Absolute Magnitude Calibrations of Population I and II Cepheids and Other Pulsating Variables in the Instability Strip of the Hertzsprung-Russell Diagram

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stars, variable, absolute magnitude calibrations, Cepheids, RR Lyraes, SX Phe stars, AHB1 stars, anomalous Cepheids

Abstract

The status of the absolute magnitude calibrations is reviewed for the long period Cepheids of population I and II, RR Lyrae stars, evolved "above horizontal branch" (AHB1) variables (periods 0.8 to 3 days), dwarf Cepheids of both populations (the Delta Scuti and SX Phoenicus variables), and the anomalous Cepheids (AC). Evidence shows that the period-color and period-luminosity (P-L) relations for population I Cepheids in the Galaxy and in the Large and Small Magellanic Clouds have different slopes and zero points. This greatly complicates use of Cepheids for the extragalactic distance scale. Strategies are discussed to patch the problem. A consensus exists for the long distance scale for RR Lyrae stars whose calibrations favor $\langle M_V(RR)\rangle = 0.52$ at $[Fe/H] = -1.5$. Exceptions exist for "second parameter" clusters where the variation of the morphology of the horizontal branch with metallicity is anomalous, the most blatant being NGC 6388 and NGC 6441. The status and calibrations of ABH1 and AC show that different evolutionary paths and masses explain the difference P-L relations for them. AC appear predominantly in the dwarf spheroidal galaxies, but are almost absent in Galactic globular clusters. AHB1 stars are absent in dwarf spheroidals but are present in globular clusters. The difference may be used to study the formation of the remote Galactic halo if it is partially made by tidal disruption of companion dwarf spheroidals.

1. INTRODUCTION

Generally, the more deeply a scientific problem is studied, the more complex becomes its solution. Although the first approximations made at the beginning can scout out a territory, as the database expands, first approximations must often be replaced. The use of Cepheid variables and RR Lyrae stars as distance indicators has now reached this point.

Recent observations have almost certainly shown that the period-luminosity (P-L) relation of classical Cepheid variables in the Galaxy has a different slope and zero point than that of Cepheids in the Large Magellanic Cloud (LMC) and Small Magellanic Cloud (SMC). Equivalent expressions of these differences are the observed offsets of the period-color relations between the Galaxy and the Clouds (due to Fraunhoffer blanketing differences plus the effects of real temperature differences in the position of the edges of the instability strip), and different slopes of the lines of constant period in the Hertzsprung-Russell (HR) diagrams of the three galaxies.

The discovery of the color differences was made 40 years ago by Gascoigne & Kron (1965) who found SMC Cepheids to be bluer than Galactic ones of the same period. The color difference was in the correct direction to be due to the known metal weakness in the SMC but was greater than could be accounted for by only the technical effect of line blanketing on the colors. However, that conclusion depended on the accuracy of the reddening corrections to the Galactic Cepheids, which were uncertain at the time.

The discovery was followed by the strong conclusion by Laney & Stobie (1986) that the temperature of the ridge-line locus of the instability strip for Cepheids in the SMC is indeed hotter than the ridge-line locus for Cepheids in the LMC and the Galaxy. Their value for the Galaxy-SMC temperature difference was 210 $\pm$ 80 K.

More recent analyses (Tammann, Sandage & Reindl 2003; Sandage, Tammann & Reindl 2004) confirm their conclusion, although the Galaxy-LMC temperature difference of 150 K is smaller than that of Laney & Stobie, and it depends on period (see figure 3 of Sandage, Tammann & Reindl 2004).

From an intricate analysis, Sandage, Tammann & Reindl (2004) concluded that the modern reddening values of Galactic Cepheids by Fernie (1990, 1994), and by 15 others reduced to the Fernie system by Fernie et al. (1995), are highly reliable after a small scale correction, removing the previous doubt for real temperature differences based on early Galactic reddening values.

Clearly, differences in the slope of the period-color relations between the Galaxy and the Clouds require that the slope of the P-L relations must also differ in the various colors. Even if the P-L relations were the same in one photometric band, they would differ in all other bands. Furthermore, if real temperature differences do exist, it is likely that the luminosity zero point of the individual P-L relations must also differ, unless unlikely compensation exists in the mass-luminosity relations such that the luminosity from the pulsation equation does not change.

This review addresses this and other problems concerning the uniqueness of the Cepheid P-L relation from galaxy to galaxy.

Since the last reviews in these pages and elsewhere about RR Lyrae (RRL) luminosities as a function of metallicity, nasty problems have also been discovered in their use as distance indicators. Until 1997, most data and analyses of period-luminosity-metallicity correlations of RRL were consistent, with their absolute magnitudes being tightly correlated with metallicity, such that if the [Fe/H] metallicity was known, then the RRL luminosity was also believed to be known to within narrow limits. The effects of evolution off the zero age horizontal branch (ZAHB) complicated the correlation, but a nearly unique period-amplitude-metallicity relation for unevolved stars (Sandage 1981a,b; Carney, Storm & Jones 1992, their equation 16; Alcock et al. 1998; Alcock 2000) was believed to be able to flag stars in this evolutionary state, permitting correction of the nonevolved calibration of the M_V(RR)-[Fe/H] relation. However, this assumption has now been challenged by the discovery of large-amplitude RRL variables with abnormally long periods in the metal rich ([Fe/H] ~ -0.5) globular clusters NGC 6388 and NGC 6441 (Rich et al. 1997; Pritzl et al. 2000, 2001). It is believed that the abnormal RRL period distribution in these clusters is due to an anomalously bright RRL HB luminosity (at the level of about 0.25 mag) compared with the canonical M_V-([Fe/H]) calibration known to be valid before 1997. This discovery showed that not all RRL follow the same luminosity-metallicity relation. There must be an unknown component to the pulsation physics (abnormal He core mass or some parameter like it) that complicates and spreads the HB zero-point luminosity values at given [Fe/H] values.

This review addresses these and other problems in the calibration and use of Cepheids and RRL variables as distance indicators. Earlier reviews of these problems include Feast & Walker (1987) and Reid (1997, 1999) in these *Annual Review* pages, and, among others, Carney, Storm & Jones (1992), Gieren, Fouque & Gomez (1998), Feast (1999, updated in 2003), Cacciari & Clementini (2003), Bono (2003), Tammann, Sandage & Reindl (2003), Fouque, Storm & Gieren (2003), and Sandage, Tammann & Reindl (2004), each with references to others to which the reader is referred. Many of the intricate details are not given again here.

Progress has also been made in testing the luminosity calibration of the Galactic classical Cepheids by (*a*) the method of angular radius measurements using stellar optical interferometers, (*b*) direct *Hipparcos* trigonometric parallaxes, (*c*) added data for the Baade (1926)-Becker (1940)-Wesselink (1946) moving atmospheres method, and (*d*) new data for the open cluster and association main sequence fittings.

New luminosity calibrations of RRL variables both in globular clusters and the general field have also been made by diverse methods including (*a*) luminosities calculated from a pulsation equation using observed inputs on period-metallicity correlations and temperature-metallicity correlations at the fundamental blue edge (FBE) of the instability strip, (*b*) main sequence fittings in the clusters, (*c*) Baade-Becker-Wesselink (BBW) moving atmosphere parallaxes for field variables, (*d*) the statistical parallax method using new radial velocities and updated proper motions, and (*e*) use of the newly discovered Delta Scuti variables (periods 0.05 to 0.25 days) in globular clusters (i.e., the SX Phe population II subclass) as stepping stones from their known trigonometric absolute magnitudes between M_V of +2 and +4 to the much brighter HB that contains the RRLs.

Progress has also been made in the luminosity calibration of the population II long period Cepheids (periods 13 to 30 days) and for the 1–3 day "above horizontal branch" (AHB1) variables in globular clusters and in the local dwarf spheroidal galaxies. The status is also reviewed of the "anomalous Cepheids" (AC) being discovered in an expanding data base.

This review contains the following: The instability strip for Cepheids is defined in Section 2 with a short history of its discovery and placement in the HR diagram. Modern period-color relations are in Section 3, based on data obtained by Berdnikov, Dambis & Voziakova (2000) for the Galaxy and by Udalski et al. (1999a,b) for the LMC and SMC. New slope and zero-point calibrations for the Cepheid P-L relations are set out in Section 4, based on a summary of data made available since Feast & Walker (1987) and Feast (1999). Comparison is made of the derived calibrations from main sequence fittings and the Baade-Becker-Wesselink moving atmosphere method with the absolute magnitudes from *Hipparcos* trigonometric parallaxes and from optical interferometer diameters of Cepheids.

New RRL luminosity calibrations are in Section 5, stressing the need for, and the evidence supporting, a nonlinear M_V-metallicity relation, both for the ZAHB for different metallicities and for evolved HB configurations. The nonuniqueness problem posed by the second parameter effect in its exaggerated form that has been signaled by the metal-rich clusters NGC 6388 and NGC 6441 is also discussed.

The status of the AHB1 variables in the field and in clusters is discussed in Section 6 as a prelude to a review of the modern calibration of the population II Cepheids relative to the cluster RRLs in Section 7.

Advances in the understanding of the anomalous Cepheids and their absolute magnitude calibration are in Section 8.

2. THE CLASSICAL CEPHEID INSTABILITY STRIP IN THE HR DIAGRAM

In the late 1800s, August Ritter (1879) began the study of the pulsation of stars by showing that the period of pulsation must be related to the mean density of a pulsating star by $P\,(\rho)^{1/2} = Q(P)$, where Q is a very weak function of period if the restoring force after displacement from the equilibrium position is gravity. The relation follows directly from dimensional analysis using Newtonian inverse square attraction and the law of inertia.

Cepheid variables had been identified as a homogeneous class in the last decade of the nineteenth century based on the similarity of light curves (see Fernie 1969 for a review), but had been shown to be pulsating stars only when Shapley (1914) gave a series of convincing arguments. An understanding of why a relatively tight P-L relation exists for Cepheids had then become clear when the Cepheids were discovered to have a highly restricted range of spectral type (temperature) at a given period (Adams & Joy 1927, Shapley 1927a). **Figure 1** shows this important conclusion from the paper by Adams & Joy. This temperature restriction (spectral type changed to temperature) at a given period, put together with the P-L relation, gives a temperature-luminosity locus in the HR diagram. This is the instability strip.

