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Absolute Magnitudes and Kinematic Parameters of the Subsystem of RR Lyrae Variables

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Abstract—The statistical parallax technique is applied to a sample of 262 RRab Lyrae variables with published photoelectric photometry, metallicities, and radial velocities and with measured absolute proper motions. Hipparcos, PPM, NPM, and the Four-Million Star Catalog (Volchkov *et al.* 1992) were used as the sources of proper motions; the proper motions from the last three catalogs were reduced to the Hipparcos system. We determine parameters of the velocity distribution for halo [$(U_0, V_0, W_0) = (-9 \pm 12, -214 \pm 10, -16 \pm 7)$ km s^{-1} and $(\sigma_U, \sigma_V, \sigma_W) = (164 \pm 11, 105 \pm 7, 95 \pm 7)$ km s^{-1}] and thick-disk [$(U_0, V_0, W_0) = (-16 \pm 8, -41 \pm 7, -18 \pm 5)$ km s^{-1} and $(\sigma_U, \sigma_V, \sigma_W) = (53 \pm 9, 42 \pm 8, 26 \pm 5)$ km s^{-1}] RR Lyrae, as well as the intensity-averaged absolute magnitude for RR Lyrae of these populations: $\langle M_V \rangle = 0.77 \pm 0.10$ and $\langle M_V \rangle = +1.11 \pm 0.28$ for the halo and thick-disk objects, respectively. The metallicity dependence of the absolute magnitude of RR Lyrae is analyzed ($\langle M_V \rangle = (0.76 \pm 0.12) + (0.26 \pm 0.26) \cdot ([\mathrm{Fe/H}] + 1.6) = 1.17 + 0.26 \cdot [\mathrm{Fe/H}]$). Our results are in satisfactory agreement with the $\langle M_V \rangle$(RR)–[Fe/H] relation from Carney *et al.* (1992) ($\langle M_V \rangle(\mathrm{RR}) = 1.01 + 0.15 \cdot [\mathrm{Fe/H}]$) obtained by Baade–Wesselink's method. They provide evidence for a short distance scale: the LMC distance modulus and the distance to the Galactic center are 18.22 ± 0.11 and 7.4 ± 0.5 kpc, respectively. The zero point of the distance scale and the kinematic parameters of the RR Lyrae populations are shown to be virtually independent of the source of absolute proper motions used and of whether they are reduced to the Hipparcos system or not. *© 2001 MAIK "Nauka/Interperiodica".*

Key words: *stars—variable and peculiar*

INTRODUCTION

RR Lyrae variables are among the tools of greatest value for determining the distances to old stellar systems both in our Galaxy and beyond, to globular clusters, Galactic center, and Local-Group galaxies. These stars are so important as one of the distance indicators in the Universe, because they have very similar luminosities [the dispersion of absolute magnitudes of RR Lyraes with the same metal abundance does not exceed $0^m_.15$ (Sandage 1990)], are easy to discover (because of substantial variability), and can be reliably classified (by low metallicity, period, and the shape of the light curve). Thus the main task to be resolved in order to successfully use these stars to confidently measure the distances is to set the zero point of their distance scale, i.e., determine the mean absolute magnitude of RR Lyraes. Because of its high homogeneity, the subsystem of RR Lyraes in our Galaxy proves to be an ideal object for the application of the statistical parallax technique, which can yield not only the kinematic parameters of a stellar population but also to constrain with high accuracy the mean absolute magnitude of its constituent stars. The latter quantity is one of the fundamental parameters that form the basis of modern distance scale in the Universe.

Almost half a century ago, Pavlovskaya (1953) published her pioneering work where she used for the first time the statistical parallax technique to determine the absolute magnitude of RR Lyrae stars (with a standard error of $\sim 0^m_.2$). Pavlovskaya's result made it necessary to reduce by $\sim 0^m_.5$ the mean absolute magnitude of RR Lyraes adopted at that time and has led to a radical revision (reduction by a factor of ~1.25) of the then adopted distance scale. Later, as the amount of observational data increased, many other authors applied the statistical parallax technique to RR Lyrae stars (van Herk 1965; Hemenway 1975; Clube and Dawe 1980a; Hawley *et al.* 1986; Strugnell *et al.* 1986; Layden *et al.* 1996; Popowski and Gould 1998a, 1998b; Fernley *et al.* 1998, and Tsujimoto *et al.* 1998), and, in spite of some scatter in the results obtained, the latter invariably favored the short distance scale. A number of authors tried to increase the accuracy of the method. To this end, they abandoned numerous simplifying assumptions by applying the maximum likelihood method (Rigal 1958; Heck and Lakaye 1978; Clube and Dawe 1980a). However, only Hawley *et al.* (1986) and Strugnell *et al.* (1986) were the first to use this technique correctly. The latter two groups used the rigorous algorithm of multi-

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dimensional minimization, also referred to as the simplex method and based on the method of Nedler and Mead (1965). Layden (1994) and Layden *et al.* (1996) used both their own observational material and the data by other authors to increase substantially the number of RR Lyrae stars with known metallicities, magnitudes, radial velocities, and proper motions. The above authors increased substantially the quality of the material used and applied the statistical parallax technique to the sample thus obtained (Layden *et al.* 1996). These efforts allowed the accuracy of both the inferred kinematical parameters of RR Lyrae type star population and the mean absolute magnitude of these stars to be increased: the standard error of the latter has been reduced to $0\overset{m}{.}12$. The result of Layden *et al.* (1996) was recently slightly refined by Popovski and Gould (1998a) (who used the same observational data) and also by Gould and Popovski (1998), Fernley *et al.* (1998), and Tsujimoto *et al.* (1998) [the latter three works made use of the proper motions measured by the Hipparcos satellite (ESA 1997)]. Note that all works published before Layden *et al.* (1996) used relative proper motions of RR Lyraes exclusively absolutized via statistical methods. The results thus obtained (especially, the kinematic parameters) could therefore depend on the assumptions used in absolutization (in particular, those concerning the kinematics of the reference stars). However, even Layden *et al.* (1996) were forced to use relative proper motions to obtain a balanced sample, because their main source of absolute proper motions—the NPM1 catalog (Klemola *et al.* 1993)—contains neither southern-sky stars (located south of $\delta = -23°$) nor stars located near the Galactic equator.

