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Classical Cepheids: Yet another version of the Baade–Becker–Wesselink method

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ABSTRACT
We propose a new version of the Baade–Becker–Wesselink technique, which allows one to independently determine the colour excess and the intrinsic colour of a radially pulsating star, in addition to its radius, luminosity, and distance. It is considered to be a generalization of the Balona approach. The method also allows the function $F(CI_0) = BC(CI_0) + 10 \times log(T_{eff}(CI_0))$ for the class of pulsating stars considered to be calibrated. We apply this technique to a number of classical Cepheids with very accurate light and radial-velocity curves and with bona fide membership in open clusters (SZ Tau, CF Cas, U Sgr, DL Cas, GY Sge), and find the results to agree well with the reddening estimates of the host open clusters. The new technique can also be applied to other pulsating variables, e.g. RR Lyraes.

Key words: Cepheids; luminosities; radii; color excess.

1 INTRODUCTION

Classical Cepheids are the key standard candles, which are used to set the zero point of the extragalactic distance scale (Freedman et al. 2001) and also serve as young-population tracers of great importance (Binney and Merrifield 1998). They owe their popularity to their high luminosities and photometric variability (which make them easy to identify and observe even at large distances) and the fact that the luminosities, intrinsic colours, and ages of these stars are closely related to such an easy to determine quantity as the variability period.

It would be best to calibrate the Cepheid period-luminosity (PL), period-colour (PC), and period-luminosity-colour relations via distances based on trigonometric parallaxes, however, the most precisely measured parallaxes of even the nearest Cepeheids remain insufficiently accurate and, more importantly, they may be fraught with so far uncovered systematic errors. Here the Baade–Becker–Wesselink method (Baade 1926; Becker 1940; Wesselink 1946) comes in handy, because it allows the Cepheid distances (along with the physical parameters of these stars) to be inferred, thereby providing an independent check for the results based on geometric methods (e.g., trigonometric and statistical parallax).

However, all the so far proposed versions of the Baade–Becker–Wesselink method (surface brightness technique (Barnes and Evans 1976), maximum-likelihood technique (Balona 1977)) depend, in one way or another, on the adopted reddening value. Both techniques are based on the same astrophysical background but make use somewhat different calibrations (limb-darkened surface brightness parameter, bolometric correction – effective temperature pair) on the normal colour. Here we propose a generalization of the Balona (1977) technique, which allows one to independently determine not only the star's distance and physical parameters, but also the amount of interstellar reddening, and even calibrate the dependence of a linear combination of the bolometric correction and effective temperature on intrinsic colour.

2 THEORETICAL BACKGROUND

We now briefly outline the method. First, the bolometric luminosity of a star at any time instant is given by the following relation, which immediately follows from the Stefan–Boltzmann law:

$$L/L_\odot = (R/R_\odot)^2 \times (T/T_\odot)^4. \quad (1)$$

Here L, R, and T are the star's bolometric luminosity, radius, and effective temperature, respectively, and the $\odot$ subscript denotes the corresponding solar values. Given that the bolometric absolute magnitude M_{bol} is related to bolometric luminosity as

$$M_{bol} = M_{bol\odot} - 2.5 \times log(L/L_\odot), \quad (2)$$

we can simply derive from Eq. (1):

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$$M_{bol} - M_{bol\odot} = -5 \times log(R/R_\odot) - 10 \times log(T/T_\odot) \quad (3)$$

Now, the bolometric absolute magnitude M_{bol} can be written in terms of the absolute magnitude M in some photometric band and the corresponding bolometric correction BC:

$$M_{bol} = M + BC, \quad (4)$$

and the absolute magnitude M can be written as:

$$M = m - A - 5 \times log(d/10\ pc). \quad (5)$$

Here m, A, and d are the star's apparent magnitude and interstellar extinction in the corresponding photometric band, respectively, and d is the heliocentric distance of the star in pc. We can therefore rewrite Eq. (3) as follows:

$$m = A + 5 \times log(d/10\ pc) + M_{bol\odot} + 10 \times log(T_\odot)$$
$$- 5 \times log(R/R_\odot) - BC - 10 \times log(T). \quad (6)$$