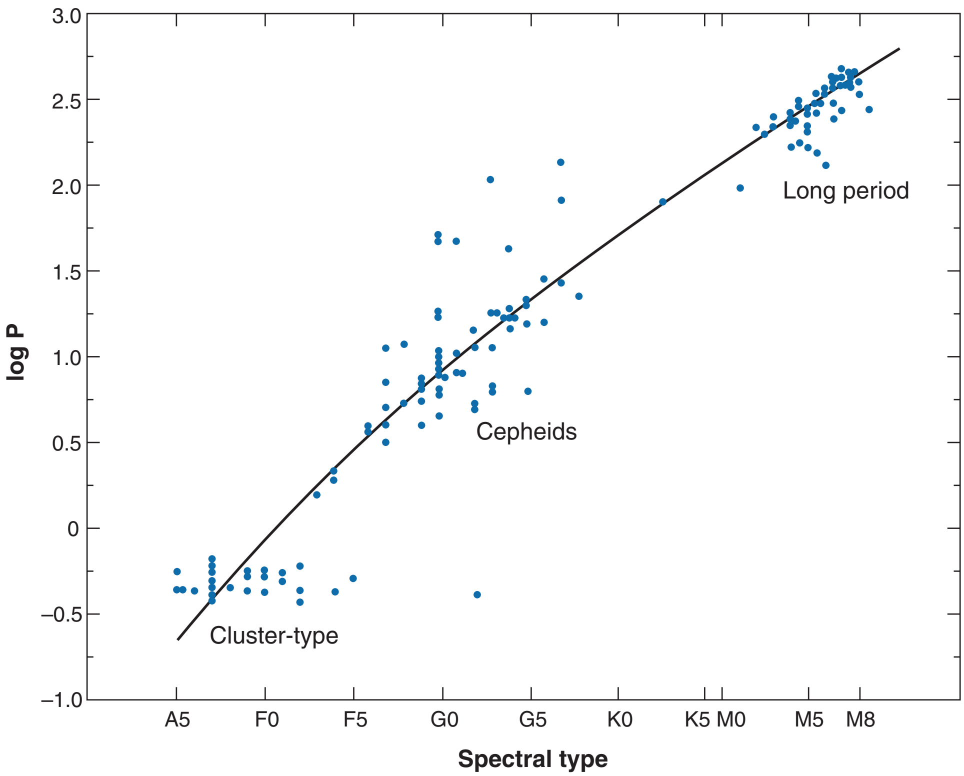


Figure 1

The relation by Adams & Joy (1927) between spectral type at maximum light and period for RR Lyrae-type variables, Cepheids, and long period Miras based on the first 15 years of sidereal observations made at Mount Wilson. The scatter at a given period is real due to the finite width of the instability strip. A similar relation using spectral type at midlight is by Shapley (1927a).

The isolation of the strip in the HR diagram was implicit in the attempts by Shapley (1927b) and Russell (1927) to understand the slope of the P-L relation. However, their papers were not particularly transparent in the modern language of the subject. They did realize that the Ritter $P\ (\rho)^{1/2}\ =\ Q(P)$ requirement would not restrict the luminosity at a given period unless there was a temperature restriction to the pulsation condition. Said differently, lines of constant period can be drawn in the HR diagram using the Ritter law. They slant over the diagram faintward and toward lower temperatures. This produces a large range of periods at a given luminosity if the temperatures of the variables can vary over an appreciable part of the HR diagram. However, the temperature restriction (the tight spectral-type, period correlation of **Figure 1**) cuts these constant period lines at discrete luminosities, producing a tight P-L relation.

Either because both the Shapley and Russell papers are semiopaque, or because interest in the problem settled on other aspects of it, these two central papers became almost lost in the archives of history between 1927 and 1950. However, the problem they solved was rediscovered once the evolutionary tracks of Cepheids across the HR diagram settled the problem of the Cepheid masses, at least in principle. Also, photoelectric observations of the Galactic Cepheids began to be obtained by Eggen (1951), permitting a color-magnitude HR diagram to be plotted (Eggen's figures 42 and 43). The language of an "instability strip" became explicit (Sandage 1958b) with its intrinsic width and lines of constant period, leading to an intrinsic scatter in the P-L relation that varies with wavelength (Sandage 1958b, 1972; Sandage & Tammann 1968, 1969).

The instability strip of the RRL variables is an extension of the Cepheid strip to fainter magnitudes and shorter periods. The RRL star position in this fainter part of the strip near $M_V = 0.5$ was discovered by Schwarzschild (1940), who showed that the RRL stars in the globular cluster M3 were confined to a narrow range of color along the cluster's HB. It was later shown that the same is true for all globular clusters and that the color boundaries of the RRL instability "gap" are a continuation of the Cepheid strip. The 1927 period-spectral relation of Adams & Joy in **Figure 1** shows the continuity of the period-spectral type relation of the RRL variables with the later spectral types of the long period Cepheids.

Hence, we are dealing with a single instability strip in the HR diagram (see Cox 1974, figure 1 for a review of the location of the various classes of variable stars in the HR diagram) that can be extended to the main sequence to include the Delta Scuti variables ultra short period Cepheids (USPC), to be discussed in Section 5.4, and even faintward to $M_V = +10$ to include the ZZ Ceti pulsating white dwarfs (figure 1 of Gautschy & Saio 1995).

2.1. Period-Color Relations for Cepheids in the Galaxy, LMC, and SMC

Since the last reviews cited earlier, two major advances have been made in the observations that permit nearly definitive determinations of the period-color and color-luminosity (i.e., the HR diagram) relations for the classical Cepheids in the Galaxy, the LMC, and the SMC.

Berdnikov, Dambis & Voziakova (2000) have published accurate B,V,I photometry on the Cape (Cousins) system as realized by Landolt (1983, 1992) for hundreds of Galactic Cepheids for which Fernie (1990, 1994) and Fernie et al. (1995) have determined E(B-V) color excesses. Tammann, Sandage & Reindl (2003) have made slight corrections to the color excess values on the Fernie system to remove a mild correlation of the initial Fernie color excess with residuals from the period-color relation; i.e., the Cepheids with large color excess on the initial Fernie system are systematically redder in their derived intrinsic colors than Cepheids of the same period with smaller excess values. The Tammann, Sandage & Reindl corrections produce the period-color relations in B-V and V-I for Galactic Cepheids of

$$(B\text{-}V)^o = (0.366 \pm 0.015) \log P + (0.361 \pm 0.013) \quad (1)$$

from 321 stars, and

$$(\text{V-I})^{o} = (0.256 \pm 0.017) \log P + (0.497 \pm 0.016) \quad (2)$$

from 250 stars.

These can be compared with similar period-color relations for Cepheids in the LMC and SMC from the extensive photometry by Udalski et al. (1999a,b). Analyzed in the same way as for the Galactic Cepheids, Tammann, Sandage & Reindl obtained period-color relations for both the LMC and SMC that differ from Equations 1 and 2. The differences are similar to those found by both Gascoigne & Kron and Laney & Stobie mentioned earlier. The comparisons in (B-V)^{o} are shown in **Figure 2**.

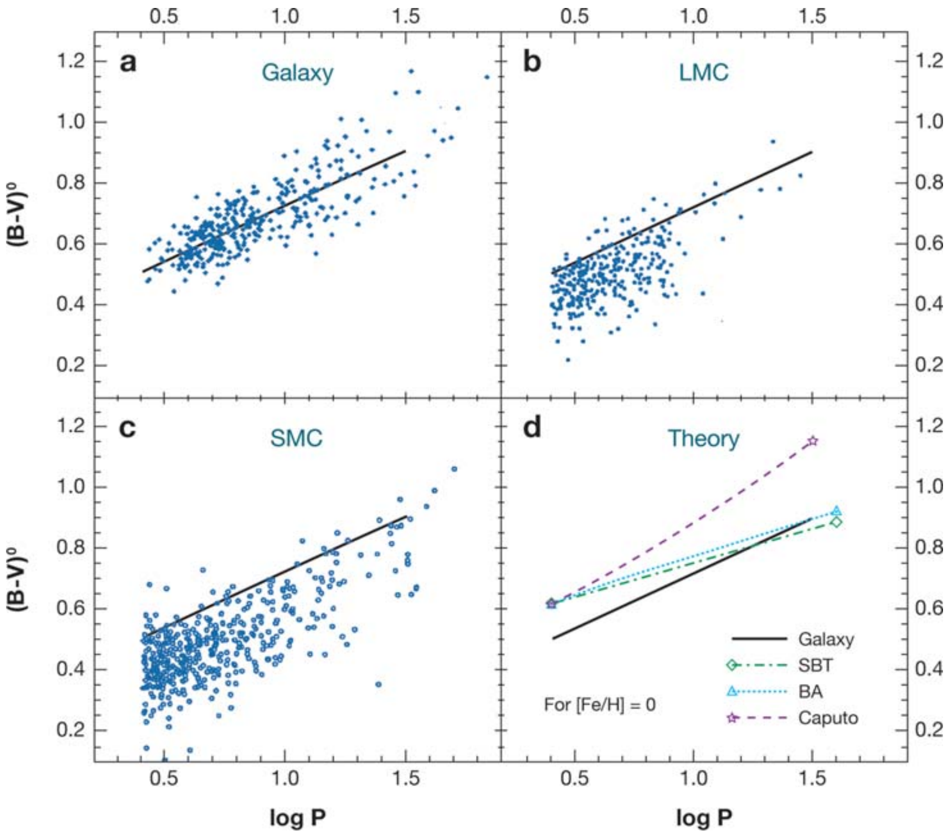


Figure 2

Comparison of the period-intrinsic (B-V)^{o} color relations for the Galaxy, LMC, and SMC from the photometry of Berdnikov, Dambis & Voziakova (2000) for the Galaxy, and of Udalski et al. (1999a,b) for the LMC and SMC. The paucity of points with log P > 1.0 for the LMC show a selection effect in the data by Udalski et al. due to photometric saturation of the detectors used for the observations. The least squares fit to the Galaxy data in the upper left panel is repeated as the solid black line in the other three panels. The offsets discovered by Gascoigne & Kron (1965) and by Laney & Stobie (1986) are evident.

Figure 2*a* gives the color-period data for the Galaxy from Berdnikov, Dambis & Voziakova (2000). A linear least squares correlation line is drawn. This line is repeated in the other three panels to illustrate the color offset using the data for the LMC and SMC from Udalski et al. (1999a,b). The paucity of points with log P > 1.0 for the LMC shows a selection effect in the Udalski et al. data due to photometric saturation of the brightest Cepheids in the LMC, hence their rejection from the Udalski et al. database.

