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New Version of the Pulsating Photospheres Method: Multiphase Temprature Measurements of Cepheids

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Abstract—We propose a new version of the (Baade–Becker–Wesselink) pulsating photospheres method based on direct spectral measurements of the effective temperatures of Cepheids carried out in different pulsation phases. By comparing the effective temperatures calculated using normal color calibrations with real spectroscopic estimates, we were able to not only determine the color excess with an accuracy of the order of $0\overset{m}{.}01$ mag, but also use all the measured effective temperature values to derive a new color calibration for the effective temperature of high luminosity stars, also taking into account the differences in metallicity [Fe/H] and surface gravity $\log g$: $\log T_{\rm eff} = 3.88 - 0.20(B-V)_0 + 0.026(B-V)_0^2 + 0.009\log g - 0.010(B-V)_0\log g - 0.051[{\rm Fe/H}] + 0.051(B-V)_0[{\rm Fe/H}]$, the relative accuracy of which is approximately 1.1%. In addition, the complete identity of the two main versions of the Baade–Becker–Wesselink method was proved: the surface brightness method (SB), first proposed by Barnes and Evans in 1976, and the maximum likelihood method (or light-curve modeling method) proposed by Balona in 1977 and later improved by Rastorguev and Dambis in 2010. This approach consists of using significantly nonlinear color calibrations for $\log T_{\rm eff}$ and bolometric correction BC and is easily applicable to the surface brightness method. This method is also applicable in studies of other types of pulsating variable stars, e.g., RR Lyrae, Mirae and δ Sct type variables with known effective temperature estimates.

Keywords: methods: data analysis, stars: fundamental parameters, stars: Cepheids

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1. INTRODUCTION

Numerous methods were proposed as solutions to problems related to constructing a universal distance scale. In particular, this applies to the "period-luminosity" relation calibrations for Cepheids, which are still considered as basic "standard candles" in galaxies. Naturally, using high-precision trigonometric parallaxes may be considered as the main and most preferable luminosity calibration method, however, at this time, trigonometric parallaxes for most Cepheids measured by the GAIA mission and included in the DR2 (Brown et al., 2018) and EDR3 (Brown et al., 2021) catalogs are still not accurate enough and carry both general and zonal systematic errors (Groenewegen, 2018). Moreover, even with precise trigonometric parallaxes available for Cepheids, some doubts remain over the reliability of the "period-luminosity" relation calibrations, due to the influence of differential interstellar extinction effects and the differences in the absorption laws for different directions in the Galaxy (see, e.g., Fitzpatrick and Massa, 2007). We shall nonetheless reference several recent studies as an example. Riess et al. (2021) attempted to estimate the Hubble constant based on GAIA EDR3 trigonometric parallaxes for 75 Galactic Cepheids with HST photometry available. The cited paper uses systematic parallax corrections that depend on the ecliptic latitude and the "average" absorption law. Ripepi et al. (2020) use the ABL (Astrometric Based Luminosity) (Arenou and Luri, 1999) approach in parallax space to estimate the luminosity of a large number of Galactic Cepheids with available GAIA DR2 data.

An alternative method of calibrating the Cepheid "period–luminosity" relation is based on the membership of Cepheids in young open clusters, the distances to which are determined by superimposing theoretical isochrones on the Hertzsprung–Russell diagrams using multicolor photometry data (see, e.g., An et al., 2007; Berdnikov et al., 1996) Compared to the trigonometric parallaxes of Cepheids, the photometric distances of open clusters containing Cepheids appear to be more reliable in terms of smaller systematic errors. However, the drawback of this method is the relatively small number of Cepheids whose membership in open clusters may be considered reliably proven, as well as the differential absorption effects in the vicinity of young open clusters.

Finally, we should note that one of the most effective means of estimating the main astrophysical characteristics of Cepheids—the radii, luminosities and distances—is still the pulsating photospheres method (the Baade–Becker–Wesselink technique, hereinafter BBW (Baade, 1926; Becker, 1940; Wesselink, 1946)), also often used for calibrating the slope and zero-point of the “period–luminosity” relation for Galactic Cepheids (Gieren et al., 2018; Groenewegen, 2007; Lazovik and Rastorguev, 2020; Molinaro et al., 2011; Storm et al., 2004, 2011a, 2011b, 2012).