In recent years, the Astrometry Division of the Sternberg Astronomical Institute has created the Four-Million Catalog of positions and proper motions of stars (Volchkov *et al.* 1992), hereafter referred to as the 4M catalog for the sake of brevity. The proper motions of more than four million stars in this catalog were determined from the differences of coordinates given in the Astrographic Catalogue (Carte du Ciel) and the Hubble Space Telescope Guide Star Catalogue, and then reduced to the proper-motion system of the PPM catalog (Roser *et al.* 1991). Below, we give the reasons why it has become necessary, in our opinion, to apply once more the statistical parallax technique to RR Lyrae stars in order to further refine the mean absolute magnitude of these objects.

First, the 4M catalog gives us the so far largest sample (236 objects) of RR(ab) Lyraes for which the proper motions determined in a *homogeneous* system are available along with photoelectric magnitudes, metallicities, and radial velocities. Note for comparison that Layden *et al.* (1996) obtained their results from a sample of 213 stars using proper motions adopted from two sources (NPM1 catalog for 171 stars and a compilation of statistically absolutized proper motions for 52 stars).

Second, the 4M catalog, which covers the entire sky, makes it possible to abandon completely the use of statistically absolutized proper motions.

Third, we can now estimate to what degree the results obtained via statistical parallaxes depend on the nature and source of the adopted proper motions: statistically absolutized proper motions, absolute proper motions computed on a reference frame defined by distant galaxies (NPM1 catalog), absolute proper motions measured on board the Hipparcos satellite on a short time interval (~3 years), and absolute proper motions from the 4M catalog determined from two to four photographic positions spanning a time interval of ~100 years.

Fourth, the Hipparcos catalog (ESA 1997), which contains almost 120 000 stars with high-precision coordinates and proper motions allows an inertial reference frame and the corresponding system of proper motions to be determined with high accuracy. The latter can be used to reduce the proper motions adopted from other catalogs and to subsequently average the reduced proper motions in order build the so far largest and at the same time homogeneous sample of RR Lyraes with known metallicities, kinematic, and photometric (photoelectric) parameters.

THE METHOD

The essence of the statistical parallax technique consists in balancing the mean kinematic parameters of a stellar group as inferred from radial velocities and from the velocity vector components perpendicular to the line of sight, i.e., in the end, from the components of proper motion. The point is that the line-of-sight velocity component (i.e., radial velocity) is obtained directly from observations, whereas the components perpendicular to the line of sight are determined by multiplying the corresponding proper-motion components by a provisional distance to the star (usually the photometric distance determined from the observed magnitude, interstellar extinction, and an assumed absolute magnitude) and are therefore directly proportional to the adopted distance scale. This fundamental difference between the line-of-sight component of the full velocity vector and the two components perpendicular to it allows a correction factor to the adopted distance scale of objects under study to be estimated. The optimum factor is considered to be the value that ensures the best agreement of the inferred kinematic parameters with both observed radial velocities and proper motions of stars.

In this paper, we use the modern, most rigorous version of the statistical parallax technique, whose detailed description can be found in Murray's (1986) book (see page 284) and in the paper by Hawley *et al.* (1986). This method consists in simultaneous determination of the set of kinematic parameters (the vector of the full space velocity and the components of the velocity dispersion tensor) and the correction factor to the initial distance scale to maximize the likelihood of the actual combination of the observed data (coordinates, relative distances, radial velocities, and proper motions).

INITIAL DATA

Metallicities, Radial Velocities, and Photometry

Our primary sources of metallicities ([Fe/H]) and radial velocities of RR Lyrae stars were the papers of Layden (1994) and Layden *et al.* (1996), which we supplemented by the data recently published by Fernley and Barnes (1997), Solano *et al.* (1997), and Fernley *et al.* (1998). We used the metallicities from the latter three papers only for the stars absent from Layden's lists [which provides the greatest [Fe/H] data set for field RR Lyraes homogeneously reduced to a single abundance scale that matches the globular-cluster metallicity scale of Zinn and West (1984)]. We reduced the corresponding data to the system adopted by Layden using the following relation:

$$[\mathrm{Fe/H}]_{\mathrm{Layden}} = [\mathrm{Fe/H}]_{\mathrm{Fernley\ and\ Solano}} - 0.16 \pm 0.13, \quad (1)$$

derived from 33 common stars.

As the main sources of radial velocities, we, on the contrary, used the lists of Fernley *et al.* (1998) and Solano *et al.* (1997), because the corresponding data are more accurate (they rely on original measurements and all earlier published data), supplementing them with the data adopted from Layden *et al.* (1996) and Layden (1994) for the stars lacking in the two lists mentioned above. Depending on the completeness of observational data available, the above authors determined the mean radial velocities by integrating radial-velocity curves, simple averaging of the measurements of different authors, or just gave the individual radial-velocity measurements (in the cases where only one measurement was available for the star). The resulting radial velocities have errors of less than 5 km s^{-1} for 97 stars, from 6 to 10 km s^{-1} for 62 stars, from 11 to 15 km s^{-1} for 38 stars, and only 65 stars have their radial velocities measured with errors exceeding 15 km s^{-1} (the maximum error is 53 km s^{-1}).