Figure 2*d* also shows predicted period-color ridge lines from three model calculations by Sandage, Bell & Tripicco (1999), Caputo, Marconi & Musella (2000b), and Baraffe & Alibert (2001), showing that the theoretical models, with their adopted transformations of temperature to B-V color, differ by ~0.2 mag at short periods but, except for the predictions of Caputo et al., agree to better than 0.05 mag in color for log P > 1.0.

Comparison of the LMC data with the Galactic Cepheids is shown in **Figure 3**. Closed circles are from Berdnikov, Dambis & Voziakova (2000). Open circles are

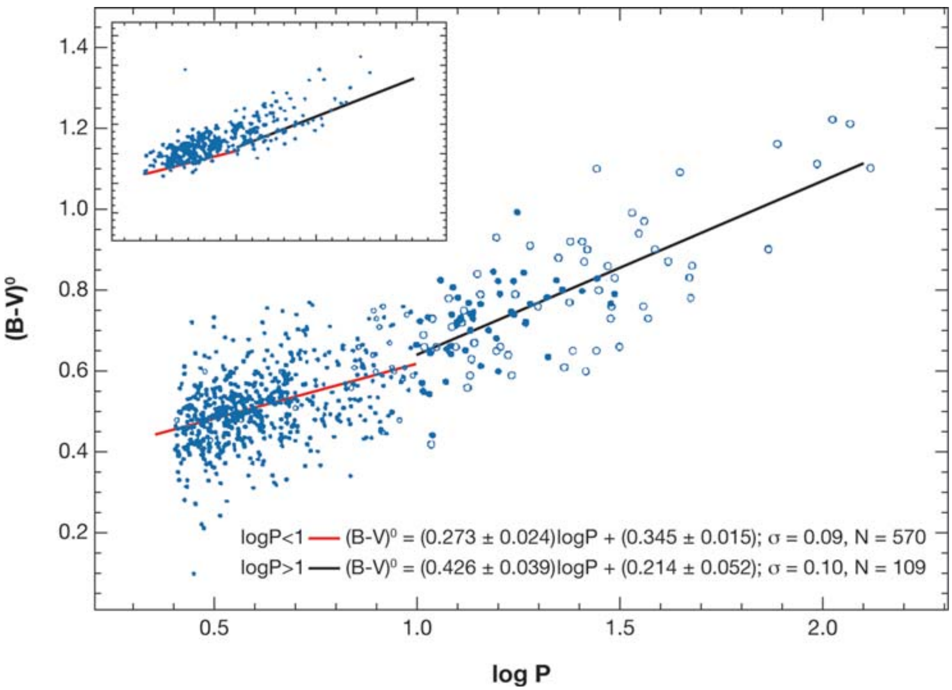


Figure 3

Comparison of the period-color relation of the Large Magellanic Cloud (LMC) with that of the Galaxy, showing the break at the 10-day period in the LMC but not in the Galaxy, and the difference in slope at all periods. The blue dots are from Udalski et al. (1999a). The open blue circles are additional data from the literature. Note the difference in the equations for the LMC (in the interior of the figure) with that of the Galaxy from Equation 1 here. The insert shows the individual Galaxy data compared with the two LMC mean lines. The difference is at the 4 sigma level. Diagram is from figure 1a of Sandage, Tammann & Reindl (2004).

additional longer period Cepheids from the literature. The difference in slope and zero point between the Galaxy and LMC is evident (shown in the insert of **Figure 3**). A similar difference exists in the period-$(V\text{-}I)^o$ relation from Sandage, Tammann & Reindl (2004), not shown here.

These color differences translate to differences in the positions of the instability strips in the HR diagrams for the Galaxy, LMC, and SMC. These can be constructed once the absolute magnitude scale is chosen. The result is shown in **Figure 4** using the true distance moduli shown in each panel, justified in Tammann, Sandage & Reindl (2003).

A more detailed comparison between the Galaxy and LMC in $(B\text{-}V)^o$ and $(V\text{-}I)^o$ is in **Figure 5**, taken from an updated analysis of the Udalski data (Sandage, Tammann & Reindl 2004, their figure 8). Four lines of constant period are shown, labeled by their log P values. The break at 10 days for the LMC is explicit. The insert sets out the individual data points for the Galaxy Cepheids, showing the offsets from the LMC ridge and the blue and red color boundaries as the lines in the insert diagrams.

The color data for the individual LMC Cepheids in **Figure 5** have been changed to temperatures by the color-temperature relations in Sandage, Bell & Tripicco (1999, their table 6) with the result shown in **Figure 6**. The two solid lines in both **Figures 5** and **6** near the middle of the strip for log P smaller and larger than 1.0 are drawn, as are the blue and red color boundaries of the strip, shown as dashed lines. Five lines of constant period in both diagrams are the sloping dotted lines crossing the instability strip, marked with their log P values. The ridge line of the Galaxy instability strip in **Figure 6** is transferred from **Figure 4** and changed to temperature.

The main conclusion from **Figure 6** is that there is a real temperature difference between the instability strips of the Galaxy and the LMC. The consequences, of course, are (*a*) there must be a difference in the zero points of the P-L relation between the Galaxy and the LMC (seen by the different log L values at the intersection of the lines of constant period with the strip boundaries and the ridge lines), and (*b*) the slopes of the P-L relations must also differ because the Galaxy and the LMC ridge lines are not parallel in **Figure 6**.

These are dire results, boding ill for the use of Cepheids as precision distance indicators at the 0.3-mag level unless corrections from one P-L relation to the other can be made. However, one wants to be convinced that the color and temperature differences shown in **Figures 2–6** are real.

The early evidence for color differences between the Galaxy and the LMC Cepheids by Gascoigne & Kron and by Laney & Stobie has been made stronger by the reddening values derived by Fernie that are confirmed by Tammann, Sandage & Reindl (2003), as discussed earlier. That evidence has been made even stronger by the recent analysis of Ngeow & Kanbur (2005).

The break in the Cepheid LMC period-color relation at 10 days and its absence in the Galaxy is the other principal difference between the two galaxies. The original suggestion of the color break by Tammann & Reindl (2002) and Tammann et al. (2002), supported by Kanbur & Ngeow (2002) and later argued by them in more detail (Kanbur & Ngeow 2004, Ngeow & Kanbur 2005, Ngeow et al. 2005), supports the reality of the difference.

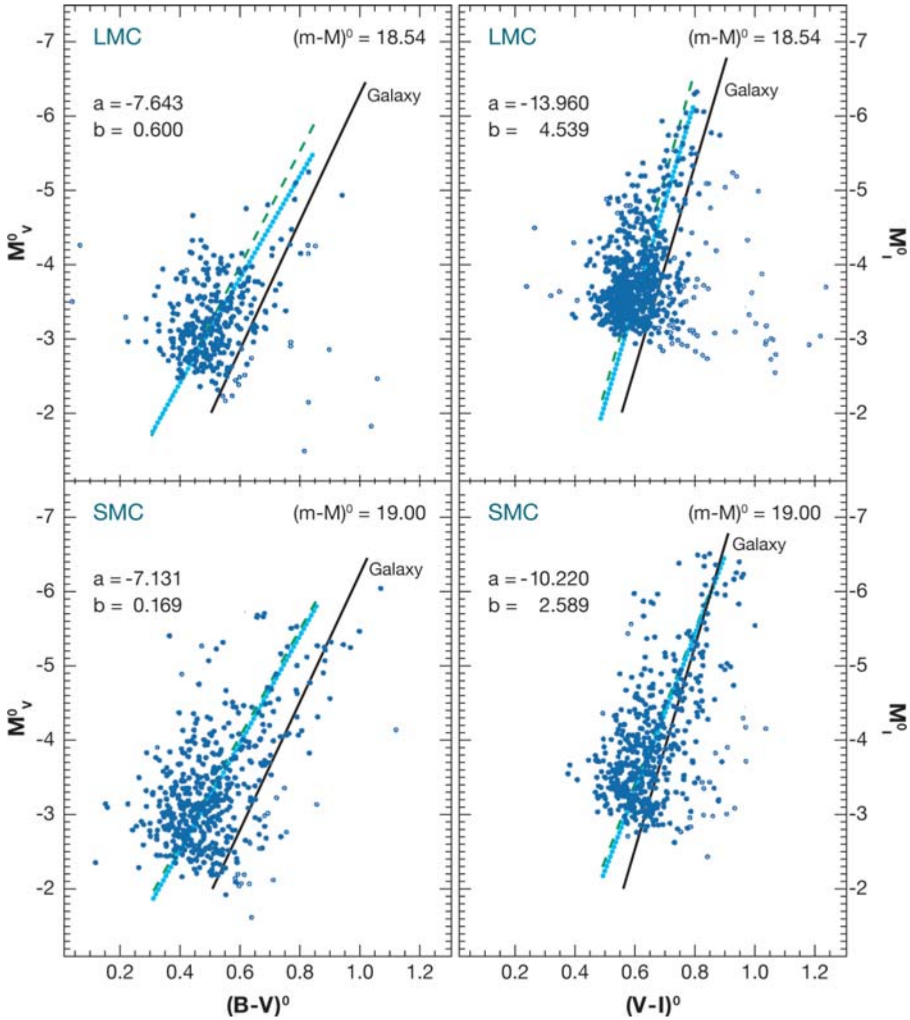


Figure 4

The instability strip in color for the Galaxy, LMC, and SMC. The ridge line for the Galaxy from Tammann, Sandage & Reindl (2003, their figure 16) is compared with the individual LMC and SMC data points showing the offset in both B-V and V-I at given $M_{V,I}$ values. The coefficients of $M_{V,I} = a(\text{color}) + b$ for the least squares fits are in each panel. The green dashed and light blue dotted lines are for two subsamples of the Udalski et al. data.

A most telling independent piece of evidence is by Tanvir et al. (2005) using Fourier decomposition of light curves into principal components to conclude that there are systematic differences in the light curve shapes at the same period between the Galaxy, LMC, and SMC.