2. VARIANTS OF THE PULSATING PHOTOSPHERES METHOD

There are two main versions of the BBW method: the Surface Brightness method (hereinafter SB), first proposed by Barnes and Evans (1976), which became the most popular, and the Maximum Likelihood method (hereinafter ML), first proposed by Balona (1977) and later significantly modified by us (Rastorguev and Dambis, 2011; Rastorguev et al., 2013) (hereinafter RD). The first method essentially comes down to modeling the radius variation of a Cepheid, and the second—to modeling the light curve. We shall show below that both these methods are completely equivalent, as they are based on the same physical foundation: the Stefan–Boltzmann law and the relation between the measured fluxes, the apparent, absolute and bolometric magnitudes, and distance.

In order to determine the physical parameters of Cepheids within the framework of the SB method, a linear relation between the so-called “surface brightness parameter” and normal color is usually used. In this approach, prior to calculating the average radius and luminosity of a Cepheid, one must first correct the light curves and colors for interstellar reddening and extinction. Such a preliminary correction of the extinction effects is not needed in the original version of the ML method, and therefore only the average Cepheid radius can be determined as a result. Note also that the original ML method variant subtly implies the existence of a linear dependence of $\log T_{\rm eff}$ and bolometric correction BC_λ on normal color, which was first noted by Rastorguev and Dambis (2011), who proposed generalizing the ML method by using nonlinear $\log T_{\rm eff}$ and BC_λ calibrations. Such a modification of the ML method, based on using well-known nonlinear approximations of $\log T_{\rm eff}$ and BC_λ from normal colors (for example, those taken from Bessell et al. (1998); Flower (1996)), allows one to determine not only the average Cepheid radius, but also the individual color excess, and compute the flux–averaged absolute magnitude and distance to the Cepheid. As a result, the modified RD method can be used for calibrating the “period–luminosity” relation for the Cepheids of the Galaxy. Let us again make notice of the fact that only the use of nonlinear relations between $\log T_{\rm eff}$ and the bolometric correction BC_λ on the one hand, and normal color on the other hand, opens the possibility of determining the color excess with an accuracy of about $0\overset{\rm m}{.}01$–$0\overset{\rm m}{.}02$. The same is true for the SB method. As is easily seen, due to the wide variety of the extinction laws within the Milky Way, noted, in particular, by Fitzpatrick and Massa (2007), the possibility of direct and independent determination of individual color excesses of Cepheids is a big advantage of our method.

3. NOTES ON USING THE PULSATING PHOTOSPHERES METHOD

Let us show the complete identity of the two popular versions of the BBW method: the surface brightness method (SB) and our variant of the light curve modeling method (RD).

3.1. RD Variant: Light Curve Modeling

As a direct consequence of the Stefan–Boltzmann law and the relation between the absolute and apparent stellar magnitudes, we can write the expression for the apparent magnitude in some phase of the pulsation cycle as (see the detailed derivation in our papers Rastorguev and Dambis (2011); Rastorguev et al. (2013)):

$$m = Y - 5\log\frac{R}{R_\odot} + \Psi(CI_0), \quad (1)$$

where R is the current radius of the Cepheid (computed by integrating the radial velocity curve), and the Y constant contains the apparent distance modulus of the Cepheid, the absolute bolometric magnitude $M_{\rm bol\odot}$, and the solar effective temperature logarithm $\log T_{\rm eff\odot}$:

$$Y = (m - M)_{\rm app} + M_{\rm bol\odot} + 10\log T_{\rm eff\odot},$$

and $\Psi(CI_0)$ is a function of normal color $CI_0 = CI - CE$ (CE—color excess) and bolometric correction BC to the apparent magnitude in the form of

$$\Psi(CI_0) = BC + 10\log T_{\rm eff}. \quad (2)$$

The $\Psi(CI_0)$ function is generally nonlinear by normal color, since the expression for the effective temperature logarithm $T_{\rm eff}(CI_0)$, as well as the expression for the bolometric correction BC, can in many cases be presented as power expansions by normal color CI_0