Our main source of intensity-mean $\langle V \rangle$ magnitudes was the paper by Fernley *et al.* (1998), whose initial data were photometric observations taken onboard Hipparcos satellite and then reduced to the scale of V magnitudes of the Johnson system. Following Gould and Popovski (1998), for the remaining stars we used first the $\langle V \rangle$ values inferred using Layden's (1994) technique from the data by Bookmeyer *et al.* (1977), Schmidt (1991), Schmidt *et al.* (1995, 1996), and Layden (1993, 1997), whose photometric systems coincide with that of Fernley *et al.* (1998). In addition, we used the $\langle V \rangle$ magnitudes inferred by Layden *et al.* (1996) from the photometry by Clube and Dawe (1980b). We transformed the latter to the system of Fernley *et al.* (1998) using the following relation:

$$\langle V \rangle_{\mathrm{Fernley}\ et\ al.} = (\langle V \rangle_{\mathrm{Clube\ and\ Dawe}} - 0.146)/0.983 \quad (2)$$

[see formula (6) of Gould and Popovski (1998)]. Thus, unlike Layden *et al.* (1996), we did not reduce the magnitudes of RR Lyraes to the system of Clube and Dawe (1980b), because Gould and Popowski (1998) found that it differs systematically not only from the magnitude scale of Bookmeyer *et al.* (1977), Schmidt (1991), Schmidt *et al.* (1995), and Schmidt and Seth (1996) [as noted by Layden *et al.* (1996)], but also from that of Fernley *et al.* (1998), whereas the magnitude scales of all above authors except Clube and Dawe (1980b) are in excellent agreement with each other. Finally, we left unchanged the $\langle V \rangle$ magnitudes of the remaining stars adopted from Layden (1994). Note that four of these stars have their $\langle V \rangle$ magnitudes determined via a careful recalibration of high-precision photographic photometry [see Layden (1997)], and those of 11 stars were determined by Layden (1994) by recomputing the intensity-mean photographic magnitudes whose photometric systems are rather unclear and must therefore be of not too high an accuracy. We included these stars into our sample, because they were used by Layden *et al.* (1996), who argue that the magnitudes of all RR Lyraes in their list have been reduced to the scale of photoelectric $\langle V \rangle$ magnitudes of the Johnson system. In any case, the number of these stars is too small (they make up about 4% of the entire sample) to distort the results substantially [see a discussion in Gould and Popowski (1998)].

Interstellar Extinction

It is well known that to correctly determine a photometric distance to a star, one must know not only its apparent and absolute magnitudes but also interstellar extinction in the photometric passband used (A_V in our case). We adopted $A_V = 3.1E_{(B-V)}$ values from Fernley *et al.* (1998), Layden (1994), and Layden *et al.* (1996), who determined most of them using HI absorption maps of Burstein and Heiles (1982) (for stars located more than 10° from the Galactic equator).

Proper Motions

The aim of this paper is to determine the kinematic parameters of RR(ab) Lyrae type stars based on the largest sample available and to compare the results obtained using proper motions adopted from different catalogs. That is why we tried to use all mass sources of absolute proper motions available to us. These are, first and foremost, the 4M catalog of positions and proper motions (Volchkov *et al.* 1992), including its reference catalog PPM (Roser *et al.* 1991). The second source is the recently released Hipparcos catalog, which contains the proper motions of about 118 000 stars. The proper motions in this catalog were determined from three-year-long high-precision position observations, and their standard errors (for the RR Lyraes included in the catalog) are on the average equal to 0″.002–0″.003/year. Our third source was the NPM1 catalog (Klemola *et al.* 1993;

Hanson 1994), which contains about 149 000 stars north of declination –23° (except for the region of low Galactic latitudes, |b| < 10°). The proper motions of stars in this catalog were determined in a frame referred to the positions of about 50 000 faint galaxies and are therefore completely independent of the proper motions of the Hipparcos and 4M catalogs.

Note, however, that the proper motions in the above catalogs have been determined using different techniques and can therefore differ systematically from each other. Therefore, before applying the statistical parallax technique to the combined sample, we must reduce all proper motions to a single system. As was noted above, we consider the Hipparcos catalog to define the most accurate proper motion system of the three catalogs, and we therefore used it as a reference to reduce the proper motions from the other two catalogs (NPM1 and 4M + PPM).

We reduced the proper motions adopted from the 4M and its reference catalog PPM to the Hipparcos system assuming that within small areas (of radius 2°) in the vicinity of each program star the systematic differences $\Delta\mu_\alpha = \mu_{\alpha(\mathrm{Hipparcos})} - \mu_{\alpha(4\mathrm{M})}$ for the stars common to both catalogs depend linearly on coordinates α and δ:

$$\Delta\mu_\alpha = a_\alpha + b_\alpha\Delta\alpha + c_\alpha\Delta\delta, \quad (3)$$

$$\Delta\mu_\delta = \alpha_\delta + b_\delta\Delta\alpha + c_\delta\Delta\delta, \quad (4)$$

where $\Delta\alpha = \alpha(*) - \alpha(\mathrm{st})$ and $\Delta\delta = \delta(*) - \delta(\mathrm{st})$ are the differences of the coordinates of a star in the vicinity of the program star (∗) and the program star proper (st). We determined parameters a_α, b_α, c_α, a_δ, b_δ, and c_δ using the least squares method based on all stars (common to the Hipparcos and 4M catalogs) within a circle of radius 2° centered on the program star (except for the program star itself even if it is included in the Hipparcos catalog). We then determined the reduced proper-motion components as follows:

$$\mu_{\alpha(\mathrm{Red})} = \mu_{\alpha(4\mathrm{M})} + a_\alpha, \quad (5)$$

$$\mu_{\delta(\mathrm{Red})} = \mu_{\delta(4\mathrm{M})} + a_\delta. \quad (6)$$

Unfortunately, this technique cannot be used to reduce the proper motions adopted from NPM1. The point is that this catalog contains only about 13 000 stars in common with the Hipparcos catalog of which less than 4000 (3760) are fainter than $V = 9\overset{m}{.}6$, the magnitude of the brightest RR Lyrae star in the NPM1 catalog. Using brighter stars to reduce the proper motions is undesirable because of a rather strong magnitude equation (no such problem exists in the case of the reduction of the proper motions from the 4M catalog, because its intersection with the Hipparcos list consists almost exclusively of the stars from the reference catalog PPM used to derive the proper motions of the 4M catalog). That is why we had to use a different technique to reduce the proper motions from the NPM1 catalog. We analyzed how the differences of proper-motion components of stars fainter than $V = 9\overset{m}{.}6$ and common to the Hipparcos and NPM1 catalogs depend on the position on the sky. We found (see Figs. 1, 2) μ_α to depend systematically on declination δ in the δ < –7° region and found no systematic dependence of μ_α on either coordinate and μ_δ on α. We therefore did not correct the NPM1 proper-motion components in right ascension and determined the corrections to μ_δ as arithmetic means of $\mu_{\delta(\mathrm{Hipparcos})} - \mu_{\delta(\mathrm{NPM1})}$ to the stars common to the NPM1 and Hipparcos catalogs, fainter than $V = 9\overset{m}{.}6$, and located within the declination interval from δ(∗) –0.25° to δ(∗) +0.25°. We then determined the reduced values as follows:

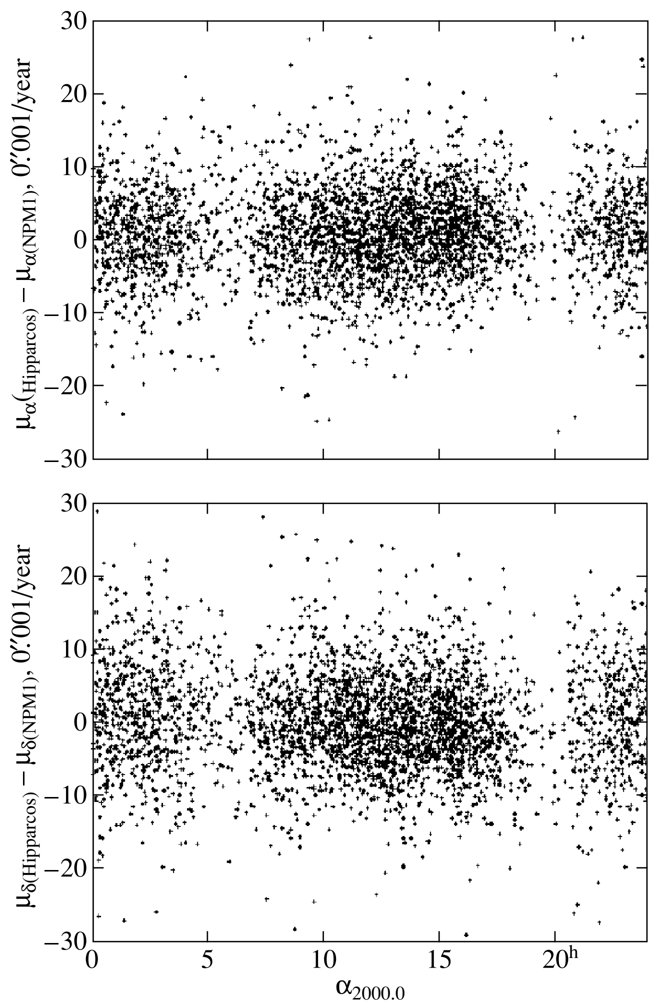

Fig. 1. The proper-motion component difference for stars fainter than $V = 9\overset{m}{.}6$ and common to the NPM1 and Hipparcos catalogs as a function of right ascension, $\alpha_{2000.0}$.

$$\mu_{\alpha(\mathrm{Red})} = \mu_{\alpha(\mathrm{NPM1})}, \quad (7)$$

$$\mu_{\delta(\mathrm{Red})} = \mu_{\delta(\mathrm{NPM1})} + \langle\mu_{\delta(\mathrm{Hipparcos})} - \mu_{\delta(\mathrm{NPM1})}\rangle. \quad (8)$$

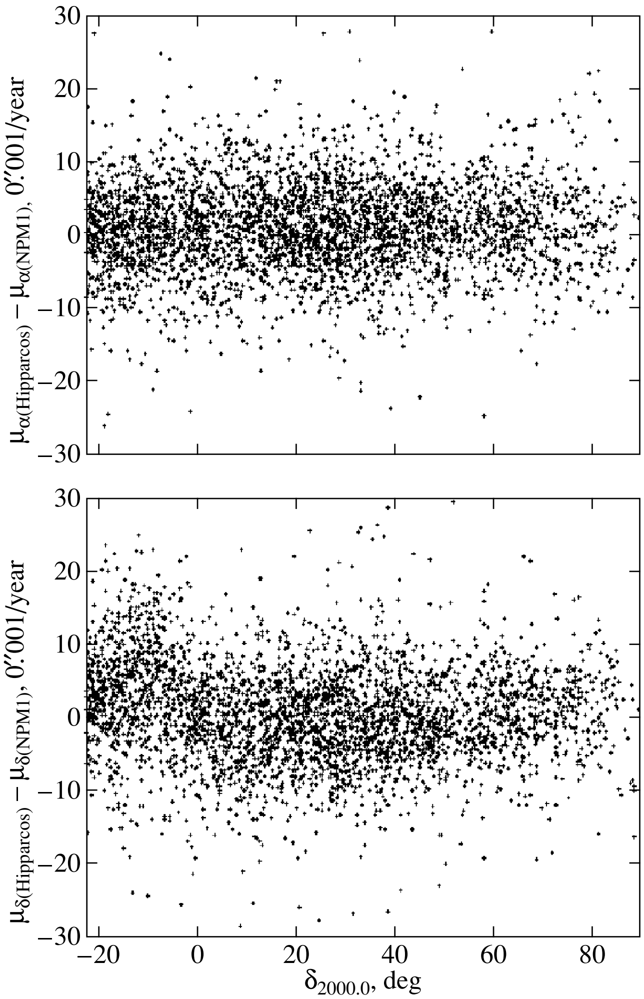

Fig. 2. The same as Fig. 1 but for declination $\delta_{2000.0}$.