Of course, the color differences require different slopes to the P-L relations, and the color break in the LMC (but not in the Galaxy) at ~10 days also produces a break

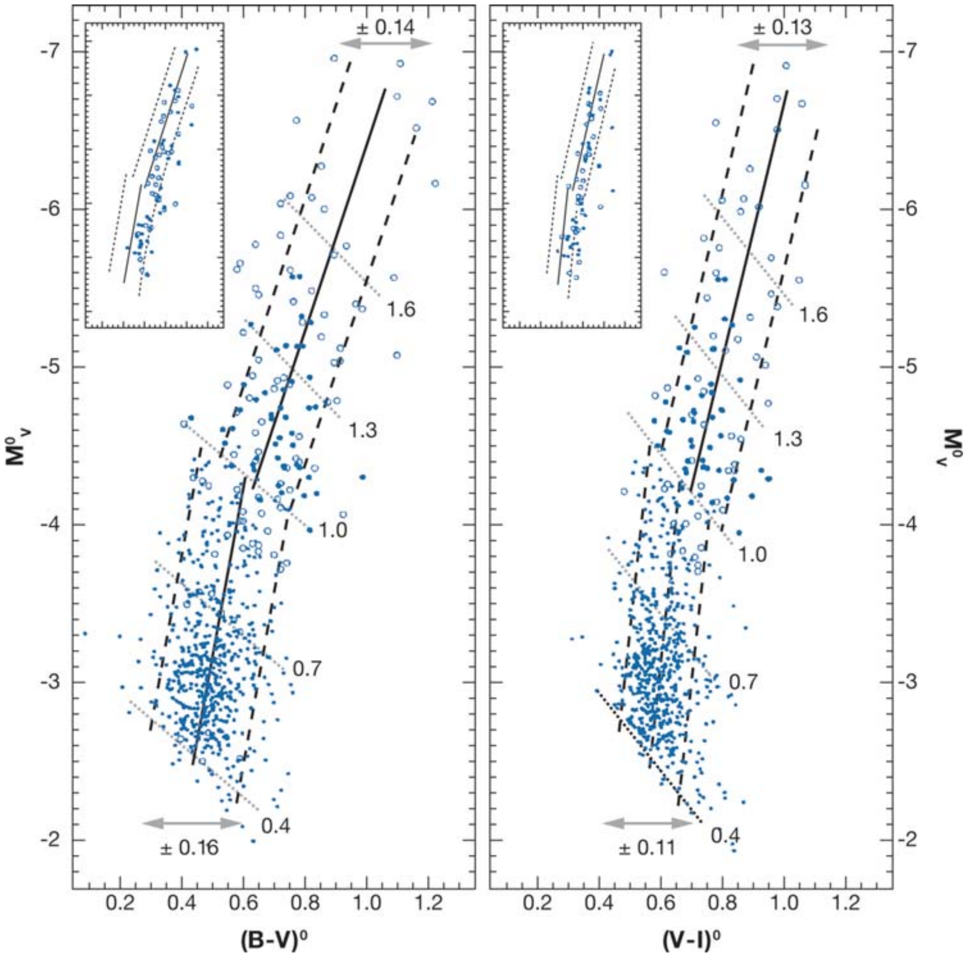


Figure 5

The instability strips for the LMC in (B-V)⁰ and (V-I)⁰ compared in the insert with the individual data for the Galactic Cepheids. Four lines of constant period, labeled by their log P values, are shown. The individual data in the main panels are for the LMC. Diagram is from Sandage, Tammann & Reindl (2004, their figure 8).

in the LMC P-L relation, but again, not in the Galaxy. It was at this point that a search for slope and zero point differences in the P-L relations became crucial.

3. CALIBRATIONS OF THE P-L RELATIONS FOR THE GALAXY, LMC, AND SMC

Before the reddening and absolute magnitude data for Galactic Cepheids became known with sufficient accuracy, it was common practice to adopt the P-L slope from Cepheids in the LMC and then to set the zero point in some way, such as by main

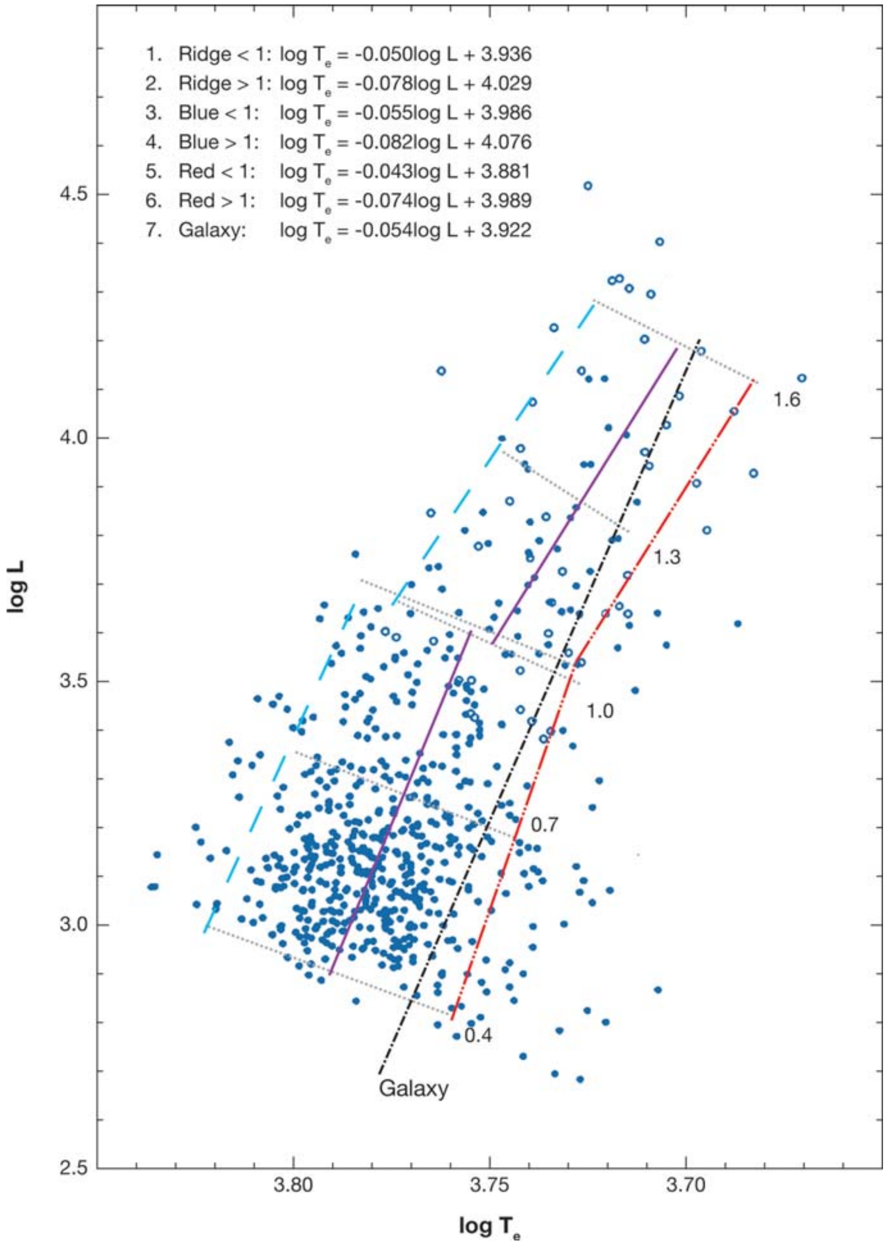


Figure 6

The instability strip in log L, log T_e for the LMC compared with the ridge-line relation for the Galaxy, shown as the black, dashed-dot line. Five lines of constant period (log P of 1.6, 1.3, 1.0, 0.7, and 0.4) are shown. The hot and cold boundary lines to the strip are the same lines as in **Figure 5** transformed to temperature. Diagram is from figure 20 of Sandage, Tammann & Reindl (2004).

sequence fittings or by some other means (Sandage & Tammann 1968). Obviously, it was impossible in this way to discover differences, if they exist, in the slopes and zero points of the Galactic and the LMC/SMC P-L relations that must be present if the color differences are real.

A fundamental advance on the problem was made beginning in the 1990s when the absolute magnitudes and interstellar absorptions of many Cepheids in the Galaxy had been determined by both main sequence fittings in clusters and associations as summarized by Feast & Walker (1987) and Feast (1999, 2003), and by Gieren et al. (1998) using a variation of the BBW moving atmospheres method introduced by Barnes & Evans (1976). Comparison of the results from the two methods were summarized in 2003 by Tammann, Sandage & Reindl. They were revised with additions to each list in a new summary by Sandage, Tammann & Reindl (2004, their tables 3 and 4). The additions to the data for the main sequence fittings since Feast (1999) are from Turner & Burke (2002) and Hoyle, Shanks & Tanvir (2003). The fiducial main sequence to which the cluster data were fitted uses a zero point based on a Pleiades modulus of $(m\text{-}M)^{o} = 5.61$ from Stello & Nissen (2001). This is within 0.06 mag of the photometric distance (5.60) by Pinsonneault et al. (1998), the moving cluster distance (5.58) of Narayanan & Gould (1999), and the latest *Hipparcos* direct parallax distance (5.55) by Makarov (2002).

The 1998 BBW listings by Gieren et al. (2005), used by Tammann, Sandage & Reindl in 2003, were replaced by Sandage, Tammann & Reindl (2004) using the revised BBW distances of Fouque et al. (2003), to which new BBW distances by Barnes et al. (2003) were added, plus the interferometer distances from angular diameter measurements as summarized by Kervella et al. (2004).

The two new independent sets of Galactic calibrators (main sequence fittings and BBW) used by Sandage, Tammann & Reindl (2004) agree in their P-L relations to within 0.05, 0.07, and 0.10 mag in B, V, and I at P = 10 days. This is highly satisfactory, and Sandage, Tammann & Reindl (2004) combined the data to form a mean Galactic calibration.