(sometimes including terms with surface gravity $\log g$ and metallicity [Fe/H]):

$$\Psi(CI_0) = a_0 + \sum_{k=1}^{N} a_k CI_0^k.$$

3.2. SB variant: Surface Brightness Method

Let us write the expression for the luminance created by a Cepheid in some color band (corrected for interstellar extinction), and the bolometric illumunance from the Cepheid and the Sun:

$$E = \frac{\pi}{4}\Phi\Theta_{\rm LD}^2;$$
$$E_{\rm bol} = \frac{\pi}{4}\Phi_{\rm bol}\Theta_{\rm LD}^2 = \frac{\sigma}{\pi}T_{\rm eff}^4; \quad (3)$$
$$E_{\rm bol\odot} = \frac{\sigma}{\pi}T_{\rm eff\odot}^4.$$

In these expressions Φ and $\Phi_{\rm bol}$ are the surface brightness of the star in the selected photometric band and the bolometric surface brightness, correspondingly (obviously, independent on distance!), σ is the Stefan–Boltzmann constant; $\Theta_{\rm LD}$ is the angular diameter of the star (with account for limb darkening), and $\Theta_\odot$ is the angular diameter of the Sun. Writing the magnitude differences $(m - m_{\rm bol})$ and $(m - m_\odot)$ in the standard form through the logarithms of the corresponding luminances, we obtain after simple algebraic transformations the following expression:

$$\log\frac{\Theta_{\rm LD}}{\Theta_\odot} = -0.2m - 2F(CI_0) + C, \quad (4)$$

where m is the magnitude of the star corrected for extinction,

$$F(CI_0) = 0.1BC + \log T_{\rm eff}; \quad (5)$$
$$C = 0.2m_{bol\odot} + 2\log T_{\rm eff\odot}.$$

In these expressions $F(CI_0)$ is the so-called "surface brightness parameter" introduced by Barnes et al. (2005); Barnes and Evans (1976)), and the constant C includes the solar parameters. It is important to note that based on our equation (2), the surface brightness parameter in expression (5) will have the form of $F(CI_0) = 0.1\Psi(CI_0)$.

Taking into consideration that

$$\frac{\Theta_{\rm LD}}{\Theta_\odot} = \frac{R}{R_\odot}\frac{1\rm AU}{D} = \frac{R}{R_\odot}\frac{1\ \rm pc}{2.063\times10^5 D},$$

after simple transformations, we can rewrite equation (4) as

$$5\log\frac{R}{R_\odot} = -m - 10F(CI_0) + X, \quad (6)$$

where the constant

$$X = (m - M)_0 + M_{\rm bol\odot} + 10\log T_{\rm eff\odot},$$

and $(m - M)_0$ is the true distance modulus. In the absence of interstellar extinction, the parameter $X = Y$, a parameter from expression (1), and both expressions for the two variants of the pulsating photospheres method—RD (equation (1) and SB (equation (6))—appear completely identical with an accuracy up to the order of the terms and some notations. It follows that when using the surface brightness method SB one should, the same as for the light curve modeling method RD, use nonlinear in terms of normal color calibrations for the surface brightness parameter $F(CI_0)$ (i.e., nonlinear color calibrations of effective temperature $\log T_{\rm eff}$ and bolometric correction BC). Evidently, using nonlinear calibrations within the framework of the surface brightness method SB will allow, as in the case of our variant of the light curve modeling method RD, to estimate the color excess independently and rather accurately (Rastorguev and Dambis, 2011; Rastorguev et al., 2013).

Additionally, we believe that under current conditions where the volume of the photometric observations of Cepheids exceeds significantly the volume of the radial velocity data, our version of the light curve modeling method RD is preferable compared to the radii variation modeling SB.