Errors of Proper Motions

To apply statistical parallax technique, we must not only know the proper-motion components but also correctly estimate (in the statistical sense) the standard errors of the latter. Of all sources mentioned above, confident *individual* estimates of the standard errors of proper motions are available only for the stars in the Hipparcos catalog. At the same time, only mean errors averaged over the entire catalog are available for NPM1 and 4M. However, the random errors in the proper-motion components given by these catalogs can and are likely to be magnitude dependent, and the mean standard error can therefore depend on the magnitude distribution of the sample considered. And this distribution for the RR Lyraes in a certain catalog can differ substantially from the magnitude distribution for the entire catalog. Moreover, even if we adopt the standard errors of the proper motions quoted by the catalog authors, these errors can characterize *only the nonreduced* proper motions and in the general case tell nothing about the accuracy of the corresponding *reduced motions*. In view of the above, we estimated the standard errors of proper motions as follows. We determine only the mean (i.e., not individual) errors for each particular catalog averaged over the corresponding subset of our combined sample of RR Lyraes. To this end, we determine (using the stars common to the Hipparcos and the catalog considered) the root mean square differences of proper-motion components $\sigma(\mu_{\alpha(\text{Hip})} - \mu_{\alpha(\text{Cat})})$ and $\sigma(\mu_{\delta(\text{Hip})} - \mu_{\delta(\text{Cat})})$, and then assume that the proper-motion errors of the two catalogs are independent to obtain the following error estimates for the catalog under study:

$$\sigma(\mu_{\alpha(\text{Cat})})^2 = \langle(\mu_{\alpha(\text{Hipparcos})} - \mu_{\alpha(\text{Cat})})^2\rangle - \langle\sigma(\mu_{\alpha(\text{Hipparcos})})^2\rangle, \quad (9)$$

$$\sigma(\mu_{\delta(\text{Cat})})^2 = \langle(\mu_{\delta(\text{Hipparcos})} - \mu_{\delta(\text{Cat})})^2\rangle - \langle\sigma(\mu_{\delta(\text{Hipparcos})})^2\rangle. \quad (10)$$

Here we substituted $\sigma(\mu_{\alpha(\text{Hipparcos})} - \mu_{\alpha(\text{Cat})})^2$ and $\sigma(\mu_{\delta(\text{Hipparcos})} - \mu_{\delta(\text{Cat})})^2$ for $\langle(\mu_{\alpha(\text{Hipparcos})} - \mu_{\alpha(\text{Cat})})^2\rangle$ and $\langle(\mu_{\delta(\text{Hipparcos})} - \mu_{\delta(\text{Cat})})^2\rangle$, respectively, to allow for eventual zero offsets.

Table 1 gives the standard errors of (nonreduced and reduced) proper-motion components of the NPM1 and 4M catalogs. It is evident from this table that our reduction indeed improved the initial proper motions (the errors of reduced motions are lower than those of the nonreduced motions).

Averaged Proper Motions

We seem to have all grounds to believe that the reduced proper motions of stars common to the 4M and NPM1 catalogs are independent of each other and of the proper motions of the Hipparcos catalog. We therefore considered it possible to determine the weighted averages of μ_α and μ_δ and the corresponding covariance matrix for the stars with proper motions available from several sources. We computed the resulting covariance matrix taking into account the correlation between the errors of μ_α and μ_δ given in the Hipparcos catalog and assuming the corresponding correlation coefficients to be zero for the other two catalogs. As a result, we obtained a combined sample of RR Lyraes containing a total of 262 stars.

INITIAL DISTANCES: DISK AND HALO STARS

Layden (1995) and Layden *et al.* (1996) showed that the kinematic population of RR Lyraes in our Galaxy breaks conspicuously into two subclasses: halo and thick-disk stars. At first approximation, the two subclasses can be separated by the metallicity value [Fe/H] = –1.0: the overwhelming majority of more metal-deficient stars exhibit a kinematic behavior typical of the objects of the Galactic halo, whereas the kinematics of more metal-rich stars is more similar to that of the Galactic disk.

Following Layden *et al.* (1996), we subdivided our sample into the halo and thick-disk subsamples in three ways using three definitions of the disk and halo (by metallicity and velocity components). To this end, we computed velocity components U, V, and W in the Cartesian Galactic system with allowance for the solar motion relative to the Local Standard of Rest (+9, +12, and +7) km s^{-1} (Mihalas and Binney 1981) and the rotation of the Sun around the Galactic center [220 km s^{-1}, Kerr and Lynden-Bell (1986)], and transformed them into the velocity components in the Galactocentric cylindrical system ($V_R V_\theta V_Z$). To transform the proper motions into the space-velocity components, we used the provisional star distances computed using the absolute-magnitude calibration $\langle M_V\rangle$(RR) – [Fe/H] of Carney *et al.* (1992):

$$\langle M_V\rangle(\mathrm{RR}) = 1.01 + 0.15[\mathrm{Fe/H}]. \quad (11)$$

We then followed the procedure described in Table 2, which gives modified definitions of the halo and disk subsamples from Layden *et al.* (1996) (see Table 3 in that paper). (We consider RW Col and IK Hya to be halo stars based on their space velocities, although they formally match D-1 and D-3 criteria of the paper mentioned above, and thus add them to AO Peg and FU Vir as exceptions to the general rule.)

Table 1. Standard errors of the proper motions estimated from stars common with the Hipparcos catalog

Catalog	Errors of nonreduced proper motions, 0″.001/year		Errors of reduced proper motions, 0″.001/year	
	$\sigma(\mu_\alpha)$	$\sigma(\mu_\delta)$	$\sigma(\mu_{\alpha(\mathrm{Red})})$	$\sigma(\mu_{\delta(\mathrm{Red})})$
PPM	6.34	5.97	4.62	5.35
4M	5.95	7.40	5.15	5.92
NPM1	5.40	7.15	5.40	6.30

Table 2. Definitions of disk and halo RR Lyrae subsamples

Definition	Description
Disk-1	All stars with $V_\theta > -400 \cdot$ [Fe/H] – 300 km s^{-1} except AO Peg, FU Vir, RW Col, and IK Hya
Halo-1	All other stars
Disk-2	All stars with [Fe/H] ≥ –1.0 $V_\theta > 80$ km s^{-1} except AO Peg
Halo-2	All other stars
Disk-3	All stars of the Disk-1sample and the stars with $\|V_R\| < 100$ km s^{-1}, $V_\theta > 80$ km s^{-1}, $\|V_Z\| < 60$ km s^{-1}, $\|Z\| < 1$ kpc, and [Fe/H] > –1.6
Halo-3	All other stars

Final Sample

Our final sample of RR Lyrae stars consists of 262 stars for which all five types of initial data, proper motions, radial velocities, metallicities, interstellar extinction, and *photoelectric or CCD* intensity-mean $\langle V\rangle$ magnitudes, are available. Our list thus contains substantially more stars than any other sample previously used to this end. Moreover, the proper motions from the 4M catalog allowed us to do without *statistically* absolutized proper motions.

