \Delta\mathbf{V}\right\}, \qquad (10)$$

where $|L_{\mathrm{obs}}|$ and L_{obs}^{-1} are the determinant and the inverse of the observed covariance tensor L_{obs} (8), respectively.

We then apply the maximum-likelihood method to determine the unknown parameters of the distribution function given above, while minimizing the likelihood function

$$-LF = -\ln F(\Delta\mathbf{V}_1, \ldots, \Delta\mathbf{V}_N) = -\ln\left[\prod_{i=1}^{N} f(\Delta\mathbf{V}_i)\right] = -\sum_{i=1}^{N} \ln f(\Delta\mathbf{V}_i),$$

The likelihood function can be written in a form that is more convenient for computations:

$$LF = \frac{3}{2} N \ln 2\pi + \frac{1}{2}\sum_{i=1}^{N} [\ln|L_{\mathrm{obs}}(i)| + (\Delta\mathbf{V}_i^{\mathrm{T}} \times L_{\mathrm{obs}}^{-1}(i) \times \Delta\mathbf{V}_i)], \qquad (11)$$

where i refers to the current object in the sample, and N is the total number of objects.

OBSERVATIONAL DATA AND COMPUTATIONS

Classical Cepheids. We adopted the distances of classical Cepheids based on period-luminosity relation (1) from a catalog by Berdnikov *et al.* (1998). This catalog gives the parameters of multicolor light curves, periods, and heliocentric distances for 449 classical Cepheids, most of which are fundamental-mode pulsators. We adopted radial velocities, V, and their standard errors from a compiled list by Dambis *et al.* (1995), which we updated using data from Gorynya *et al.* (1996), Pont *et al.* (1997), and Metzger *et al.* (1998). We computed components μ_l and μ_b from HIPPARCOS data and, for 19 Cepheids absent in HIPPARCOS, from the TRC catalog. The latter gives proper motions for ~990 thousand stars in the supporting TYCHO catalog. These proper motions have been computed from the differences of TYCHO and the Astrographic catalog positions reduced to the HIPPARCOS system (Kuimov *et al.* 1998).

We used individual proper-motion errors quoted in the catalog for all HIPPARCOS Cepheids and assigned a single root mean square error ($\sigma_\mu = \sigma_{\mu l} = \sigma_{\mu b}$), which we inferred by applying statistical-parallax technique to the same sample of Cepheids with TRC proper motions (a total of 203 stars), to a few stars whose proper motions have been adopted from the TRC. We found = 0.0036 arcsec yr^{-1}, in good agreement with the median error quoted in the TRC catalog. (Note that, when applied to HIPPARCOS proper motions, this procedure yields a mean error of 0.0022 arcsec yr^{-1}). Furthermore, to make our sample more complete, we expanded it by including less accurate proper motions adopted from the Four-Million Star Catalog (Kuimov *et al.* 1992), below referred to as 4M. The standard errors of these proper motions have been estimated by Glushkova *et al.* (1996).

Our initial sample thus consisted of a total of 270 classical Cepheids with heliocentric distances < 6 kpc with both radial velocities and proper motions available. Proper-motion sources are distributed as follows: HIPPARCOS—230; TRC—19, and 4M catalog—21. We performed separate analysis for each of the two Cepheid subsamples: short-period stars with $P_{\mathrm{pls}} < 9^{\mathrm{d}}$ and long-period stars with $P_{\mathrm{pls}} > 9^{\mathrm{d}}$. These subsamples differ in the mean age, kinematical parameters (Dambis *et al.* 1995) and, possibly, pulsation mechanism (Fadeev 1994). The results are summarized in Table 1.

We did not attempt to simultaneously refine the distance scale and infer the distance to the Galactic center because these two parameters are strongly correlated. When refining the distance scale, we adopted R_0 = 7.5 kpc, which agrees within the errors with recent results by different authors ranging from 7 to 8 kpc (Nikiforov 1994; Nikiforov and Petrovskaya 1994;

Table 1. Kinematical parameters and correction factors to the Cepheid distance scale