4. USING MULTIPHASE EFFECTIVE TEMPERATURE MEASUREMENTS

4.1. The Idea Behind the Method and Observational Material

In this work we shall describe in more detail another approach to the problem of determining Cepheid radii, color excesses and luminosities based on using multiphase measurements of their effective temperatures (Rastorguev et al., 2019). The results of spectroscopic effective temperature measurements obtained by the method of spectral line pairs depth ratio (Line Depth Ratio, LDR) (Gray and Brown, 2001) from high resolution echelle spectroscopy data (R = 40000–60000), as well as surface gravity values $\log g$ and metallicities [Fe/H] are published in a series of papers by Andrievsky et al. (2005); Kovtyukh et al. (2005, 2008); Luck (2018); Luck and Andrievsky (2004); Luck et al. (2008). The master catalog containing 1127 spectroscopic estimates (effective temperatures, chemical composition, surface gravity) for 435 Cepheids of the Galaxy is presented in Luck (2018). The data on 52 Cepheids with five or more effective temperature measurements are of superior value. The typical root-mean-square error of effective temperature determination by the LDR method from

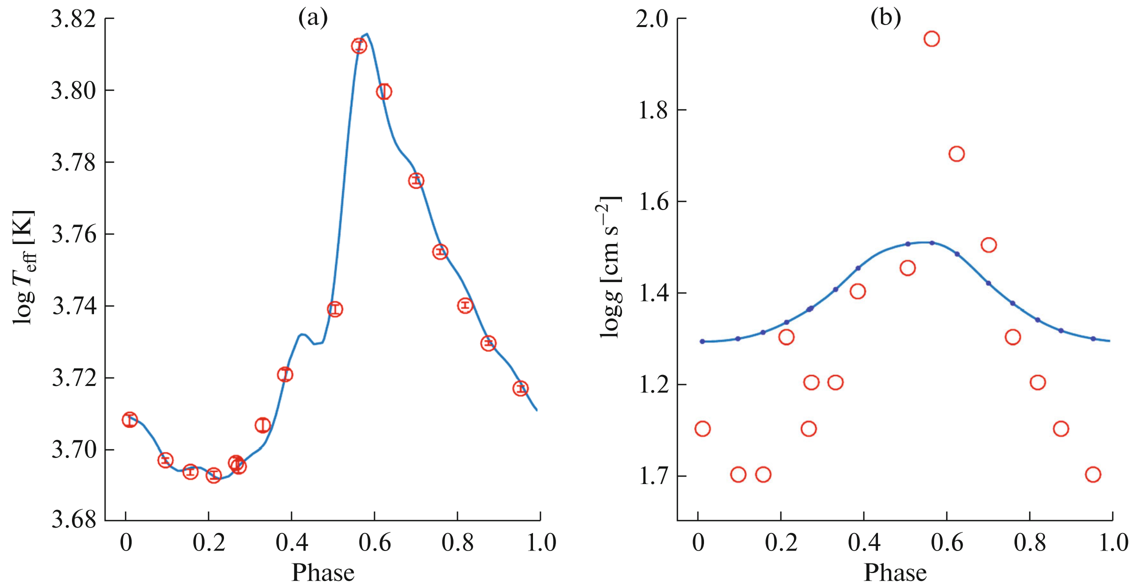

Fig. 1. Cepheid CD Cyg. (a) A comparison of the measured (circles) and calculated using the calibration of Flower (1996) (solid curve) effective temperatures. The measurement errors shown in the figure do not exceed the sizes of the circles. (b) A comparison of the measured (circles) and computed (solid curve) $\log g$ values.

several tens of spectral line pairs usually ranges from 30–40 to 100 K.

The currently available effective temperature calibrations (e.g., those presented in Bessell et al. (1998); Flower (1996)) are rather sensitive to color index. The main idea behind the new version of the RD algorithm consists of determining the color excess CE that gives the best agreement between the spectroscopically measured and pre-computed effective temperatures $\log T_{\text{eff}}$, calculated by applying "$\log T_{\text{eff}} - CI_0$" calibrations to the observed normal color index variation curve: $CI_0(\varphi) = CI(\varphi) - CE$, where φ is the pulsation phase. To model the $\log T_{\text{eff}}$ variations, we used the two above-mentioned calibrations for the effective temperature and bolometric correction by normal color $(B-V)_0$ (Bessell et al., 1998; Flower, 1996). As was shown earlier by Rastorguev and Dambis (2011), they facilitate the best agreement between the observed Cepheid light curves and those modeled using the RD method. The pulsation radius variations were computed simultaneously by means of the light curve modeling technique (Balona, 1977), but with nonlinear color decomposition (Sachkov et al., 1998).