Table 3[1] gives the list of the 262 RR Lyraes used in this paper together with their parameters. The latter include the accurate equatorial coordinates for the epoch and equinox of J2000.0; proper-motion components in right ascension and declination adopted from the Hipparcos (with the corresponding standard errors and the correlation coefficient), 4M + PPM, and NPM1 catalogs; the 4M + PPM and NPM1 proper-motion components reduced to the Hipparcos system; intensity-mean $\langle V\rangle$ magnitudes; [Fe/H]; and mean radial velocities and their standard errors. For three stars AE Dra, BD Dra, and BK Eri, we adopted the NPM1 proper motions refined by Layden *et al.* (1996). Finally, we excluded from our sample a number of stars that are classified as RR Lyraes in the GCVS, but must actually be of different variability types [see comments to Table 3 of this paper and to Table 1 of Fernley *et al.* (1998)].

[1] Table 3 is published in electronic form only and is available from the Strasbourg Data Center at ftp: cdsarc.u-strasbg.fr/pub/cats/J (130.79.128.5) or http://cdsweb.u-strasbg.fr/pub/cats/J.

RESULTS

We first applied the statistical parallax technique described above to nine samples of Galactic halo RR Lyraes that differ from each other by the source of proper motions (Hipparcos, NPM1, and 4M + PPM) and the method of separation of halo and thick-disk stars. Table 4 gives the resulting parameters for the RR Lyrae samples thus defined. Column 1 gives the name of the halo sample according to the definition in Table 2; column 2, the source (catalog) of proper motions; column 3, the number of stars in the sample; column 4, the mean [Fe/H] averaged over the sample; columns 5 to 7, the mean velocity components, U_0, V_0, and W_0 (in the direction to the Galactic center, Galactic rotation, and North Galactic Pole, respectively); columns 8 to 10, the corresponding diagonal components of the velocity dispersion tensor, σ_U, σ_V, and σ_W [we assume the nondiagonal components to be equal to zero; see Layden *et al.* (1996)]; column 11, the inferred correction to the initial mean absolute magnitude $\langle M_V\rangle$ given by formula (11); column 12, the mean absolute magnitude $\langle M_V\rangle$ at [Fe/H] = –1.60; and column 13, the corresponding LMC distance modulus. Columns 5 to 13 also give the standard errors of the corresponding parameters for each sample. The results obtained for different samples can be seen to agree excellently with each other: the inferred parameters are virtually independent of both the adopted criterion of halo and disk separation [this result agrees with that of Layden *et al.* (1996)] and the

Table 4. Kinematic parameters of the subsystem of halo RR Lyraes inferred using nonreduced proper motions

Sample	Catalog	N	$\langle[Fe/H]\rangle$	U_0, km s^{-1}	V_0, km s^{-1}	W_0, km s^{-1}	σ_U, km s^{-1}	σ_V, km s^{-1}	σ_W, km s^{-1}	$\Delta\langle M_V\rangle$	$\langle M_V\rangle$ (–1.60)	DM (LMC)
Halo-1	Hipparcos	100	–1.568	–19 ±17	–217 ±14	–8 ±10	167 ±15	106 ±9	98 ±9	–0.08 ±0.14	+0.69 ±0.14	18.30 ±0.15
Halo-2	Hipparcos	101	–1.558	–17 ±17	–213 ±14	–9 ±10	166 ±15	109 ±9	97 ±8	–0.07 ±0.14	+0.70 ±0.14	18.29 ±0.15
Halo-3	Hipparcos	95	–1.572	–18 ±18	–222 ±15	–9 ±10	171 ±16	106 ±10	100 ±9	–0.08 ±0.15	+0.69 ±0.15	18.30 ±0.16
Halo-1	4M + PPM	185	–1.592	–4 ±13	–206 ±10	–17 ±8	170 ±12	104 ±7	97 ±7	+0.02 ±0.11	+0.79 ±0.11	18.20 ±0.12
Halo-2	4M + PPM	196	–1.564	–4 ±12	–198 ±10	–18 ±7	165 ±11	107 ±7	95 ±7	+0.02 ±0.11	+0.79 ±0.11	18.20 ±0.12
Halo-3	4M + PPM	179	–1.600	–5 ±13	–209 ±11	–18 ±8	172 ±12	105 ±8	98 ±7	+0.03 ±0.12	+0.80 ±0.12	18.19 ±0.13
Halo-1	NPM1	134	–1.593	–8 ±14	–204 ±13	–13 ±8	159 ±13	101 ±9	87 ±7	+0.04 ±0.13	+0.81 ±0.13	18.18 ±0.14
Halo-2	NPM1	139	–1.589	–9 ±14	–195 ±13	–14 ±8	157 ±13	109 ±9	86 ±7	+0.04 ±0.13	+0.81 ±0.13	18.18 ±0.14
Halo-3	NPM1	184	–1.593	–7 ±15	–207 ±13	–13 ±8	160 ±13	100 ±9	89 ±8	+0.05 ±0.14	+0.82 ±0.14	18.17 ±0.15

proper-motion source used (which we consider to be a fact of greater or at least equal importance).