N	R_0, kpc	P_{pls}, days	Δr, kpc	u_0, km s^{-1}	v_0, km s^{-1}	w_0, km s^{-1}	σ_U, km s^{-1}	σ_V, km s^{-1}	σ_W, km s^{-}	ω_0, km s^{-1} kpc^{-1}	ω_0', km s^{-1} kpc^{-2}	ω_0'', km s^{-1} kpc^{-3}	p
Cepheids with proper motions from HIPPARCOS and TRC													
249	8.0	All	0–6	−8.95	−12.10	−7.67	15.25	10.28	8.55	28.46	−4.15	0.92	0.879
249	7.5	All	0–6	−8.90	−11.73	−7.51	15.24	10.26	8.36	28.73	−4.54	1.05	0.899
Errors:				±1.58	±1.28	±1.23	±1.24	±0.91	±1.24	±1.16	±0.27	±0.18	±0.057
168	8.0	<9	0–6	−9.97	−12.07	−8.68	14.62	10.33	8.29	30.02	−4.13	0.89	0.822
168	7.5	<9	0–6	−9.87	−11.70	−8.51	14.55	10.37	8.13	30.29	−4.50	1.00	0.839
Errors:				±1.82	±1.48	±1.44	±1.39	±1.05	±1.49	±1.50	±0.36	±0.23	±0.064
81	8.0	>9	0–6	−7.41	−13.28	−5.79	15.70	10.64	9.53	26.62	−4.36	1.13	0.997
81	7.5	>9	0–6	−7.42	−12.83	−5.67	15.75	10.54	9.30	26.85	−4.76	1.29	1.018
Errors:				±3.26	±2.71	±2.66	±2.42	±1.90	±2.99	±2.10	±0.50	±0.41	±0.115
213	7.5	All	0–3	−8.89	−11.16	−7.59	15.13	9.59	8.38	28.65	−4.45	0.79	0.895
149	7.5	<9	0–3	−10.20	−11.54	−8.70	14.74	9.40	8.31	29.49	−4.25	0.83	0.821
64	7.5	>9	0–3	−6.30	−11.63	−5.46	15.28	11.11	8.48	28.03	−5.13	0.93	1.099
187	7.5	All	1–6	−9.29	−12.97	−8.33	14.77	10.64	11.88	28.52	−4.67	1.18	0.917
115	7.5	<9	1–6	−9.77	−12.59	−9.13	14.97	10.75	12.31	30.21	−4.67	1.13	0.872
72	7.5	>9	1–6	−9.00	−14.15	−7.17	13.77	10.50	11.18	26.52	−4.75	1.34	0.990
Cepheids with proper motions from HIPPARCOS, TRC, and the Four-Million Star Catalog													
270	7.5	All	0–6	−8.87	−11.52	−7.42	15.27	10.14	8.27	29.08	−4.58	1.00	0.914
180	7.5	<9	0–6	−9.45	−11.26	−8.50	14.73	10.29	8.16	30.66	−4.48	0.88	0.840
90	7.5	>9	0–6	−8.11	−13.33	−5.62	15.34	10.45	8.97	26.93	−4.89	1.41	1.061

Rastorguev *et al.* 1994; Dambis *et al.* 1995; Layden *et al.* 1996, and Glushkova *et al.* 1998). A further analysis showed (Table 1) that increasing R_0 to 8 kpc has virtually no effect on either the retrieved kinematical parameters of the entire sample (except for the Oort constant A and the second derivative of the angular velocity) or on the inferred distance-scale factor p.

Young open clusters. Our sample includes 117 open clusters that are younger than 100 Million years and located at heliocentric distances < 5 kpc (as inferred by Dambis (1998) using Kholopov's (1980) ZAMS with allowance for evolutionary deviation curves). The initial absolute proper motions of these clusters have been computed by Glushkova *et al.* (1996, 1997). We adopted the radial velocities of 40 clusters from a list compiled by Hron (1987). The radial velocities of the remaining clusters were derived partly through a critical analysis of the data for individual cluster members given in Mermilliod' (1988, 1992) data base, and partly inferred from our own radial-velocity measurements (Glushkova and Rastorguev 1991).

To reduce the absolute proper motions of open clusters to the HIPPARCOS system, we applied zonal corrections for the mean open-cluster proper motions which Glushkova *et al.* (1996, 1997) computed from the 4M catalog data. We considered two variants of reduction.

(1) Using HIPPARCOS stars. There are few HIPPARCOS stars among cluster members and therefore we computed zonal corrections from all stars in common (whether members or not) for HIPPARCOS and the 4M catalog and located within 2° × 2° fields centered on each cluster (in some cases we increased the field size to 3° × 3°). For these stars we computed individual proper-motion differences $\Delta\mu_\alpha = \mu_\alpha$(HIPPARCOS) – μ_α(4M) and $\Delta\mu_\delta = \mu_\delta$(HIPPARCOS) – μ_δ(4M) and applied the median values of $\Delta\mu_\alpha$ and $\Delta\mu_\delta$ to reduce the initial proper motions of open clusters to the HIPPARCOS system. There were, on the average, 10 to 20 stars in common for both catalogs for each open cluster.

(2) Using stars of the preliminary version of the TRC catalog (Volchkov 1997). This catalog includes more than 1 million stars with proper motions computed in the same way as in the final TRC version and, like TRC data, have been reduced to the HIPPARCOS system. The preliminary version (below referred to as TYC) differs from the TRC catalog mainly in the reduction algorithm applied to Astrographic Catalog data. We computed the corrections to the initial proper motions using stars within 2° × 2° fields (100 to 150 stars per cluster) centered on open clusters and applying the same technique as described above for the first reduction variant. The distribution of individual proper-motion differences fits well a Gaussian law with an average

standard deviation of 0.0035–0.0045 arcsec yr^{-1}. The open cluster data are summarized in Table 2.