The results are in **Figure 7**, taken from figure 5 of Sandage, Tammann & Reindl (2004). The open circles are from the BBW method; the closed circles are from main sequence fittings. The dashed lines are the calculated envelope boundaries to the P-L ridge lines whose width is given by the product of the width of the instability strip and the slope of the constant period lines. There is no break in slope in the 10-day period in these Galactic data, differing in that respect from that in LMC. The B, V, and I equations of the combined Galactic P-L relations are:

$$M_B = -(2.692 \pm 0.093) \log P - (0.575 \pm 0.107), \qquad (3)$$

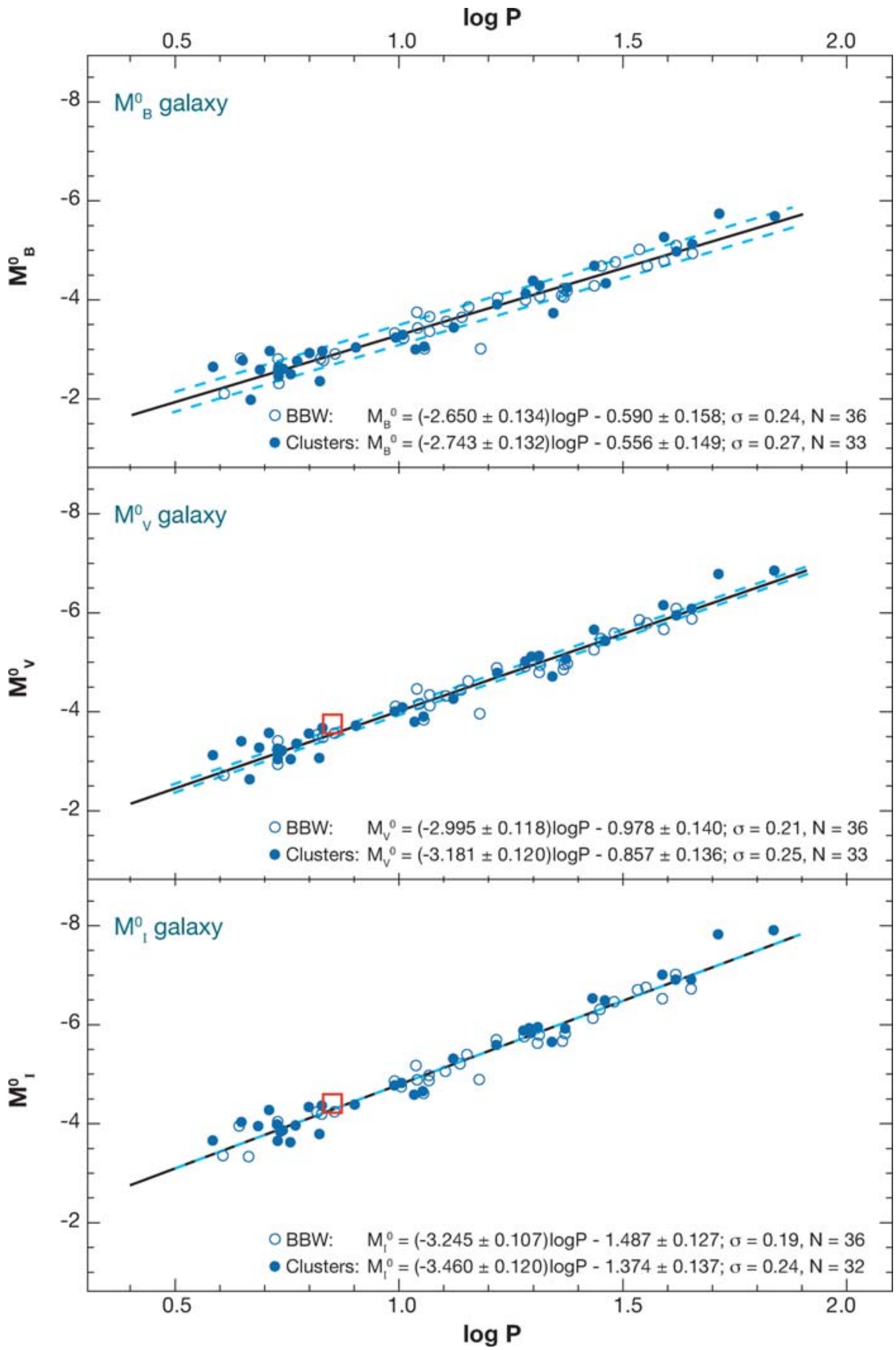
with rms = 0.25 mag per star,

$$M_V = -(3.087 \pm 0.085) \log P - (0.914 \pm 0.098), \qquad (4)$$

rms = 0.23 mag per star, and

$$M_I = -(3.348 \pm 0.083) \log P - (1.429 \pm 0.097), \qquad (5)$$

rms = 0.23 mag per star.


log P
0.5
1.0
1.5
2.0
M⁰B galaxy
M⁰V galaxy
M⁰I galaxy
M⁰B
M⁰V
M⁰I
-8
-6
-4
-2
○ BBW: M_B⁰ = (-2.650 ± 0.134)logP - 0.590 ± 0.158; σ = 0.24, N = 36
● Clusters: M_B⁰ = (-2.743 ± 0.132)logP - 0.556 ± 0.149; σ = 0.27, N = 33
○ BBW: M_V⁰ = (-2.995 ± 0.118)logP - 0.978 ± 0.140; σ = 0.21, N = 36
● Clusters: M_V⁰ = (-3.181 ± 0.120)logP - 0.857 ± 0.136; σ = 0.25, N = 33
○ BBW: M_I⁰ = (-3.245 ± 0.107)logP - 1.487 ± 0.127; σ = 0.19, N = 36
● Clusters: M_I⁰ = (-3.460 ± 0.120)logP - 1.374 ± 0.137; σ = 0.24, N = 32

The P-L data in B,V, and I for the LMC, transformed to absolute magnitude using an LMC modulus of $(m\text{-}M)^o = 18.54$ that is independent of Cepheid data (summarized in table 6 of Tammann, Sandage & Reindl 2003), are shown in **Figure 8**, taken from figure 4 of Sandage, Tammann & Reindl (2004). The data for $\log P < 1.5$ are from Udalski et al. (1999). Data for longer period Cepheids, not measured by Udalski et al. because of saturation of the CCD chips for such bright Cepheids, are from previous photoelectric measurements published in the literature, summarized in Tammann, Sandage & Reindl (2003).

Figure 8 shows the change of slope at $P = 10$ days for the LMC Cepheids. The lower right panel shows the difference between the P-L relations in the Galaxy and the LMC. The individual data points in that panel are for the Galaxian Cepheids. The lines are from the other three panels. The equations for the LMC P-L relations for periods smaller and larger than 10 days are shown within the borders of **Figure 8**.

The absolute V magnitudes of Galactic Cepheids according to Equation 4 and the LMC Cepheids from the equations in **Figure 8** differ significantly at the short and long period ends of the P-L relation. At $\log P = 0.5$ ($P = 3$ days) Cepheids in the Galaxy are 0.36 mag fainter than in the LMC in V. At $\log P = 1.7$ ($P = 50$ days), Galactic Cepheids are 0.16 mag brighter in V. The consequences of these differences in using Cepheid distances to determine the Hubble constant are profound.

The evidence in the lower right panel of **Figure 8** for the reality of the difference is strong if the Galactic Cepheid data are not plagued by systematic error. That both the main sequence fitting method and the BBW method used in **Figure 7** give the same slope and zero point to better than 1 sigma of the statistics gives support to the reality of a difference. Furthermore, there is no break in the Galactic data at 10 days, but it is clearly evident in the LMC data.

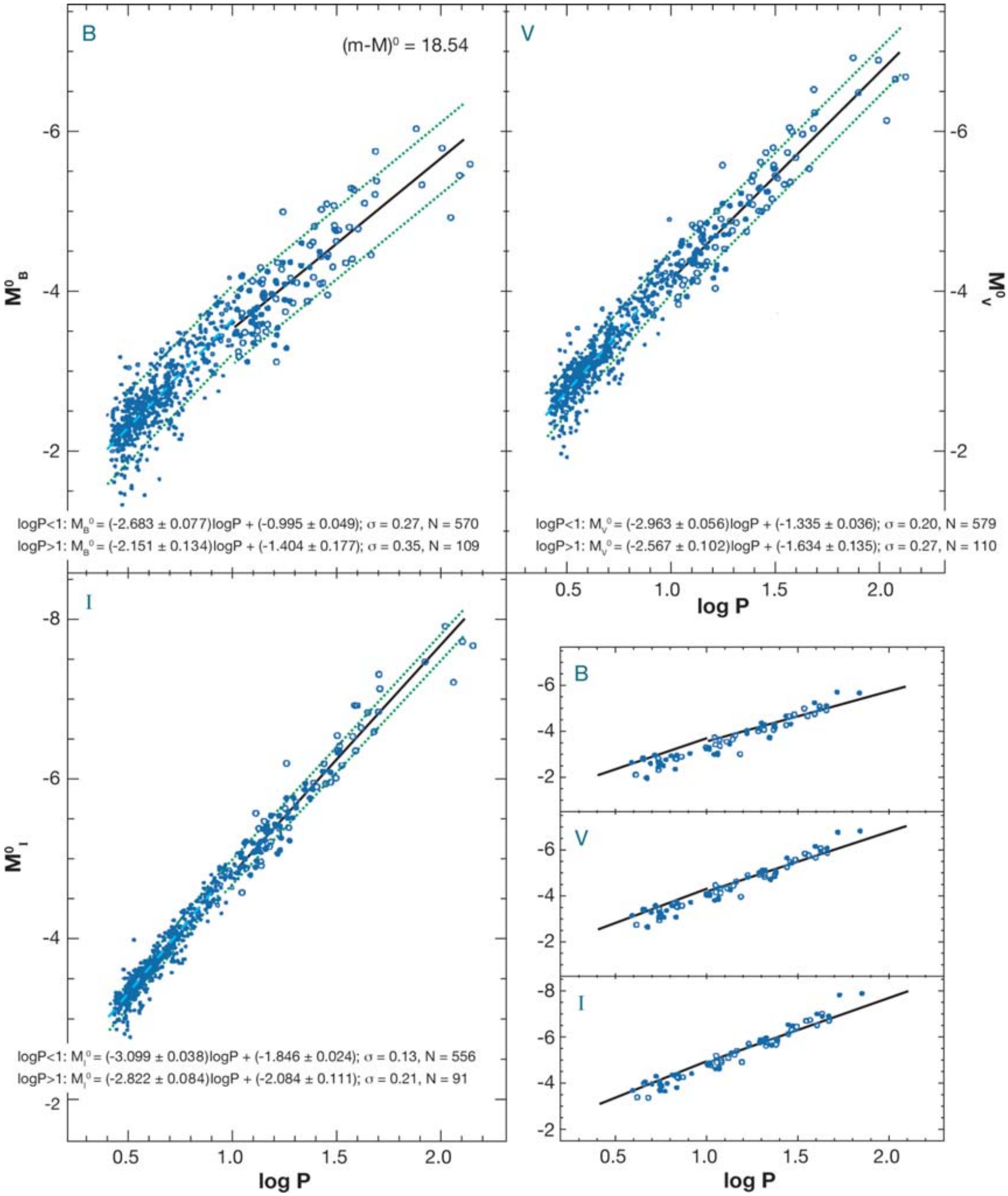
In an attempt to reconcile the Galactic and the LMC data, Gieren et al. suggested at the 2005 Rome conference on pulsating variables and in the literature (Gieren et al. 2005) that a break in his Galactic P-L relation at 10 days could be produced if the velocity projection factor, p, necessary in the BBW method, is not a constant but varies with period. Although that might be made to reconcile the Galaxy and the LMC P-L slopes, no such fix is possible in the main sequence fitting method, and the excellent agreement of the two methods for Galactic Cepheids militates against such a fix. A refutation of the Gieren et al. (2005) conclusion has been made by Ngeow et al. (2005).