In all computations we used photoelectric and CCD BV-band observations of the sample Cepheids from the VizieR Online Data Catalog II/285 (Berdnikov, 2008), multiple high-precision radial velocity measurements for northern sky Cepheids presented in Gorynya et al. (1992, 1996, 1998, 2002), and multiphase spectroscopic measurements of effective temperatures from the catalog of Luck (2018). When selecting photometric and spectral data, preference was given to quasi-synchronous observations, in order to exclude as much as possible the possibility of systematic errors of color excess, radius and luminosity determination due to the influence of evolutionary Cepheid period variations on the phase shift between the light curves and the radial velocity curves. As was shown by Sachkov et al. (1998), the phase shift errors are the main source of significant systematic errors that reach 30% of the average radius.

4.2. Effective Temperature Color Calibration

Owing to the high sensitivity of the computed effective temperature to the adopted color excess, the $E(B-V)$ color excess values were determined with an accuracy of about $0\overset{m}{.}01$–$0\overset{m}{.}02$ for 32 isolated Cepheids from our sample of a total of 52. Figure 1a shows, as an example of an outstanding agreement between the measured and computed temperatures, the results of modeling temperature data for the Cepheid CD Cyg, for which the color excess was estimated as $E(B-V) \approx 059 \pm 001$ using the calibration of Flower (1996). The measurements for effective temperatures $\log T_{\text{eff}}$ and surface gravities $\log g$ were taken from the spectral works of Andrievsky et al. (2005); Kovtyukh et al. (2005, 2008); Luck (2018); Luck and Andrievsky (2004); Luck et al. (2008). The complete list of Cepheids, including the known spectroscopic binaries, is given in Table 1 in Lazovik and Rastorguev (2020).

We also modeled surface gravity values. To estimate the current $\log g$ values we used the Cepheid radii computed for each pulsation phase and approximate estimates of their evolutionary masses made from Padova evolutionary tracks (Bressan et al., 2012). For comparison with the evolutionary tracks, the absolute magnitudes M_V we estimated from the "period–luminosity" relation (Berdnikov et al., 1996). Most Cepheids are at the stage of second or third crossing of the instability strip, and their masses were estimated from the approximate formula $\log M_{\rm cep}/M_\odot \approx 0.40 + 0.37\log P_0$, where P_0 is the period of the fundamental pulsation mode. The accuracy of the evolutionary mass estimate amounts to $0.3-0.5M_\odot$ and one can easily show that the typical $\log g$ estimate error is about 0.01–0.05 for Cepheid radii greater than $50R_\odot$, and therefore, our approximation is sufficiently good. In our computations, we adopted the often-used approximation with a period dependence $pf \approx 1.376 - 0.064\log P_0$ for the projection factor (Nardetto et al., 2007).

Figure 1b shows a comparison of the $\log g$ values measured spectroscopically and those calculated by us. The average CD Cyg radius is approximately $90R_\odot$, and the half-amplitude of its variation is about $10R_\odot$. From this, a straightforward estimation can be made of the maximum variation value: $\Delta\log g \approx 0.2$, which is precisely what is demonstrated by the solid curve that varies within the range of 1.3 to 1.5. At the same time, the spectroscopically measured $\log g$ values show an unrealistically large scatter in the interval from 0.7 to 2.0, although the phases of maximum and minimum $\log g$ values practically coincide with the computed curve. Such differences between the measured and calculated $\log g$ values are exhibited by all of our program Cepheids without exceptions. We believe that the reason behind the unrealistically large variations of the measured $\log g$ at different pulsation phases is the not quite correct Voigt spectral line profile decomposition performed in Andrievsky et al. (2005); Kovtyukh et al. (2005, 2008); Luck (2018); Luck and Andrievsky (2004); Luck et al. (2008). That is why the spectral calibration derived in Kovtyukh et al. (2008) for the normal colors of FGK supergiants and Cepheids in the form of