Like in the case of the TRC catalog, we applied the statistical-parallax technique (11) to the entire cluster sample in order to derive a preliminary estimate of the errors in the reduced proper motions of open clusters (we fixed the distance-scale factor at $p = 1$). Standard errors of proper-motion components, σ_μ, included in the observed covariance tensor proved to be equal to 0.0042 and 0.0043 arcsec yr^{-1} for the first and the second reduction algorithm, respectively, in good agreement with the scatter of individual proper-motion differences. Somewhat lower error inferred in the second version must be due to a much greater number of stars used in the reduction. Note that minimizing likelihood function (11) with both p and σ_μ treated as free parameters has little effect on the resulting σ_μ values, which we used in subsequent computations.

The distance-scale factors and kinematical parameters inferred for the open-cluster sample are summarized in Table 3. Note that the relatively low quality of open-cluster proper motions involved did not allow us, in contrast to the Cepheid case, to estimate either the minor axis σ_w of the velocity ellipsoid or (for three versions listed in Table 3) the mean vertical velocity component of the sample objects. We therefore fixed these quantities at $\sigma_w = 8.5$ km s^{-1} and $w_0 = -7$ km s^{-1}, respectively, assuming that the values of these parameters are the same for the Cepheid and open-cluster sample. In support of this assumption, we point out that, according to Tables 1 and 3, the remaining solar-velocity and velocity-dispersion tensor components of the two samples are virtually identical.

Figure 2 gives a plot of the proper-motion component μ_l as a function of galactic longitude, l, showing a well-defined effect of differential-rotation pattern. Table 4 lists the inferred rotation-curve parameters for the open-cluster sample computed separately from radial velocities and proper motions using the technique applied by Dambis *et al.* (1995): we first estimated u_0 and v_0 solar-motion components from radial velocities and then fixed the resulting values and solved the proper-motion equation set.

We analyzed the possible biases in the parameters returned by the statistical-parallax technique. To this end, we simulated the open-cluster sample by fixing the actual coordinates and distances of individual clusters and the initial values of kinematical parameters and adding random errors to cluster velocities. We set the standard errors of simulated radial velocities equal to the quoted errors and varied the proper motion errors from 0.002 to 0.004 arcsec yr^{-1}. The simulations showed the resulting distance-scale coefficient, p, to be slightly biased toward negative values, on the average, no more than by 0.02. Moreover, the scatter of the p values returned by statistical-parallax technique applied to simulated samples turned out to be well below the standard error. The negative bias of the derivative ω'_0 did not exceed 0.10, resulting in the Oort constant A being underestimated by 0.38. No significant biases were found in the other parameters.

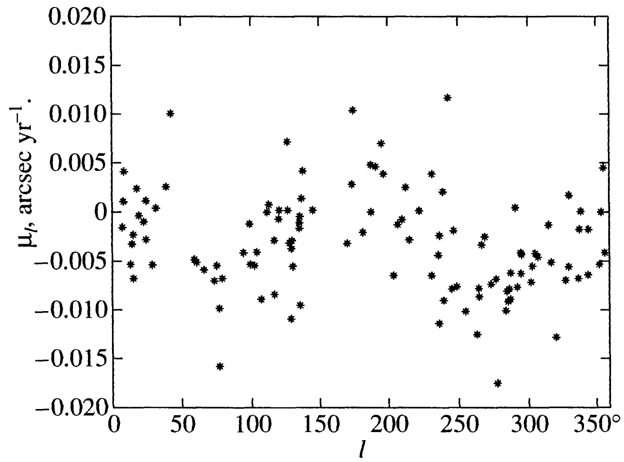

Fig. 2. Proper-motion components μ_l of the open clusters as a function of the galactic longitude.

To minimize the nonlinear likelihood function (11), we used the Gauss–Newton and Levenberg–Marquardt algorithms with interpolation and/or gradient search and a slower Nelder–Mead algorithm of deformable polyhedrons (also called the simplex-algorithm), which, however, requires no evaluation of the derivatives of the function to be minimized [see pp. 298, 593 in Press *et al.* (1987)].

To estimate the standard errors in the inferred parameters, we used the following algorithm suggested by Hawley *et al.* (1986). Let α_i^0 be one of the derived parameters and let LF_0 be the minimum value of the likelihood function at the true solution. We fix the parameter in question at $\alpha_i = \alpha_i^0 + \Delta\alpha_i$, where $\Delta\alpha_i$ is a small deviation from $\Delta\alpha_i$. We then allow the other parameters to converge to a new minimum solution with the likelihood function equal to $LF_i > LF_0$. The standard error of the parameter α_i is then given by:

$$\sigma_i^2 = \frac{\Delta\alpha_i^2}{LF_i - LF_0}.$$

RESULTS AND DISCUSSION

An analysis of Table 1 shows that the two Cepheid subsamples—the short-period and the long-period ones (with a boundary set at period 9^d)—yield significantly different distance-scale factors in all solutions (which differ in the heliocentric distance intervals covered). On the average, Cepheids with $P_{pls} < 9^d$ yield $p \approx 0.82$–0.87, whereas $p \approx 0.99$–1.10 for Cepheids with $P_{pls} > 9^d$. The entire Cepheid sample yields intermediate values