Nevertheless, the value of the p projection factor is a central component in the BBW method. In a discussion on its determination and its uncertainties, Fernley

←

Figure 7

The Galactic P-L relations for 36 Cepheids with cluster distances (*blue dots*) and 33 with BBW moving atmosphere distances (*open blue circles*). The mean midpoint ridge lines for the combined data are expressed by Equations 3, 4, and 5 of the text. Separate equations for the main sequence fittings and the BBW distances are in the borders of the figure. The open red square is the *Hipparcos* calibration by Groenewegen & Oudmaijer (2000). The light blue dashed lines are the expected envelope lines due to the finite width of the instability strip. They are calculated assuming strip widths of 0.13 mag in B-V and 0.10 mag in V-I and the slopes of the lines of constant period derived by Sandage, Tammann & Reindl (2004).


B
(m-M)^0 = 18.54
V
M^0_B
M^0_V
-6
-4
-2
logP<1: M_B^0 = (-2.683 ± 0.077)logP + (-0.995 ± 0.049); σ = 0.27, N = 570
logP>1: M_B^0 = (-2.151 ± 0.134)logP + (-1.404 ± 0.177); σ = 0.35, N = 109
logP<1: M_V^0 = (-2.963 ± 0.056)logP + (-1.335 ± 0.036); σ = 0.20, N = 579
logP>1: M_V^0 = (-2.567 ± 0.102)logP + (-1.634 ± 0.135); σ = 0.27, N = 110
I
M^0_I
-8
logP<1: M_I^0 = (-3.099 ± 0.038)logP + (-1.846 ± 0.024); σ = 0.13, N = 556
logP>1: M_I^0 = (-2.822 ± 0.084)logP + (-2.084 ± 0.111); σ = 0.21, N = 91
0.5
1.0
1.5
2.0
log P

(1994) reviews the reasons that the factor is expected to be a function of (*a*) the Fraunhofer line strengths for the lines used to measure the pulsation velocities, (*b*) the velocity amplitude of the pulsation, and (*c*) the temperature of the star at each observation. A single factor for all temperatures and at all phases of the cycle is not theoretically correct. Hence, systematic errors at the level of 0.1 mag still seem possible in the BBW method due to mishandling of the p factor alone. The problem in another context has also been discussed by Gieren et al. (2005), but, again, the main objection for a p-factor revision comes from the excellent agreement of the slopes in B, V, and I of the BBW unrevised results with those from main sequence fittings. In addition, the differences still remain between the Galaxy and the LMC—the temperature difference in the instability strip with its different slope, and in the break in the P-L relation at 10 days.

However, problems in the main sequence fitting method are also present at the 0.1-mag level if the reddening corrections are known only to 0.02 mag, because the main sequence slope at the fitting point is about $dM_V/d(B\text{-}V) = 6$. However, random errors in the reddening can only increase the scatter in the P-L relation, not the slope. The method of removing a slight scale error in the previous Galactic reddenings by Tammann, Sandage & Reindl (2003) has been discussed above.

The error in the Pleiades main sequence placement in M_V may still be at the 0.04-mag level. Added to this uncertainty is the effect of variation of the metallicity of the individual Cepheids and of their parent clusters and associations. The adopted absolute magnitudes from the main sequence fitting method are based on making no correction for variations of the metallicities from that of the Pleiades. Feast & Walker (1987) discuss this potential problem, concluding that, for their list of clusters and associations containing Cepheids, "there is little direct evidence that the calibrations clusters in general have [Fe/H] significantly different from" that of the Pleiades to which the main sequence fit has been made. And "when zero points for each calibrating Cepheid were recalculated assuming a galactic metallicity gradient of delta $[Fe/H] = -0.1\ kpc^{-1}$, the change in the Cepheid luminosities was insignificant." Nevertheless, the change of main sequence position with metallicity is steep near the solar metallicity of $[Fe/H] = 0$. A summary (Sandage & Cacciari 1990, their figure 4) of the determinations of main sequence position in the HR diagram using models prior to 1990 gave the gradient of $dM_V/d[Fe/H] \sim 0.1$, showing that an error of 0.1 mag in M_V would be made by neglecting a metallicity change of 0.1 dex.

However, the Feast & Walker test for the absence of an effect of variations in [Fe/H] on the luminosities of the cluster main sequences is supported by other

Figure 8

The period-luminosity (P-L) relations in B, V, and I for Cepheids in the LMC fitted to two linear regressions broken at 10 days. The dashed envelope lines have the same meaning as in **Figure 7**. Comparisons with the calibration for Galactic Cepheids from Equations 3–5 are shown in the lower right panel. The lines in the insert are the regressions for the LMC from the equations in the borders of the B, V, and I main panels. The individual Galactic Cepheids are shown as the data points from **Figure 7**. The differences in the P-L slopes are evident. The diagram is from figure 4 of Sandage, Tammann & Reindl (2004).

evidence. Consider first a straw-man possibility of a set of circumstances suggested by J. Kormendy (private communication) that could artificially give an incorrect slope and zero point to the Cepheid P-L relation determined from main sequence fittings in the presence of a Galaxian [Fe/H] gradient with geocentric distance.

If long period Cepheids, being brighter than those of short period, have larger geocentric distances on average, then, because of the Galactic metallicity gradient, a systematic error could occur in the derived main sequence fitted absolute magnitudes that is progressive with period. This, in fact, is the test made by Feast & Walker. They recalculated the absolute magnitude of each calibrating Cepheid using a metallicity gradient of $d[Fe/H]/dR(gal) = -0.1\ kpc^{-1}$, and found no mean effect. This is because there is, in fact, no systematic difference in R(gal) with period for the Feast/Walker calibrators. But what about the zero point?

[Fe/H] values are available for 15 of the 25 calibrating Cepheids from Fry & Carney (1997), Andrievsky et al. (2002), and Luck (2003). There is overlap between the three lists, and, although [Fe/H] scatters considerably, the mean is $\langle[Fe/H]\rangle = -0.02 \pm 0.02$ (rms $= 0.09$) and there is no significant trend of [Fe/H] with period, which is the crucial point. Hence, the mean [Fe/H] of the Cepheid-bearing clusters is solar on average. If this is correct also for the Pleiades, which sets the zero point of the Cepheid scale, the main sequence fitting does not introduce a systematic metallicity related error in either the slope or the zero point of the Galaxian P-L relation.

Furthermore, the Galaxian gradient of $-0.1\ kpc^{-1}$ assumed by Feast & Walker in their test is a worst case value. Modern values are smaller at $-0.06\ kpc^{-1}$ (Luck et al. 2003; Kovtyukh, Wallerstein & Andrievsky 2005; Chen & Hou 2005).

There is still a further test for the absence of a main sequence fitting error. If such an error exists, and if there is an appreciable range of [Fe/H] for the calibrating Cepheids, the rms scatter of the P-L relation for cluster-fitted Cepheids (*blue dots* in **Figure 7**) should be larger than for the BBW calibrators (*open blue circles* in **Figure 7**) where no error due to [Fe/H] variations exists. However, the rms scatter about the P-L relation in Tammann, Sandage & Reindl (2003, their tables 3 and 4) is nearly identical for the main sequence fittings (rms $= 0.259$ mag) and the BBW data (rms $= 0.245$ mag) in the V band. Furthermore, these values are what is expected from the intrinsic spread due to the finite width of the instability strip.

Finally, the agreement in the P-L slopes and zero points between the blue dots and the open circles in **Figure 7**, based on two totally independent methods, is evidence in favor of an absence of systematic errors in each.

Potentially useful are the angular diameter measurements of Cepheids during their cycle, and their subsequent use to determine distances and therefore absolute magnitudes. A review of the results to 2004 by Kervella et al. (2004) gives absolute V and K magnitudes for seven long period Cepheids determined by the optical interferometric method. At this writing, the large statistical errors of the interferometer absolute magnitudes confirm the Equation 4 zero point, but only at the level of 0.3 mag. But great promise is believed to be in store for the method eventually.

Feast (2003) again discusses the problem of the Galactic calibration, including the *Hipparcos* trigonometric parallaxes discussed first by Feast & Catchpole (1997), statistical parallaxes, pulsation parallaxes, and water maser parallax for NGC 4258, which is also discussed by Saha et al. (2005). His conclusion is that the mean calibration for the Galactic Cepheids is

$$M_V = -2.81 \log P - 1.35 \pm 0.05. \quad (6)$$

This differs from Equation 4 both in slope and zero point, because Feast uses the slope of the LMC to set the slope of the Galactic Cepheids, as was the tradition before 2003. Equation 6 cannot be correct if the slopes are indeed different as indicated by **Figures 7** and **8**. However, at $\log P = 1.5$ ($P = 32$ days) where many of the Cepheids in the HST Cepheid programs lie, Equations 4 and 6 are the same within 0.02 mag. But the analysis by Feast of the calibration of the Galactic Cepheids should be redone using the slope of $dM_V/d\log P = -3.087$ as in Equation 4 rather than adopting the slope of the LMC Cepheids, and zero-pointing it by the Galactic data.

If the slopes of the LMC and Galactic Cepheids do indeed differ, the conclusion of Feast (2003) "that the calibration presented [using the LMC slope] is valid to about 0.1 mag (rms) at least for Cepheids near solar metallicity" is not correct. Equation 6 differs from Equation 4 by 0.3 mag at $\log P = 0.5$ and by 0.12 mag in the opposite direction at $\log P = 2.0$. Feast emphasizes, "To reduce the uncertainty substantially below ~0.1 mag will require extensive work on metallicity effects."