Let us introduce the function $F(CI_0) = BC + 10 \times \log(T)$, the apparent distance modulus $(m - M)_{app} = A + 5 \times log(d/10\ pc)$, and rewrite Eq. (7) as the light curve model:

$$m = Y - 5 \times log(R/R_\odot) - F. \quad (7)$$

where constant

$$Y = (m - M)_{app} + M_{bol\odot} + 10 \times log(T_\odot).$$

We now recall that interstellar extinction A can be determined from the colour excess CE as $A = R_\lambda \times CE$, where R_λ is the total-to-selective extinction ratio for the passband-colour pair considered, whereas $M_{bol\odot}$, $R_\odot$, and $T_\odot$ are rather precisely known quantities. The quantity $F(CI_0) = BC + 10 \times log(T)$ is a function of intrinsic colour index $CI_0 = CI - CE$. Balona (1977) used a very crude approximation for the effective temperature and bolometric correction, reducing the right-hand of the light curve model (Eq. 7) to the linear function of the observed colour, with the coefficients containing the colour excess in a latent form. It should be noted that Sachkov et al. (1998); Sachkov (2002) used non-linear approximation in Eq. (7) to calculate Cepheid radii.

The key point of our approach is that the values of function F are computed from the already available calibrations of bolometric correction $BC(CI_0)$ and effective temperature $T(CI_0)$ (Flower 1996; Bessell et al. 1998; Alonso et al. 1999; Sekiguchi and Fukugita 2000; Ramirez and Melendez 2005; Biazzo et al. 2007; Gonzalez Hernandez and Bonifacio 2009). These calibrations are expressed as high-order power series in the intrinsic colour:

$$F(CI_0) = a_0 + \sum_{k=1}^{N} a_k \cdot CI_0^k, \quad (8)$$

with known $\{a_k\}$ and N amounting to 7; in some cases the decomposition also includes the metallicity ($[Fe/H]$) and/or gravity ($log\ g$) terms.

As for the star's radius R, its current value can be determined by integrating the star's radial-velocity curve over time ($dt = (P/2\pi) \cdot d\varphi$):

$$R(t) - R_0 = -pf \cdot \int_{\varphi_0}^{\varphi} (V_r(t) - V_\gamma) \cdot (P/2\pi) \cdot d\varphi, \quad (9)$$

where R_0 is the radius value at the phase φ_0 (we use mean radius, $< R >= (R_{min} + R_{max})/2$); V_γ, the systemic radial velocity; φ, the current phase of the radial velocity curve; P, the star's pulsation period, and pf is the projection factor that accounts for the difference between the pulsation and radial velocities. Given the observables (light curve – apparent magnitudes m, colour curve – apparent colour indices CI, and radial velocity curve – V_r) and known quantities for the Sun, we end up with the following unknowns: distance d, mean radius $< R >$, and colour excess CE, which can be simply found by the least-squares or maximum likelihood technique.

In the case of Cepheids with large amplitudes of light and colour curves ($\Delta CI \geqslant 0.4^m$) it is also possible to apply a more general technique by setting the expansion coefficients $\{a_k\}$ in Eq. (8) free and treating them as unknowns. We expanded the function $F = BC + 10 \times log(T)$ in Eq. (7) into a power series about the intrinsic colour index CI_0^{st} of a well-studied "standard" star (e.g., α Per or some other bright star) with accurately known T^{st}:

$$F = BC^{st} + 10 \times log(T^{st}) + \sum_{k=1}^{N} a_k \cdot (CI - CE - CI_0^{st})^k \quad (10)$$

The best fit to the light curve is provided with the optimal expansion order $N \simeq 5 - 9$. We use this modification to calculate the physical parameters and reddening CE of the Cepheid, as well as the calibration $F(CI_0) = BC(CI_0) + 10 \times log(T_{eff}(CI_0))$ for the given metallicity $[Fe/H]$.