Table 2. Open cluster data

Cluster	l	b	r, kpc	V_r, km s^{-1}	σ_V, km s^{-1}	HIPPARCOS		TYCHO	
						μ_α, 0.001 arcsec yr^{-1}	μ_δ, 0.001 arcsec yr^{-1}	μ_α, 0.001 arcsec yr^{-1}	μ_δ, 0.001 arcsec yr^{-1}
1	2	3	4	5	6	7	8	9	10
BAS3	111.4	–0.2	2.53	–81.0	2.0	3.2	–1.3	0.5	–1.3
BE86	76.7	1.3	0.78	–19.3	7.8	–7.1	–11.7	–10.3	–12.1
BOCHUM2	212.1	–1.3	6.42	66.3	4.4	8.8	–0.2	5.5	2.0
CR121	235.4	10.4	0.63	35.0	1.0	–5.7	3.1	–2.7	1.1
CR135	248.8	11.2	0.27	33.7	8.0	–9.4	7.5	–11.1	1.2
CR140	245.2	–7.9	0.34	10.5	0.7	–11.0	2.8	–8.0	4.7
CR197	261.7	8.9	1.06	33.1	3.2	–22.3	24.1	–21.3	24.8
CR223	286.2	–1.9	2.13	2.0	1.0	–13.3	–2.0	–9.1	2.4
CR228	287.5	–1.0	2.29	–12.0	6.0	–11.9	1.5	–6.3	1.5
CR394	14.7	–9.0	0.60	6.0	0.3	–0.3	–5.9	–3.4	–5.8
Carina	287.5	–0.6	2.58	–20.0	5.0	–14.0	2.3	–9.2	1.8
DO25	211.9	–1.3	2.99	70.9	30.0	3.1	0.0	7.8	1.3
HAFFNER19	243.0	0.5	4.09	68.0	6.0	5.6	–21.9	5.0	–24.0
HOGG16	307.5	1.3	1.90	–35.8	0.4	–6.0	–2.2	–3.9	–5.5
IC1805	134.7	1.0	1.93	–26.5	23.0	0.2	–0.3	–0.3	2.0
IC1848	137.2	0.1	1.89	–22.0	13.0	4.0	–3.0	2.9	–3.5
IC2395	266.6	–3.8	0.66	0.5	2.0	–4.0	3.2	–1.2	3.2
IC2581	284.6	0.0	2.16	–6.0	7.0	–12.9	–4.2	–11.7	–3.3
IC2944	294.6	–1.4	2.07	–0.9	6.0	–2.8	–4.4	–4.8	–1.4
IC4665	30.6	17.1	0.33	–13.0	3.0	0.8	–4.5	4.8	–2.0
IC4725	13.6	–4.5	0.53	3.0	0.3	–1.9	–5.7	0.9	–4.1
IC4996	75.4	1.3	1.35	–22.0	29.0	3.2	–9.1	2.9	–8.5
LY6	330.4	0.3	2.04	–59.4	0.6	3.6	1.0	1.8	0.4
MARK18	269.2	–1.8	1.52	8.5	2.0	–3.0	0.6	–4.2	–0.2
MARK38	12.0	–0.9	1.71	–18.0	0.4	–2.1	–6.4	–2.2	–4.8
MARK6	134.7	0.0	0.67	–8.0	15.0	–7.4	2.8	–8.6	4.1
NGC103	119.8	–1.4	2.75	–11.0	30.0	1.8	–2.4	–0.4	–2.3
NGC129	120.3	–2.6	1.56	–36.8	0.3	3.7	–1.1	0.5	–3.7
NGC1502	143.6	7.6	0.71	–18.0	13.0	2.0	1.3	1.0	0.8
NGC1647	180.4	–16.8	0.41	–2.0	4.4	–3.4	2.4	–1.1	1.8
NGC1778	168.9	–2.0	1.52	10.0	2.0	–0.5	0.6	2.2	5.7
NGC1893	173.6	–1.7	2.63	–4.7	4.4	–2.1	–6.4	–0.1	–3.4
NGC1960	174.5	1.0	1.09	–4.0	13.0	17.5	–7.8	14.7	–2.7
NGC2129	186.6	2.2	0.76	–6.3	1.4	8.0	–2.8	6.2	–1.9
NGC2168	186.6	0.1	1.65	17.7	0.2	–4.4	–4.3	–3.0	–1.7
NGC2169	195.6	–2.9	0.99	16.6	6.0	–3.9	–4.9	–2.7	–5.8
NGC2175	190.2	0.4	1.79	22.0	7.7	–1.6	–7.0	1.2	–4.6
NGC2232	214.4	–7.7	0.38	20.0	10.0	–5.7	–1.4	–3.5	1.5
NGC2244	206.4	–2.0	1.43	36.6	5.5	–0.3	–0.5	–1.3	0.7
NGC2264	202.9	2.2	0.72	25.5	5.5	–9.6	0.0	–8.6	2.8
NGC2323	221.7	–1.2	0.91	9.5	4.0	1.6	0.6	3.2	1.6
NGC2362	238.2	–5.5	1.27	42.2	0.2	–1.9	–3.0	–1.5	–3.0

Table 2. (Contd.)