We then applied the statistical parallax technique to the six RR Lyrae samples with reduced proper motions [two sources (4M + PPM and NPM1 catalogs) and three halo criteria (the proper motions from the Hipparcos catalog)], naturally, were not reduced. The corresponding results are summarized in Table 5, which has the same layout as Table 4. Again, the inferred parameters can be seen to agree well with those based on Hipparcos data. Moreover, as one would expect, the agreement is even better than in the case when nonreduced proper motions were used. This result serves to justify the averaging of proper motions from the three catalogs considered (naturally, with weights inversely proportional to the squared errors of the corresponding values) in order to build a combined RR Lyrae sample consisting of a total of 262 stars with absolute proper motions on the Hipparcos system. We subdivided this sample, too, into halo and thick-disk subsamples using three criteria from Layden *et al.* (1996) in a slightly modified form (see Table 2). We then applied statistical parallax technique to this combined sample to infer three (most accurate) sets of parameter values for both the halo and the thick-disk RR Lyrae populations. The results thus obtained are given in Table 6, whose layout, too, is identical to that of Table 4, except for the second part, which gives the results for the thick-disk RR Lyrae samples: the corresponding absolute magnitudes $\langle M_V\rangle$ refer not to [Fe/H] = –1.60 but to the mean $\langle[Fe/H]\rangle$ for the corresponding sample as given in column 3. Based on the same reasons as Layden *et al.* (1996), we consider the results obtained using Halo-3 sample to be most representative of the halo population. However, we prefer the Halo-2 sample solution as a more justified option in the case of the thick-disk population, because the corresponding sample is least "contaminated" by halo stars. We thus finally adopt $(U_0, V_0, W_0) = (-9 \pm 12, -214 \pm 10, -16 \pm 7)$ km s^{-1}, $(\sigma_U, \sigma_V, \sigma_W) = (164 \pm 11, 105 \pm 7, 95 \pm 7)$ km s^{-1}, and $\langle M_V\rangle = 0.77 \pm 0.10$ for halo objects (201 stars) and $(U_0, V_0, W_0) = (-16 \pm 8, -41 \pm 67, -18 \pm 5)$ km s^{-1}, $(\sigma_U, \sigma_V, \sigma_W) = (53 \pm 9, 42 \pm 8, 26 \pm 5)$ km s^{-1}, and $\langle M_V\rangle = +1.11 \pm 0.28$ for thick-disk objects (46 stars), which agrees very well with the results of Layden *et al.* (1996).

$\langle M_V\rangle$([Fe/H]) RELATION

Our list is superior in size to all other RR Lyrae samples previously used and it must therefore allow the statistical parallax technique to be used for the first time to obtain confident conclusions about the behavior of the heavy-element abundance dependence of RR Lyrae luminosities. The previous such attempt by Layden *et al.* (1996) failed: these authors found $\Delta\langle M_V\rangle/\Delta[Fe/H] = +0.09 \pm 0.38$. We subdivided the Halo-2 sample from Table 6 into four approximately equal subsamples with mean metallicities $\langle[Fe/H]\rangle_1 = -2.02$, $\langle[Fe/H]\rangle_2 = -1.53$, $\langle[Fe/H]\rangle_3 = -1.43$, and $\langle[Fe/H]\rangle_4 = -1.17$, and applied the statistical parallax technique to each of them. The results are summarized in Table 7. We thus obtained four halo RR Lyrae absolute-magnitude estimates at four metallicity values. We now add to them the $\langle M_V\rangle$

Table 5. Kinematic parameters of the subsystem of halo RR Lyraes inferred using proper motions reduced to the Hipparcos system

Sample	Catalog;	N	$\langle$[Fe/H]$\rangle$	U_0, km s^{-1}	V_0, km s^{-1}	W_0, km s^{-1}	σ_U, km s^{-1}	σ_V, km s^{-1}	σ_W, km s^{-1}	$\Delta\langle M_V\rangle$	$\langle M_V\rangle$ (–1.60)	DM (LMC)
Halo-1	4M + PPM	187	–1.588	–10 ±13	–206 ±10	–16 ±7	166 ±11	105 ±7	97 ±7	0.00 ±0.11	+0.77 ±0.11	18.22 ±0.12
Halo-2	4M + PPM	194	–1.573	–9 ±12	–199 ±10	–18 ±7	164 ±11	109 ±7	96 ±7	+0.00 ±0.11	+0.77 ±0.11	18.22 ±0.12
Halo-3	4M + PPM	176	–1.605	–10 ±13	–213 ±11	–17 ±8	171 ±12	105 ±7	100 ±7	0.00 ±0.11	+0.77 ±0.11	18.22 ±0.12
Halo-1	NPM1	136	–1.596	–7 ±14	–204 ±13	–16 ±8	158 ±13	105 ±9	87 ±7	+0.04 ±0.13	+0.81 ±0.13	18.18 ±0.14
Halo-2	NPM1	138	–1.593	–6 ±14	–197 ±13	–16 ±8	156 ±13	110 ±9	86 ±7	+0.06 ±0.13	+0.83 ±0.13	18.16 ±0.14
Halo-3	NPM1	130	–1.610	–6 ±15	–208 ±13	–16 ±8	161 ±13	105 ±9	89 ±8	+0.04 ±0.14	+0.81 ±0.14	18.18 ±0.15

value obtained for thick-disk RR Lyraes (see solution Disk-2 in Table 6) to make up a total of five data points on the [Fe/H]–$\langle M_V\rangle$ diagram (see Table 7 and Fig. 3), which yield the following least-squares fit:

$$\langle M_V\rangle = (0.76 \pm 0.12) + (0.26 \pm 0.26) \cdot ([\mathrm{Fe/H}] + 1.6), \tag{12}$$

or

$$\langle M_V\rangle = 1.17 \pm 0.26 \cdot [\mathrm{Fe/H}]. \tag{13}$$

This result is, on the whole, consistent with formula (11) derived for field RR Lyraes using the Baade–Wesseling technique (Carney *et al.* 1992). We inferred the slope of the [Fe/H]($\langle M_V\rangle$) relation one and a half times more accurately than Layden *et al.* (1996), although, unfortunately, the error remains too high.

DISTANCE SCALE

As was noted above, the mean absolute magnitude of RR Lyrae type stars forms one of the bases of the distance scale in the Universe and can be used, in particular, to measure the distances to the LMC and the Galactic center, because both objects contain RR Lyraes.

The Distance to the LMC

If applied to photometric observations of RR Lyraes in seven globular clusters in the LMC (Walker 1992), our calibration of the zero point of the [Fe/H]($\langle M_V\rangle$) relation (Halo-2 in Table 6: $\langle M_V\rangle$([Fe/H] = –1.60) = +0.77 ± 0.10) yields an LMC distance modulus of $DM_{\mathrm{LMC}} = 18.22 \pm 0.11$, which is in excellent agreement with the result of Layden *et al.* (1996) and the value of 18.25 ± 0.05 (Berdnikov *et al.* 1996) obtained using the period-luminosity relation for classical Cepheids (in terms of the short distance scale).