3 OBSERVATIONAL DATA, CONSTANTS, AND CALIBRATIONS

Our sources of data include Berdnikov's extensive multicolor photoelectric and CCD photometry of classical Cepheids (Berdnikov 1995, 2008) and very accurate radial-velocity measurements of 165 northern Cepheids (Gorynya et al. 1992, 1996, 1998, 2002) taken in 1987-2009 (about 10500 individual observations) with a CORAVEL-type spectrometer (Tokovinin 1987). These data sets are nearly synchronous, to prevent any systematic errors in the computed radii and other parameters due to the evolutionary period changes resulting in phase shifts between light, colour and radial velocity variations (Sachkov et al. 1998). We adopt $T_\odot = 5777$ K, $M_{bol\odot} = +4.76^m$ (Gray 2005). We proceeded from $(V, B - V)$ data and found the best solutions for the V-band light curve and $(B - V)$ color curve to be those computed using the $F((B - V)_0)$ function based on two calibrations (Flower 1996; Bessell et al. 1998) of similar slope (see Fig. 2 e); the poorer results obtained using the other cited calibrations can be explained by the fact that the latter involved insufficient number of supergiant stars.

4 THE PROJECTION FACTOR

There is yet no consensus concerning the projection factor (PF) value to be used for Cepheid variables (Nardetto et al. 2004; Groenewegen 2007; Nardetto et al. 2007, 2009). Different authors use constant values ranging from 1.27 to

1.5, as well as PFs depending on the pulsation period and other parameters. Different approaches lead to small systematic differences in the inferred Cepheid parameters, first of all, in the radii. Based on geometrical considerations, Rastorguev (2010) derived phase-dependent PFs as simple three-parametric analytic expressions depending on the pulsation velocity, limb darkening coefficient, and spectral line broadening, adjusted to CORAVEL radial velocities of Cepheids. We suspect that the period dependence reflects mainly the dependence of the PF on limb darkening. To compare our results with other calculations, we finally adopted a moderate dependence of PF on the period advocated by Nardetto et al. (2007):

$$p = (-0.064 \pm 0.020) \times log(P, days) + (1.376 \pm 0.023), \quad (11)$$

though we repeated all calculations with other variants of PF dependence on the period and pulsation phase to assure the stability of the calculated colour excess.

5 PRELIMINARY RESULTS

To test the new method, we used the maximum likelihood technique to solve Eq. (7) for the V-band light curve and $B-V$ colour curve for several classical Cepheids residing in young open clusters: SZ Tau (NGC 1647), CF Cas (NGC 7790), U Sgr (IC 4725), DL Cas (NGC 129), GY Sge (anonymous OB-association (Forbes 1982)) as well as for approximately 30 field Cepheids from our sample. We found two $log(T_{eff})$ calibrations – those of Flower (1996) and Bessell et al. (1998) – combined with the $BC(V)$ calibration as a function of normal colour $(B-V)_0$ proposed by Flower (1996) – to yield the best fit to the observed V-band light curve via Eq. (7). A weak sensitivity of calculated reddening, E_{B-V}, to the adopted PF value (constant or period/phase–dependent) and to the derived $< R >$ value can be explained by very strong dependence of the light curve's amplitude on the effective temperature, $\sim 10 \times log(T)$, and, as a consequence, on the dereddened colour.

Though the internal errors of the reddening E_{B-V} seem to be very small, the values determined using the two best calibrations, Flower (1996) and Bessell et al. (1998), may differ by as much as $0.03 - 0.05^m$, due to the systematic shift between these two calibrations (Fig. 2 e). Table 1 summarizes the inferred parameters for the cluster Cepheids studied. Fig. 1 shows the observed and smoothed data and the final fit to the V-band light curve for U Sgr Cepheid. Our reddenings seem to agree well with the corresponding WEBDA values, particularly if we remember that the errors of the adopted cluster reddening estimates are as high as $\pm 0.05^m$. Our next step will be to make use of the calibrations of T_{eff} and BC as a function of red and infrared colours ($V-R$, $V-I$, $V-K$) and to compare derived reddening ratios with the conventional extinction laws.

Note that the inferred radius and luminosity of SZ Tau are too large for its short period; this Cepheid probably pulsates in the 1st or even in the 2nd overtone, as may be indirectly evidenced by its low colour amplitude (about 0.15^m).