1	2	3	4	5	6	7	8	9	10
NGC2367	235.6	−3.9	1.99	40.6	2.8	−7.2	−1.4	−8.8	0.4
NGC2384	235.4	−2.4	3.16	50.6	7.0	−13.6	5.8	−7.0	9.2
NGC2414	231.4	2.0	3.16	64.5	1.0	0.6	−3.6	5.0	−1.6
NGC2422	231.0	3.1	0.42	36.0	4.0	−7.5	3.3	−5.6	4.4
NGC2439	246.4	−4.4	3.80	68.0	0.8	−0.5	1.0	−0.5	1.9
NGC2467	243.1	0.4	4.57	65.7	11.0	4.2	−9.0	4.0	−11.0
NGC2516	273.9	−15.9	0.37	22.0	3.6	−2.9	2.4	−3.6	6.3
NGC2546	254.9	−2.0	0.51	15.0	3.0	−9.8	10.7	−1.8	11.0
NGC2547	264.6	−8.6	0.38	14.5	3.0	−11.1	2.5	−8.8	3.7
NGC3114	283.3	−3.8	0.95	1.0	2.0	−9.9	3.5	−8.8	5.1
NGC3293	285.9	0.1	2.15	−13.0	8.0	−12.6	−2.4	−10.6	−2.4
NGC3572	290.7	0.2	2.39	−4.1	1.7	−0.5	2.7	1.5	2.6
NGC3590	291.2	−0.2	2.18	−1.0	30.0	−8.2	3.0	−7.2	2.6
NGC3766	294.1	0.0	1.69	−19.0	7.0	−3.0	−1.3	−3.6	2.2
NGC436	126.1	−3.9	2.59	−74.4	0.3	1.9	−6.1	−0.4	−6.7
NGC4463	300.7	−2.0	1.01	−14.6	4.0	−11.0	−2.1	−7.3	−0.8
NGC457	126.6	−4.4	2.42	−34.0	9.0	8.3	9.9	8.0	8.1
NGC4755	303.2	2.5	1.89	−20.2	6.5	−7.1	−3.6	−5.6	−2.6
NGC5606	314.9	1.0	1.80	−37.1	3.0	−0.7	0.4	−3.3	4.8
NGC5662	316.9	3.5	0.72	−23.2	1.0	−5.6	−7.9	−3.4	−5.3
NGC581	128.0	−1.8	2.60	−37.0	10.0	−5.5	0.0	−3.0	0.0
NGC6067	329.8	−2.2	1.73	−39.9	0.2	−10.8	−2.0	−8.4	0.7
NGC6087	327.8	−5.4	0.82	5.7	0.2	−7.2	−4.6	−6.6	−3.2
NGC6178	338.4	1.2	0.88	4.5	3.0	0.4	−1.8	−0.2	−2.3
NGC6193	336.7	−1.6	1.19	−24.3	10.0	−2.7	−6.8	−2.4	−7.0
NGC6204	338.6	−1.2	2.39	−52.7	6.0	−1.5	0.7	−2.0	1.7
NGC6231	343.5	1.2	1.40	−25.0	8.0	2.1	−4.9	−1.5	−1.1
NGC637	128.5	1.7	2.37	−46.0	10.0	−2.9	4.1	−3.3	1.5
NGC6383	355.1	0.1	1.18	6.8	7.0	2.4	−0.6	5.2	1.8
NGC6396	354.0	−1.9	1.11	−29.0	2.5	0.2	−2.8	4.0	−2.7
NGC6405	356.6	−0.7	0.45	10.4	1.1	−1.1	−6.2	−2.6	−3.3
NGC6514	7.0	−0.3	0.83	−22.4	20.0	0.7	−2.6	4.5	−1.3
NGC6530	6.1	−1.4	1.27	−30.0	11.0	1.4	−1.7	2.6	−3.3
NGC6531	7.7	−0.4	1.03	−16.0	6.0	3.0	3.4	1.6	3.8
NGC654	129.1	−0.4	2.45	−33.8	1.4	−1.0	1.6	−3.0	−0.4
NGC6604	18.3	1.7	2.39	18.5	14.0	1.3	−0.6	0.4	−0.6
NGC6611	17.0	0.8	1.74	23.5	6.5	2.4	−0.8	3.4	0.8
NGC6613	14.2	−1.0	1.13	−14.0	9.0	−0.7	−2.3	2.3	−3.8
NGC663	129.5	−1.0	1.81	−32.0	2.0	−4.4	−2.6	−5.8	−0.7
NGC6649	21.6	−0.8	1.66	−8.8	0.8	−6.8	0.0	−4.0	1.0
NGC6664	24.0	−0.5	1.25	17.8	1.0	5.1	−4.8	4.0	−5.3
NGC6694	23.9	−2.9	1.32	−9.2	0.5	1.5	1.4	1.1	0.8
NGC6709	42.2	4.7	0.82	−21.0	14.0	1.7	3.7	−2.8	12.7
NGC6755	38.6	−1.7	1.88	18.7	5.8	5.6	0.3	7.3	−0.9
NGC6823	59.4	−0.1	1.91	11.0	6.0	1.5	−4.5	0.7	−6.3

Table 2. (Contd.)