To this end, the problem should become clearer when the P-L relations in SMC, IC 1613, and other dwarf galaxies of low and intermediate metal abundance are analyzed in the same way as was done in Tammann, Sandage & Reindl (2003) and Sandage, Tammann & Reindl (2004) for the Galaxy and LMC. In addition, if there are metallicity gradients across the faces of giant-to-intermediate luminosity galaxies, as in M101, M33, and perhaps NGC 300, an analysis of the deviations from one another of separate P-L relations for separate spatial regions in these galaxies can be expected to clarify the role of metallicity in the P-L differences.

Early attempts to make this test are by Freedman & Madore (1990) in M31, and by Kennicutt et al. (1998) in M101. The difficulty of such a test is to separate the effects of metallicity on the P-L scatter from the severe differential absorption that will dominate the scatter. To date, the efforts to extract the metallicity effect from the observed P-L data have not been convincing at the necessary level of accuracy. However, the attempt to be made by Pietrzynski et al. (2002) in NGC 300, a galaxy with small internal absorption, is anticipated.

It can also be expected that the DIRECT program to use eclipsing binaries and Cepheids to refine the distances of M31 and M33 may be capable of making a more definitive test. A description of the program is by Macri (2004) with references to the first nine papers of the series.

4. USE OF CEPHEIDS FOR THE EXTRAGALACTIC DISTANCE SCALE

The differences in the slopes and zero points of the P-L relations in the Galaxy and LMC greatly complicate the use of Cepheids to determine Cepheid distances to galaxies using HST, and now also complicate the use of the large ground-based telescopes using adaptive optics (e.g., Thim et al. 2003 for NGC 5236 = M83). Until the reason is understood, effective correction methods to overcome the differences cannot be established in any definitive manner. Yet, if Cepheids are to be used to gauge cosmic distances in this new era of uncertainty of what P-L relation to use, we must attempt to understand, even at the 0.3-mag level in distance moduli.

At this point, the most reasonable hypothesis is that the difference in slope in the P-L relations between the Galaxy and LMC is related to metallicity differences, because the difference in metallicity between the Galaxy and LMC is well established to be about delta [Fe/H] $= 0.4$ dex. It is also known that the instability strip in the LMC is hotter on average than that for the Galaxy (see **Figure 6**, from figure 20 of Sandage, Tammann & Reindl 2004) and that the slopes of the boundary lines of the strips differ (**Figure 8**, *lower right panel*), requiring, from the pulsation equation, that the P-L slopes must also differ.

That these differences are due to differences in the chemical compositions is supported by theoretical models of the positions of the fundamental blue edges of the instability strip as functions of the abundances of hydrogen, helium, and the metals. These variations were first calculated by Iben & Tuggle (1975), and later by many others, among whom are Chiosi et al. (1992) and Saio & Gautschy (1998). A review by Sandage, Bell & Tripicco (1999) discussed the effect of variations in the Y and Z chemical abundance variations on the position of the fundamental blue edge. Indeed, changes in the temperature of the edge are predicted by the theoretical models for the edge position for reasonable changes of Y and Z. Hence, the principle is established that changes in chemical composition can change the position of the instability strip, and hence the slope and zero point of the P-L relation, but we must proceed empirically to avoid making decisions between the various conflicting models.

For any application of the P-L relations to Cepheid data, we must decide which P-L relation to use—that for the Galaxy or the LMC, or some interpolation between them for intermediate metallicities, or extrapolations for metallicities weaker than for the LMC, on the assumption that metallicity change is the cause of the differences.

Or, alternatively, one can proceed as done in Gibson et al. (2000) and Freedman et al. (2001) by adopting a single fiducial P-L relation (that, as usual before 2003, uses the LMC slope and Galactic calibrators) and then by correcting for metallicity variations by a Kennicutt et al. (1998)-like correction that is linear in delta [Fe/H], or by a Sakai et al. (2004)-like correction in delta [O/H]. The 2002–2005 literature contains both approaches.

The resulting distance scale by Freedman et al. (2001) differs from distances determined by interpolating between the Galaxy and LMC according to the measured mean metallicity of the parent galaxy. The details of this interpolation method have been developed in detail by Saha et al. (2005) and applied there to the same galaxies used by Gibson that were included in the original program galaxies in the HST SNe Ia calibration campaign led in a consortium by Tammann (Saha et al. 2005, Sandage et al. 2006).

Comparison of the two Cepheid distance scales differ from each other at the 0.2-mag level. The distance moduli of the eight supernovae calibrating galaxies measured by the Tammann consortium and 26 other galaxies in common with Gibson et al. (2000) average 0.20 mag more distant. The detailed comparison set out in Sandage et al. (2006) shows that part of this is due to the different treatments of the P-L relations, and part due to the different treatment of the correction of the Cepheid data for internal absorption in the parent galaxies, showing the extreme difficulties in photometry, even with HST, at V = 25 (Saha et al. 2005 for details). Because this review is not concerned with the extragalactic distance scale per se, but rather only with the Cepheid P-L relation, readers interested in the detailed reasons for the difference between the Freedman et al. (2001) and the Sandage et al. (2006) distance scales are referred to the literature references just cited.

Clearly, much work lies ahead. We are only at the beginning of a new era in distance determinations using Cepheids. It can be expected that much will be discovered and illuminated in the years to come.

5. ABSOLUTE MAGNITUDE CALIBRATIONS OF RR LYRAE STARS IN GLOBULAR CLUSTERS AND IN THE FIELD

5.1. Linear Correlations of M_V(RR) with Metallicity

When it was discovered that the division of globular clusters into two groups with mean periods of the RRL stars differing at ⟨P⟩ = 0.55 and ⟨P⟩ = 0.65 days (Grosse 1932; Hachenberg 1939; Oosterhoff 1939, 1944) was also a division by metal abundance (Arp 1955, verified by Kinmann 1959), it became obvious that a difference in mean luminosity between the two groups of about 0.2 mag could explain the observations. Temperature differences between the two Oosterhoff period groups were not sufficient (Sandage 1958). It was further found that the Oosterhoff period difference with metallicity was also present in the field RRL stars and that the correlation of mean period with metallicity was a continuum rather than a division into two discrete period groups (Preston 1959). It was further found that the period shifts in globular clusters existed star by star when the cluster variables were compared between clusters of different metallicity and where the RRL parameters were read at the same amplitude, or the same temperature, or the same rise time (Sandage 1981b, 1982). Hence, the Oosterhoff effect is not caused by differences in ensemble averages over different distributions of periods in different clusters, which was an early usual way of discussing the Oosterhoff effect, but is a star-by-star effect. Further evidence of the star-by-star explanation is in two general summaries (Sandage 1990a,b, 1993a,b).

It became customary to assume a linear relation between absolute magnitude and metallicity of the form M = a + b[Fe/H]. The calibration problem then reduced to finding a and b by whatever calibration method was used. The three most popular have been (*a*) the traditional way through statistical parallaxes, (*b*) the BBW moving atmosphere method, and (*c*) main sequence fitting.

The abundant literature on the first two methods had been reviewed by Smith (1995) in his textbook on RRL stars, and by Cacciari & Clementini (2003) using the extensive later literature. The results are listed in their table 6.2.

They give $\langle M_V(RR)\rangle = 0.78 \pm 0.12$ at [Fe/H] = −1.5 from the statistical parallax method, based primarily on the modern analysis of new radial velocities and proper motions by Layden et al. (1996) and by Gould & Popowski (1998). These statistical parallax values are near the faint end of the new calibrations. They are representative of the short distance scale for the RRLs, being about 0.25 mag fainter than the calibrations defining the long RRL distance scale, among which are as follows.

The Cacciari/Clementini (2003) listing for the BBW moving atmospheres method is $M_V = 0.55 \pm 0.12$ at $[Fe/H] = -1.5$, based on a high weight determination of RR Cet by Clementini et al. (1995) and Fernley et al. (1998a). This differs from the summary review by Fernley et al. (1998b) who derive the calibration of $M_V = (0.98 \pm 0.05) + (0.20 \pm 0.04)[Fe/H]$ from many other BBW determinations, giving $\langle M_V \rangle = 0.68$ at $[Fe/H] = -1.5$, midway to the short distance scale. The difference between these two values using the BBW method (0.56 mag versus 0.68 mag) illustrates the level of systematic differences in different applications of the BBW method for RRLs, which is at about the 0.15-mag level, although in better agreement are many calibrations favoring the long distance scale to be set out below. But consider first the value of $\underline{b}$.

A large literature exists on the measurement of $\underline{b}$. Among the largest values of $\underline{b}$ is from a linear pulsation calibration by Sandage (1993b) who used an empirical correlation of log P with [Fe/H] and adopted variations of mass and T_e with [Fe/H] to give $M_V = 0.94 + 0.30[Fe/H]$. This gives $M_V = 0.49$ at $M_V = -1.5$. This is among the brightest of the calibrations but is supported by McNamara (1997a,b) who derived $M_V(RR) = 0.96 + 0.29[Fe/H]$ from his application of the BBW method, giving $M_V = 0.53$ at $[Fe/H] = -1.5$. McNamara (1999) also derived $M_V = 1.00 + 0.31[Fe/H]$ using RRLs of different metallicities in the Galactic bulge from data by Alcock et al. (1998) on the period-amplitude-metallicity relation, giving $M_V = 0.54$ at $[Fe/H] = -1.5$. A high value of $\underline{b}$ was also obtained by Feast (1997) who gave $M_V = 1.13 + 0.37[Fe/H]$, or $M_V = 0.58$ at $M_V = -1.5$. In a rediscussion of Fernley's (1993, 1994) BBW results, McNamara (1999) derived $1.06 + 0.32[Fe/H]$, also giving $M_V = 0.58$ at $[Fe/H] = -1.5$.

Smaller values of $\underline{b}$ are more common. Fernley (1993) derived $\underline{b} = 0.19$ using $(V-R)^o$ colors. He later obtained $\underline{b} = 0.21$ from his assessment (Fernley 1994), mentioned earlier, of the velocity projection factor, $\underline{p}$, needed in the BBW method. Fernley et al. (1998a) later derived $\underline{b} = 0.18$ from new BBW data.

Carretta et al. (2000) derived $M_V(RR) = 0.74 + 0.18[Fe/H]$, from main sequence fitting (see below), giving $M_V(RR) = 0.47$ at $[Fe/H] = -1.5$.

From a study of eight clusters in M31 that have RRL photometric data, Fusi Pecci et al. (1996) obtained the shallow slope of $\underline{b} = 0.13$. They later revised the value to 0.22 (Rich et al. 2001) from a larger sample of M31 clusters.