Fig. 2 shows the observed data, the fit to the V-band light curve, and the inferred calibration $F = 10 \times log(T_{eff}) + BC(V)$ vs $(B-V)_0$) calculated for TT Aql Cepheid (as a 5th-order expansion in the normal colour). The inferred calibration is very close to that of Flower (1996). We used α Per as the "standard" star, with $T^{st} \approx (6240 \pm 20)$ K, $[Fe/H] \approx -0.28 \pm 0.06$ (Lee et al. 2006), $(B-V)^{st} \approx 0.48^m$ and $E_{B-V} \approx 0.09^m$ (WEBDA, for α Per cluster). To take into account the effect of metallicity on the zero-point $F(CI_0)^{st}$, we estimated the gradient $dF(CI_0)^{st}/d[Fe/H] \approx +0.24$ from the calibrations by Alonso et al. (1999); Sekiguchi and Fukugita (2000); Gonzalez Hernandez and Bonifacio (2009). For TT Aql, $E_{B-V} \approx (0.65 \pm 0.03)^m$. In some cases (with large amplitude of color variation) the "free" calibration (Eq. 10) can markedly improve the model fit to the observed light curve of the Cepheid variable. Fig. 2 f shows the example of calibrations of the F functions derived from nine Cepheids with different metallicity and surface gravity values. Temperature difference at $T_{eff} \sim 6600 - 5100$ K is amounted to $3-5\%$.

When applied to an extensive sample of Cepheid variables with homogeneous photometric data and detailed radial velocity curves, the new method is expected to give a completely independent scale of reddenings, a new Period - Colour - Luminosity relation, and a new distance scale for the Milky-Way Cepheids.

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REFERENCES

Alonso, A., Arribas, S., Martinez-Roger, C., 1999, AsAp-Suppl, 140, 261
Baade, W., 1926, AN, 228, 359
Barnes, T.G., Evans, D.S., 1976, MNRAS, 174, 489
Balona, L.A., 1976, MNRAS, 178, 231
Becker, W., 1940, ZA, 19, 289
Berdnikov, L. N., 1995, Astrophysical applications of stellar pulsation. Proceedings of IAU Colloquium 155 held in Cape Town, South Africa, 6-10 February 1995; eds. Stobie, R. S. and Whitelock, P. A., ASP Conference Series, 83, 349
Berdnikov, L. N., 2008, VizieR On-line Data Catalog: CDS II/285
Bessell, M. S., Castelli, F., Plez, B., 1998, AsAp, 333, 231
Biazzo, K., Frasca, A., Catalano, S., Marilli, E., 2007, AN, 328, 938
Binney, J. and Merrifield, M., 1998, Galactic astronomy, Princeton, NJ : Princeton University Press
Flower , Ph. J., 1996, ApJ, 469, 355
Forbes, D., 1982, AJ, 87, 1022
Freedman, W. L., Madore, B. F., Gibson, B. K., et al. 2001, ApJ, 553, 47
Gonzalez Hernandez, J. I., Bonifacio, P. 2009, AsAp, 497, 497

Gorynya, N. A., Irsmambetova, T. R., Rastorguev, A. S., Samus', N. N., 1992, SvAL, 18, 316
Gorynya, N. A., Samus', N. N., Rastorguev, A. S., Sachkov, M. E., 1996, AstL, 22, 175
Gorynya, N. A., Samus', N. N., Sachkov, M. E., Rastorguev, A. S., Glushkova, E. V., Antipin, S. V., 1998, AstL, 24, 815
Gorynya, N. A., Samus, N. N., Sachkov, M. E., Rastorguev, A. S., Glushkova, E. V., Antipin, S. V., 2002, VizieR Online Data Catalog: III/229
Gray, C. D. F., 2005, The Observation and Analysis of Stellar Photospheres, Cambridge: Cambridge University Press
Groenewegen, M. A. T.,2007, AsAp, 474, 975
Lee, B.-C., Galazutdinov, G. A., Han, I., Kim, K.-M., Yushchenko, A. V., Kim, J., Tsymbal, V., Park., M.-G., 2006, PASP, 118, 636
Nardetto, N., Fokin, A., Mourard, D., Mathias, Ph., Kervella, P., Bersier, D., 2004, AsAp, 428, 131
Nardetto, N., Mourard, D., Mathias, Ph., Fokin, A., Gillet, D., 2007, AsAp, 471, 661
Nardetto, N., Gieren, W., Kervella, P., Fouque, P., Storm, J., Pietrzynski, G., Mourard, D., Queloz, D., 2009, AsAp, 502,951
Ramirez, I., Melendez, J., 2005, ApJ, 626, 465
Rastorguev, A. S., 2010, Variable Stars, the Galactic halo and Galaxy Formation, Proceedings of an international conference held in Zvenigorod, Russia, 12-16 October 2009; eds. Sterken, Chr., Samus, N., Szabados, L., Published by Sternberg Astronomical Institute of Moscow University, Russia, 225; arXiv:1001.1648
Sachkov, M. E., Rastorguev, A. S., Samus', N. N., Gorynya, N. A., 1998, AstL, 24, 377
Sachkov, M. E. 2002, AstL, 28, 589
Sekiguchi, M., Fukugita, M., 2000, AJ, 120, 1072
Tokovinin, A. A., 1987, SvA, 31, 98
Wesselink, A. J., 1946, BAN, 10, 91