1	2	3	4	5	6	7	8	9	10
NGC6834	65.7	1.2	2.14	–6.8	1.4	–2.5	–3.8	–3.0	–5.0
NGC6871	72.6	2.1	1.51	–7.7	3.2	0.3	–8.8	–0.7	–7.8
NGC6910	78.7	2.0	1.42	–30.0	7.0	–1.8	–1.7	–4.3	–5.2
NGC6913	76.9	0.6	1.15	–34.0	14.0	–0.2	–11.6	–3.8	–9.3
NGC7086	94.4	–5.5	0.97	–5.6	1.7	–13.0	7.4	–13.8	10.3
NGC7160	104.0	6.5	0.67	–20.0	10.0	–2.4	–2.2	–4.4	–1.1
NGC7235	102.7	0.8	2.61	–52.0	4.0	–5.6	1.2	–7.6	1.4
NGC7243	98.9	–5.6	0.76	–16.0	9.0	1.2	–2.8	–0.1	–2.0
NGC7380	107.1	–0.9	2.55	–39.0	14.0	–11.7	–7.4	–6.7	–6.3
NGC7654	112.8	0.5	1.44	–35.5	1.0	1.7	2.1	–0.6	3.9
NGC7788	116.4	–0.8	2.29	–55.0	30.0	0.3	–1.6	–3.1	0.6
NGC7790	116.6	–1.0	2.66	–78.0	0.5	–6.4	–4.4	–8.0	–2.8
NGC869	134.6	–3.7	1.87	–52.7	6.0	–0.9	–4.4	–1.5	0.4
NGC884	135.1	–3.6	1.99	–40.5	8.0	–1.5	–2.4	–0.5	0.0
NGC957	136.2	–2.7	1.72	–36.0	13.0	3.8	–1.2	0.5	–2.4
Orion(CR69)	195.1	–12.0	0.36	35.0	1.0	3.1	–2.1	5.5	–4.8
PIS16	277.8	0.7	2.08	11.0	4.4	–15.1	7.4	–17.7	6.0
PIS20	320.5	–1.2	2.42	–49.3	15.0	–13.9	–6.9	–11.2	–6.4
PIS6	264.8	–2.9	1.72	20.8	3.0	–5.5	7.8	–3.9	7.9
ROS3	58.8	–4.7	1.21	–4.7	3.0	–1.0	–5.4	–2.9	–4.0
RU127	352.9	–2.5	1.38	–29.9	4.0	–4.3	–4.7	–4.8	–3.3
RU55	250.7	0.8	6.79	96.2	3.0	–4.3	–0.9	0.5	1.0
RU79	277.1	–0.8	3.81	21.4	1.2	–11.5	–1.7	–11.5	–2.7
STOCK14	295.2	–0.6	2.16	–4.0	2.0	–6.5	–0.6	–5.8	2.6
STOCK16	306.1	0.1	1.85	–45.0	20.0	–5.2	–2.0	–4.4	0.2
TR1	128.2	–1.1	2.69	–65.0	30.0	–7.9	2.7	–10.7	2.0
TR10	262.8	0.6	0.36	21.5	3.0	–15.6	6.2	–15.1	4.0
TR24	344.4	1.7	1.40	–4.0	1.0	–0.7	–4.3	–5.2	–4.1
TR35	28.3	0.0	1.58	–4.7	0.7	–5.6	–7.4	–4.4	–3.7
TR7	238.3	–3.9	1.34	33.6	6.0	–9.3	4.3	–7.3	6.3
Tr37	99.3	3.7	0.81	22.3	0.2	–6.5	–0.8	–5.8	–1.5
VDB1	208.6	–1.8	1.44	18.9	1.0	–14.5	–6.6	–15.0	–6.8

$p \approx 0.88$–0.91, mainly because the short-period Cepheids outnumber the long-period ones by a factor of about 2 in the sample considered. The trend persists even if we expand the sample by adding 21 Cepheids with less accurate proper motions adopted from the Four-Million Star Catalog. This leads us to conclude that the distances of short-period Cepheids should be increased, on the average, by 15–20%, whereas the distance scale of long-period Cepheids, on the whole, is correct and requires no significant extension argued for by Feast and Catchpole (1998) and Feast *et al.* (1998). The luminosities of short-period Cepheids must be increased, on the average, by $\delta M_V \approx -0\overset{m}{.}30$–$0\overset{m}{.}40$.

One way of interpreting the higher luminosity of short-period Cepheids is to suggest that the slope of the period-luminosity relation should be reduced to 2.3–2.5 [instead of 2.87 inferred by Berdnikov *et al.* (1996)]. We cannot rule out this possibility, because the adopted period-luminosity relation has been derived from a small number of Cepheid members in a few open clusters, whose distance errors could have substantially biased the inferred slope. However, it is widely believed that the period-luminosity relations in the Galaxy and in the LMC are unlikely to have very different slopes. Note that an earlier version of the period-luminosity relation by Berdnikov and Efremov (1985) adopted a slope inferred from LMC Cepheids.