$$\begin{aligned}(B-V)_0 &= (57.984 \pm 4.485)\\ &- (10.3587 \pm 0.9797)(\log T_{\rm eff})^2\\ &+ (1.67572 \pm 0.17631)(\log T_{\rm eff})^3\\ &- (3.356 \pm 0.461)\log g\\ &+ (0.0321 \pm 0.0024)V_t\\ &+ (0.2615 \pm 0.0301)[\mathrm{Fe/H}]\\ &+ (0.8833 \pm 0.1229)\log g \log T_{\rm eff}\end{aligned} \quad (7)$$

cannot be considered correct, since its derivation included the not quite correct spectroscopic $\log g$ determinations.

Based on the most accurate color excess estimates for 32 Cepheids with multiple (more than ten) spectral measurements of effective temperatures, we made an attempt of a more correct derivation of the $\log T_{\rm eff}$ color calibration, also taking into account the differences in metallicity [Fe/H] and $\log g$. We have at our disposal for the 32 Cepheids a total of 407 effective temperature $\log T_{\rm eff}$ and metallicity [Fe/H] measurements, $(B-V)_0$ color indices corrected for reddening, and the $\log g$ values calculated by us. All computations—determinations of color excesses, radii, luminosities, and $\log g$ variations—were carried out for the two best calibrations (Bessell et al., 1998; Flower, 1996), used as calibrations for the first approximation. In order to obtain a refined calibration, we used 407 corrected $(B-V)_0$ color indices and $\log g$ surface gravity values computed for the moments (pulsation phases) of $\log T_{\rm eff}$ spectral measurements. All these data were used for each of the mentioned calibrations of the first approximation to determine using the optimization method the coefficients of the new calibration and their errors. As was expected, both first approximation calibrations yielded practically the same results.

As a result, we found the best relation in terms of accuracy between the normal color, surface gravity, metallicity and effective temperature in the following form:

$$\begin{aligned}\log T_{\rm eff} &= 3.88(\pm0.01) - 0.20(\pm0.02)(B-V)_0\\ &+ 0.026(\pm0.008)(B-V)_0^2 + 0.009(\pm0.004)\log g\\ &- 0.010(\pm0.006)(B-V)_0\log g\\ &- 0.051(\pm0.017)[\mathrm{Fe/H}]\\ &+ 0.051(\pm0.022)(B-V)_0[\mathrm{Fe/H}].\end{aligned} \quad (8)$$

The new calibration that also takes into account the differences in metallicity and surface gravity in Cepheids gives a relative error of reconstructing the measured effective temperature of the order of: $\sigma_T/T \approx 1.1\%$.

5. CONCLUSIONS

In this work we continue modifying the BBW method variant first proposed in our papers (Rastorguev and Dambis, 2011; Rastorguev et al., 2013) and based on using significantly nonlinear color calibrations for the effective temperature $\log T_{\rm eff}$ and bolometric correction BC, which opens up the possibility of estimating the color excesses of Cepheids independently and rather accurately. We propose a new modification of the method that consists of using multiphase spectral effective temperature estimates for Cepheids and comparing them with the temperatures pre-computed using the known calibrations. We derived a new calibration from 407 effective temperature measurements for 32 Cepheids that also takes into account the dependence on metallicity [Fe/H] and surface gravity $\log g$, and provides an accuracy of the effective temperature estimate of the order of 1%. The method was successfully used in the work of Lazovik and Rastorguev (2020) to derive a new "period—luminosity" dependence for Cepheids.

We show that both variats of the BBW method—the surface brightness method and the light curve modeling method—are completely equivalent and are actually based on the color calibrations of the so-called "surface brightness parameter", which is a linear combination of $\log T_{\rm eff}$ and BC. Both variants, when used with nonlinear by normal color calibrations for the surface brightness parameters allow one to determine not only the average radius of a Cepheid, but independently estimate its color excess and average absolute magnitude. At the same time, using the light curve modeling method is preferable due to the higher number of brightness and color index measurements compared to the volume of spectral observations. Evidently, the proposed method that uses effective temperature measurements is applicable not only to Cepheids but to other types of pulsating stars: RR Lyrae, Mirae, and δ Sct type variables.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

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