Table 1. Physical parameters, distances, and interstellar reddening values for the cluster Cepheids analysed using the new version of the BW method. Reddening values from WEBDA data base (http://www.univie.ac.at/webda/) are also shown for comparison; asterisk: E_{B-V} adopted from Forbes (1982). Distances are calculated using $R_V = 3.3$.

Star	Cluster	Period (d)	Distance (pc)	E_{B-V}	E_{B-V}(WEBDA)	$< R > /R_{\odot}$	M_V
SZ Tau	NGC 1647	3.149	796±90	0.40±0.02	0.370	57.0±7.0	-4.32±0.25
CF Cas	NGC 7790	4.875	3585±87	0.54±0.02	0.531	46.7±0.9	-3.41±0.05
U Sgr	IC 4725	6.745	613±25	0.50±0.03	0.475	54.2±1.8	-3.90±0.08
DL Cas	NGC 129	8.001	2067±58	0.47±0.05	0.548	69.3±1.6	-4.12±0.06
GY Sge	Anon OB	51.78	2136±163	1.44±0.05	1.29±0.06 (*)	208±11	-6.27±0.15

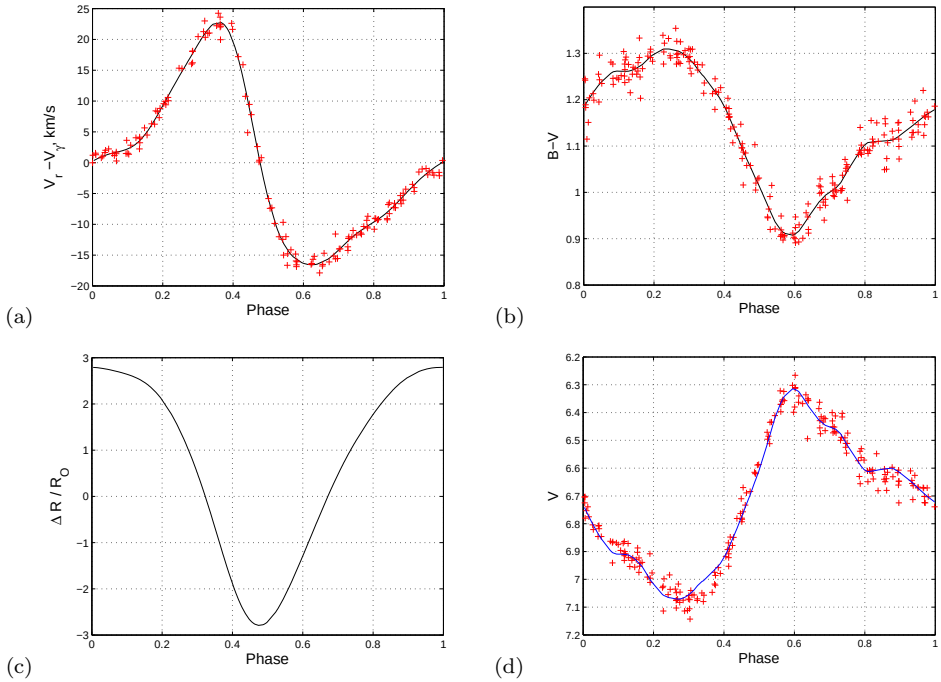

Figure 1. Panel (a): Observed and fitted radial-velocity curve of U Sgr. Standard deviation $\sigma_{Vr} = 1.1\ km/s$. Panel (b): Observed and smoothed colour curve. Panel (c): Radius variation with phase. Panel (d): Observed and fitted light curve.