It is our opinion that the increase of the adopted Cepheid luminosity can be partly due to a "contamination" of the short-period sample by unidentified first-overtone pulsators, mistakenly considered to be fundamental-mode Cepheids (Rastorguev *et al.* 1998). There is no unambiguous way to determine the pulsation

Table 3. Kinematical parameters and correction factors to the open-cluster distance scale

N	R_0, kpc	Δr, kpc	u_0, km s^{-1}	v_0, km s^{-1}	w_0, km s^{-1}	σ_U, km s^{-1}	σ_V, km s^{-1}	σ_W, km s^{-1}	ω_0, km s^{-1} kpc^{-1}	ω_0', km s^{-1} kpc^{-2}	ω_0'', km s^{-1} kpc^{-1}	p
					Open clusters with proper motions from HIPPARCOS							
115	7.5	0–4	−11.0	−11.2	−8.9	14.6	10.8	8.5*	31.5	−4.44	0.64	0.87
	Errors:		±2.6	±2.5	±3.5	±2.6	±2.6	–	±3.7	±0.8	±0.5	±0.15
117	7.5	0–5	−10.7	−11.1	−9.1	14.9	10.8	8.5*	29.0	−4.28	0.64	0.86
111	7.5	0–3	−12.1	−10.1	−9.5	14.9	10.5	8.5*	31.0	−4.17	0.23	0.79
104	7.5	0.5–4	−10.8	−11.9	−7.4	14.5	11.7	8.5*	33.8	−5.27	0.85	1.06
					Open clusters with proper motions TYC							
115	7.5	0–4	−10.4	−10.5	−4.7	14.9	10.8	8.5*	30.6	−4.27	0.71	0.83
	Errors:		±2.7	±2.6	±3.6	±2.7	±2.6	–	±3.6	±0.7	±0.4	±0.14
111	7.5	0–3	−11.3	−9.7	−5.0	15.1	10.4	8.5*	30.0	−4.01	0.40	0.75
104	7.5	0.5–4	−10.0	−10.3	−7.0*	14.2	8.8	8.5*	32.4	−4.78	0.82	0.94
82	7.5	1–4	−12.4	15.6	−7.0*	15.4	8.8	8.5*	34.2	−5.71	1.70	1.09
100	7.5	0.5–3	−11.0	−9.5	−7.0*	15.9	10.9	8.5*	31.6	−4.45	0.44	0.85

Note: Fixed parameters are marked by asterisks.

Table 4. Separate open-cluster rotation-curve solutions using radial velocities and proper motions

N	R_0, kpc	Δr, kpc	u_0, km s^{-1}	v_0, km s^{-1}	w_0, km s^{-1}	ω_0, km s^{-1} kpc^{-1}	ω_0', km s^{-1} kpc^{-2}	ω_0'', km s^{-1} kpc^{-3}
				Radial-velocity solution				
114	7.5	0–4	−9.7	−13.2	10.7	–	−5.01	0.90
	Errors:		±2.0	±2.2	±16.5	–	±0.34	±0.48
				Solution based on HIPPARCOS proper motions				
113	7.5	0–4	−9.7*	−13.2*	−7.5	32.2	−4.77	0.81
	Errors:		–	–	±1.7	±3.4	±0.78	±0.67
	Residuals		$s_{\mu l} \approx 0.0045$ arcsec yr^{-1}; $s_{\mu b} \approx 0.0042$ arcsec yr^{-}					
				Solution based on TYC proper motions				
113	7.5	0–4	−9.7*	−13.2*	−4.2	30.4	−4.66	1.74
	Errors:		–	–	±1.7	±3.2	±0.74	±0.64
	Residuals		$\sigma_{\mu l} \approx 0.0043$ arcsec yr^{-}; $\sigma_{\mu b} \approx 0.0044$ arcsec yr^{-}					

Note: Asterisks mark the parameters inferred from a radial-velocity analysis which we fixed when solving Bottlinger equations for proper motions (Dambis *et al.* 1995)].

modes of galactic Cepheids, although the use of the period-radius relation seems to be a rather promising technique in this respect (Sachkov 1997). We now estimate the luminosity bias due to assigning a wrong pulsation mode to a star. It is well known that the first-overtone to fundamental-mode period ratio is close to $P_1/P_0 \approx 0.71$. Assigning first-overtone mode to fundamental-mode Cepheid causes the absolute magnitude of the star to be underestimated by about $\Delta M_V \approx 2\overset{m}{.}87\log(P_1/P_0) \approx -0\overset{m}{.}43$ [if we adopt the slope from relation (1)]. Moreover, since we estimate the intrinsic color of the Cepheid using the period-color relation for fundamental-mode pulsators:

$$\langle B\text{–}V\rangle_0 = 0\overset{m}{.}27 + 0\overset{m}{.}46\log P_{\rm pls},$$

(Dean *et al.* 1978), the resulting color excess is overestimated by $\Delta E_{(B-V)} \approx 0\overset{m}{.}07$ and the total absorption is overestimated by $\Delta A_V \approx 0\overset{m}{.}23$. Therefore the true distance modulus of the Cepheid should be increased by $\Delta \mathrm{Mod}_0 \approx \Delta A_V - \Delta M_V \approx 0\overset{m}{.}65$ and the distance should be multiplied by a factor of $10^{0.65/5} \approx 1.35$. The above considerations allow the fraction of short-period first-overtone pulsators (and which are erroneously assumed to pulsate in the fundamental mode) in our sample to be crudely estimated at 0.45–0.65.