The Distance to the Galactic Center

Layden *et al.* (1996) applied their calibration of the [Fe/H]($\langle M_V\rangle$) relationship to the data of Walker and Mack (1988) to infer a distance of $R_0 = 7.6 \pm 0.6$ kpc to the Galactic center. Our $\langle M_V\rangle$ is $0\overset{m}{.}06$ fainter than that obtained by Layden *et al.* (1996) and somewhat more accurate, and we therefore find R_0 = 7.4 ± 0.5 kpc, which agrees excellently with the estimates inferred from (1) the rotation-curve analysis of classical Cepheids based on the *short* distance scale [$R_0 = 7.1 \pm 0.5$ kpc (Dambis *et al.* 1995)]; (2) the rotation-curve analysis of a more extensive sample of young Galactic objects, also based on the short distance scale [$R_0 = 7.3 \pm 0.3$ kpc (Glushkova *et al.* (1998)]; and (3) the only available

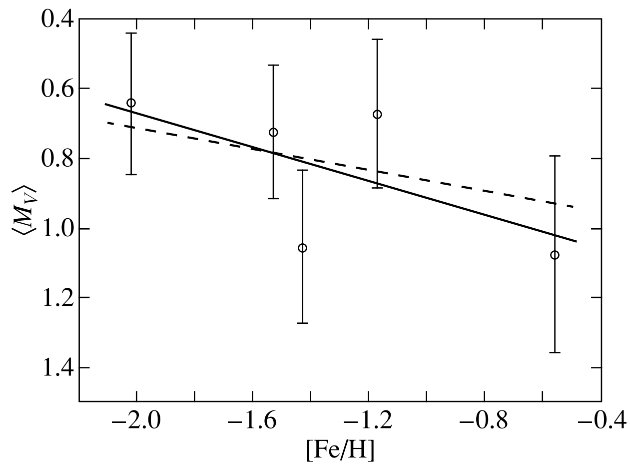

Fig. 3. Metallicity ([Fe/H]) dependence of the mean absolute magnitude $\langle M_V\rangle$ of RR Lyraes. Dots are the values inferred using statistical parallax technique. Solid and dashed lines are the relation of Carney *et al.* (1992) [formula (11)] and linear least squares fit to our results [formula (13)], respectively.

Table 6. Kinematic parameters of the subsystem of halo RR Lyraes inferred using proper motions reduced to the Hipparcos system and averaged over three catalogs

Sample	N	⟨[Fe/H]⟩	U_0, km s^{-1}	V_0, km s^{-1}	W_0, km s^{-1}	σ_U, km s^{-1}	σ_V, km s^{-1}	σ_W, km s^{-1}	$\Delta\langle M_V\rangle$	$\langle M_V\rangle$ (–1.60)	DM (LMC)
Halo-1	211	–1.574	–9	–208	–15	161	106	93	–0.01	+0.76	18.23
			±11	±10	±7	±10	±7	±6	±0.10	±0.10	±0.11
Halo-2	216	–1.565	–8	–202	–15	158	109	92	+0.01	+0.78	18.21
			±11	±10	±7	±10	±7	±6	±0.10	±0.10	±0.11
Halo-3	201	–1.585	–9	–214	–16	164	105	95	0.00	+0.77	18.22
			±12	±10	±7	±11	±7	±7	±0.10	±0.10	±0.11
Sample	N	⟨[Fe/H]⟩	U_0, km s^{-1}	V_0, km s^{-1}	W_0, km s^{-1}	σ_U, km s^{-1}	σ_V, km s^{-1}	σ_W, km s^{-1}	$\Delta\langle M_V\rangle$	$\langle M_V\rangle$(⟨[Fe/H]⟩)	
Disk-1	51	–0.622	–12	–34	–16	47	35	24	+0.40	+1.35	
			±7	±6	±4	±8	±6	±5	±0.28	±0.28	
Disk-2	46	–0.560	–16	–41	–18	53	42	26	+0.16	+1.11	
			±8	±7	±5	±9	±8	±5	±0.28	±0.28	
Disk-3	61	–0.740	–12	–49	–17	54	47	26	–0.02	+0.92	
			±7	±7	±4	±9	±7	±5	±0.23	±0.24	

Table 7. Kinematic parameters of the subsystems of halo RR Lyraes with different metallicities (subsamples of Halo-2 sample) inferred using proper motions reduced to the Hipparcos system and averaged over three catalogs

min([Fe/H])	max([Fe/H])	⟨[Fe/H]⟩	N	U_0, km s^{-1}	V_0, km s^{-1}	W_0, km s^{-1}	σ_U, km s^{-1}	σ_V, km s^{-1}	σ_W, km s^{-1}	$\Delta\langle M_V\rangle$	$\langle M_V\rangle$
–2.84	–1.80	–2.020	46	–29	–228	+4	154	130	89	–0.07	+0.64
				±21	±23	±13	±20	±16	±11	±0.20	±0.20
–1.79	–1.52	–1.526	55	–25	–210	–33	174	107	96	–0.05	+0.72
				±24	±18	±13	±20	±13	±12	±0.19	±0.19
–1.51	–1.33	–1.434	48	+12	–201	–33	158	86	83	+0.26	+1.05
				±23	±19	±13	±21	±11	±12	±0.22	±0.22
–1.32	–0.80	–1.172	57	+9	–173	0	141	103	89	–0.16	+0.67
				±19	±18	±13	±19	±13	±11	±0.21	±0.21

direct estimate based on the measurement of the proper motions of an H_2O maser [$R_0 = 7.2 \pm 1.3$ kpc (Reid 1993)].

CONCLUSION

The application of the statistical-parallax technique to the largest sample of RR Lyraes with known kinematic parameters (a total of 262 stars, i.e., a 25% increase over the largest of such samples used so far) leads us to the following conclusions:

(1) Our analysis provides further and more precise evidence in favor of the short distance scale: the implied LMC distance modulus and the distance to the Galactic center are equal to 18.22 ± 0.11 and 7.4 ± 0.5 kpc, respectively.

(2) The conclusion about the short distance scale is independent of the proper-motion source used.

(3) The inferred slope of the [Fe/H](⟨M_V⟩) relation is close to that determined using the Baade–Wesselink method.

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