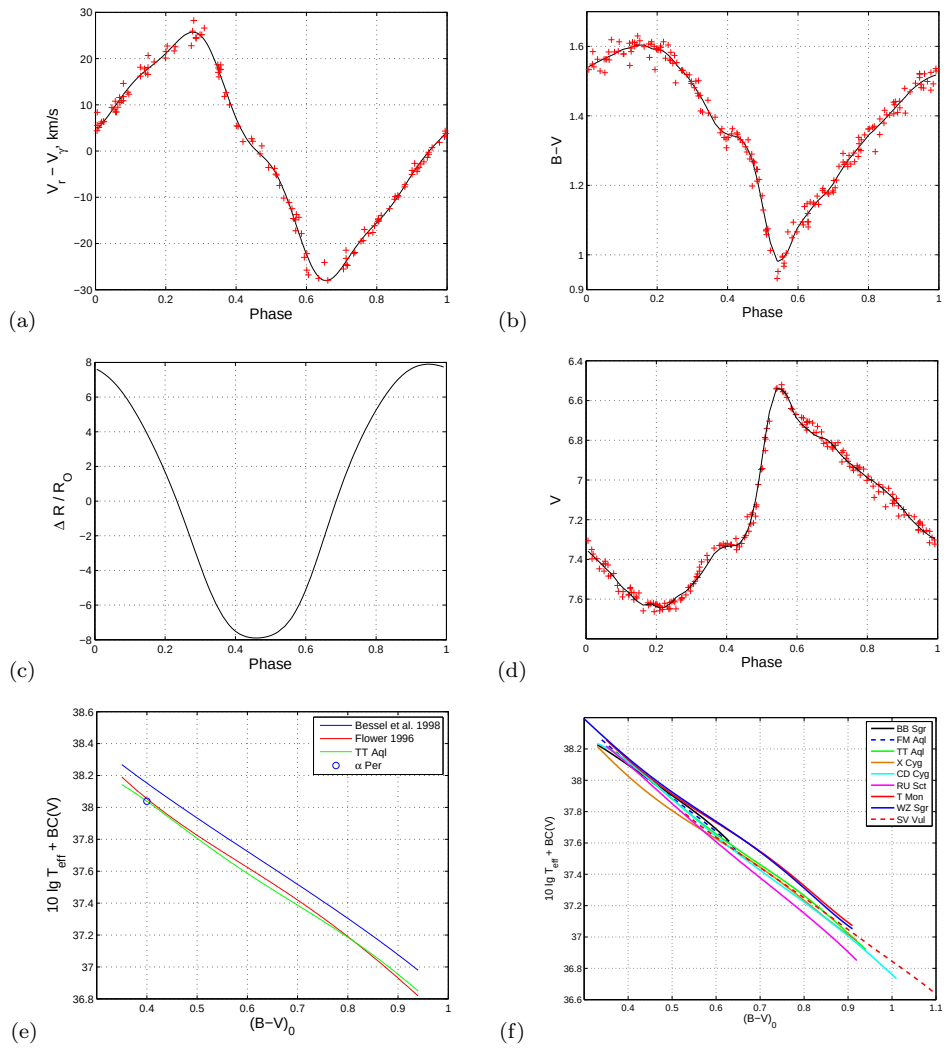

Figure 2. Panel (a): Observed and fitted radial-velocity curve of TT Aql. Standard deviation $\sigma_{Vr} = 1.3\ km/s$. Panel (b): Observed and smoothed colour curve. Panel (c): Radius variation with phase. Panel (d): Observed and fitted light curve. Panel (e): Calculated calibration for TT Aql (function $F = 10 \times log(T_{eff}) + BC(V)\ vs\ (B-V)_0)$) and calibrations by Flower (1996) and Bessell et al. (1998). Also shown is the position of the standard star α Per corrected for metallicity difference. Panel (f): Calculated calibration (function $F = 10 \times log(T_{eff}) + BC(V)\ vs\ (B-V)_0)$) for 9 Cepheids with large amplitudes of the colour curves and different metallicities.