This is an upper estimate, which can be lowered by adopting a somewhat longer Cepheid distance scale. Thus, if we adopt the period-luminosity relation of Berdnikov and Efremov (1985), which yields for short-period Cepheids luminosities that are $0.15–0\overset{m}{.}18$ higher than those given by relation (1), the fraction of unknown first-overtone pulsators decreases to a more realistic value of 0.2–0.3. It seems plausible that the fraction of first-overtone pulsators increases with decreasing period. Thus, a subsample of 70 Cepheids with periods less than 5^d yields a distance-scale factor as low as 0.7. However, poor statistics prevents more definitive conclusions. Obviously, a slight decrease of the adopted slope of the period-luminosity relation allows the fraction of first-overtone pulsators to be further reduced. Our results for long-period Cepheids with periods greater than 10^d lead us to conclude that the distances to these stars are in overall agreement with the distance scale of Berdnikov and Efremov (1985).

A possible correction to the Cepheid distance scale must be harmonized with the distances to young open clusters. As the results given in Table 3 suggest, the distance-scale factor for open clusters lies within the $p \approx$ 0.8–1.1 interval, depending on the type of the solution. Unfortunately, the low accuracy of available radial velocities and proper motions allow the distance-scale factor to be estimated only with a considerable error (about ±0.15), preventing unambiguous selection of the distance scale. Table 3 can be used to derive another estimate of the distance-scale correction. Radial-velocity solution yields a derivative ω_0' that depends strongly on the adopted distance scale, whereas the parameter returned by the proper-motion solution is virtually scale independent. Adopting the mean value of this derivative inferred from proper motions, $\omega_0' \approx -4.72$, allows us to estimate the distance-scale correction factor at 0.94, which corresponds to a distance-modulus increase by $0\overset{m}{.}12$. All the above estimates inferred for the open-cluster sample agree with each other within the quoted errors.

CONCLUSIONS

We now summarize the main conclusions of this paper:

(1) Twenty to forty per cent of short-period Cepheids should be first-overtone pulsators and which are erroneously assumed to pulsate in the fundamental mode. The latter must provide the main contribution to the effect of underestimation of the luminosities of this group of Cepheids, thereby explaining the conclusion of Feast and Catchpole (1998) and Feast *et al.* (1998) that the LMC distance modulus should be increased to $18\overset{m}{.}70$.

(2) The entire body of results of statistical-parallax analysis as applied to 249 classical Cepheids and 117 young star clusters can be harmonized just by increasing the distance moduli of all objects by no more than $0\overset{m}{.}15$ compared to the adopted distance scale [set by Eq. (1)], implying an LMC distance modulus of $18\overset{m}{.}40$. The distance scale of Berdnikov and Efremov (1985), which relies on period-luminosity relation $\langle M_V \rangle_I = -1\overset{m}{.}24 - 2\overset{m}{.}79 \log P_{pls}$, provides a good compromise solution.

(3) The adopted distance scale for Cepheids with periods > 10^d agrees with both statistical parallaxes and the distance scale of Berdnikov and Efremov (1985). Extragalactic distances rely on just such long-period Cepheids and therefore require no decrease of the Hubble constant, which is estimated from extragalactic Cepheid observations. This result leaves the fundamental problem of large globular-cluster ages unsolved.

(4) Assuming that the galactocentric distance of the Sun is $R_0 = 7.5$ kpc, we inferred from Cepheid data the following values of the kinematical parameters, which, however, provide a satisfactory fit to the open-cluster sample as well:

Solar-motion components $(U_0, V_0, W_0) \approx (9, 12, 7)$ km s^{-1} (±1 km s^{-1}).

Velocity-ellipsoid axes $(\sigma_U : \sigma_V : \sigma_W) \approx (15.0 : 10.3 : 8.5)$ km s^{-1} (±1 km s^{-1}).

The angular velocity of rotation of the subsystem, $\omega_0 \approx 28.7 \pm 1$ km s^{-1} kpc^{-1}.

The Oort constant $A \approx 17.4 \pm 1.5$ km s^{-1} kpc^{-1}.

The second derivative $\omega_0'' \approx 1.15 \pm 0.2$km s^{-1} kpc^{-3}.

The axial ratio of the velocity ellipsoid is equal to 1.76:1.21:1.0. The inferred ratio $\frac{\sigma_U}{\sigma_V} \approx 1.45$ agrees well with the theoretical ratio derived from the Lindblad formula: $\frac{\sigma_U}{\sigma_V} = \sqrt{\frac{\omega_0}{\omega_0 - A}} \approx 1.57$.

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