The most secure determination to date is by Clementini et al. (2003) from their LMC sample of RRL where they derive $\underline{b} = 0.214 \pm 0.047$, and a zero point of $M_V = 0.52$ at $[Fe/H] = -1.5$. Their linear calibration is $M_V = 0.84 + 0.214[Fe/H]$, zero pointed using an LMC modulus of $(m\text{-}M)^o = 18.54$, which can be made independent of any long period Cepheid data (Tammann, Sandage & Reindl 2003, their table 6).

Jones et al. (1992) and Carney et al. (1992) derived $\underline{b} = 0.16$ in reviews of the BBW results to 1992.

COMPARISON OF CEPHEIDS AND RR LYRAE VARIABLES IN GALAXIES WHERE BOTH APPEAR TOGETHER

It has often been hoped that the Cepheid and RR Lyrae (RRL) calibrations can be tested relative to one another by comparing the apparent magnitude levels of each in galaxies that contain them both. This seemed straightforward and powerful until the Cepheid P-L relation was shown to be variable from galaxy to galaxy by the arguments set out in Section 2. Nevertheless, it can still be expected that comparisons between the RRL and Cepheids in a given galaxy will yet prove to be useful, perhaps by turning the problem around to determine the Cepheid P-L relation using RRL, if the second parameter problem of the RRL can be solved (see below). Recent use of the method of joint comparisons, made before the results of Section 2 or the blatant second parameter problem of the RRL from NGC 6388 and NGC 6441 was fully appreciated, has been made by Fusi-Pecci et al. (1996) for M31; van den Bergh (1995) and Sandage, Bell & Tripicco (1999) for M31, the LMC, the SMC, and IC 1613; Smith et al. (1992) and Walker & Mack (1988) for SMC; and Dolphin et al. (2001) for IC 1613.

Comprehensive reviews of many of the extant linear calibrations are by Gratton et al. (1997), Gratton (1998), and Carretta et al. (2000).

Many of the calibrations cited earlier for RRL are consistent with $\langle M_V \rangle = 0.82 + 0.20[Fe/H]$, giving $M_V = 0.52$ at $[Fe/H] = -1.5$, which fits most of the cited calibrations to within about 0.05 mag.

However, the evidence is strong that the luminosity-metallicity relation is not linear, but rather that $\underline{b}$ is a function of [Fe/H] in the sense of being larger at the metal-rich end than at the metal-poor end of the distribution. The evidence is from (*a*) theoretical models of the HB (both at zero age and in an evolved state), (*b*) the pulsation equation using the observed input parameters of mass-[Fe/H] and log T_e (from colors) as functions of metallicity relations, (*c*) and from semiempirical use of observational data.

5.2. Nonlinear Calibrations of M_V(RR)/([Fe/H])

Most theoretical HB models made after ~1990 predict a nonlinear relation for M_V with [Fe/H] for the ZAHB. Examples are the models of Lee, Demarque & Zinn (1990); Castellani, Chieffi & Pulone (1991); Bencivenni et al. (1991); Dorman (1992); Caputo et al. (1993); Caloi, D'Antona & Mazzitelli (1997); Salaris, Degl'Innocenti & Weiss (1997); Cassisi et al. (1999); Ferraro et al. (1999); Demarque et al. (2000); VandenBerg et al. (2000, their figures 2, 3, and 20); and Catelan, Pritzl & Smith (2004). A graphical summary showing several of these calibrations is given by Cacciari & Clementini (2003). To illustrate the theme of all these models we show the prediction of the VandenBerg et al. (2000) models (their figure 3) in **Figure 9**, and a summary

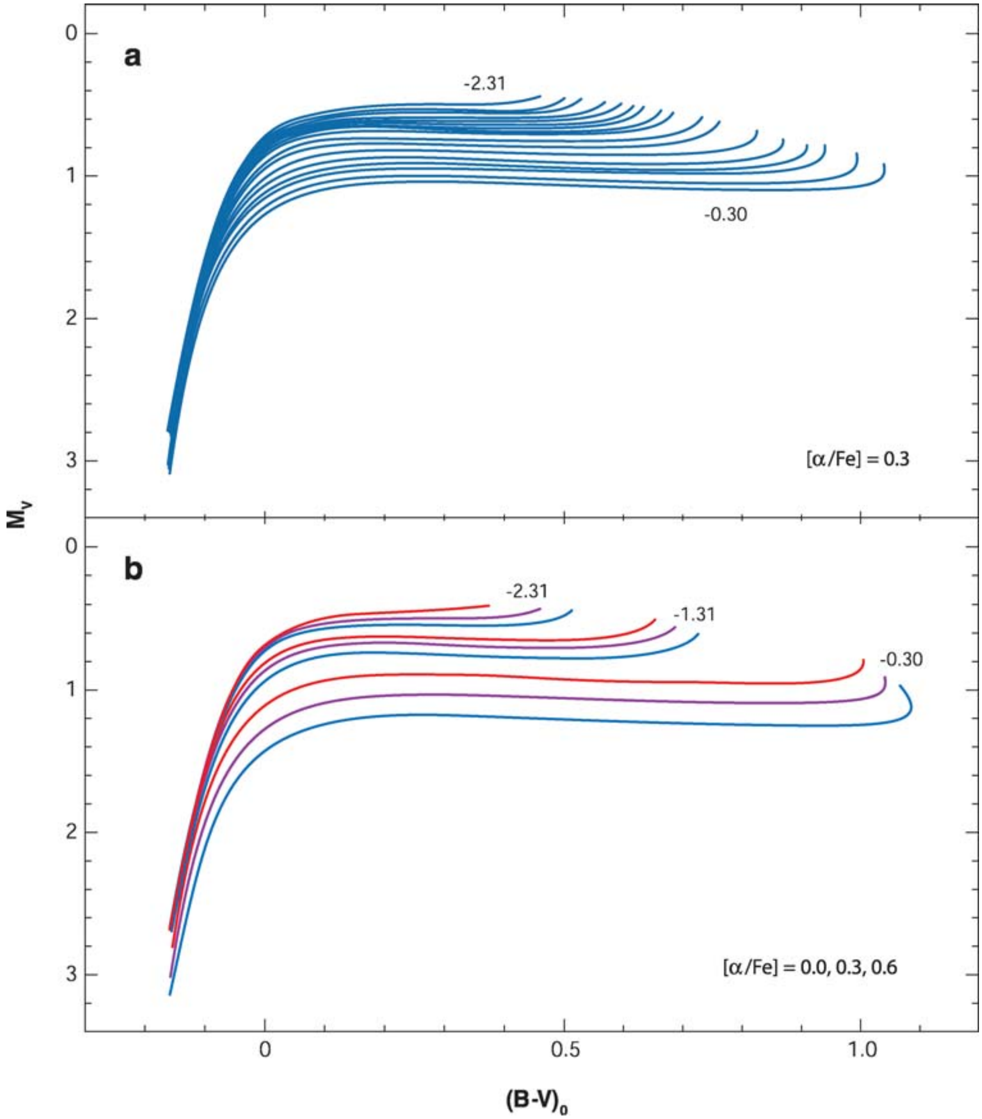


Figure 9

(*a*) Predicted level of the zero age horizontal branch (ZAHB) by VandenBerg et al. (2000) for 17 different metallicities in steps of 0.15 in [Fe/H] for an alpha element enhancement of 0.3 dex over solar. (*b*) The sensitivity of the horizontal branch level to variations of the alpha element enhancements for three values of [alpha/Fe]. The higher the alpha element enhancement, the fainter is the ZAHB. Diagram is from figure 3 of VandenBerg et al. (2000).

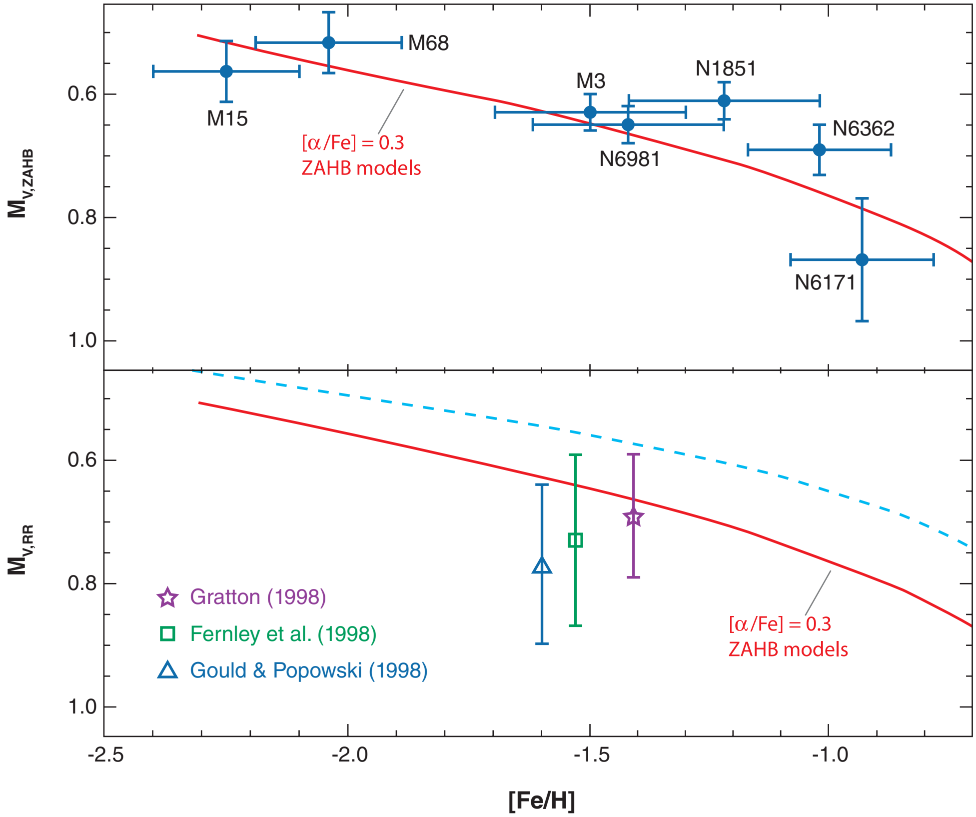


Figure 10

Top panel is the zero age horizontal branch (ZAHB) level in M_V from the analysis of DeSantis & Cassisi (1999) of individual data in selected globular clusters, based on the pulsation equation and their adopted input parameters of mass and temperature. The curve is the predicted calibration by VandenBerg et al. (2000) for an alpha element enhancement of [alpha/Fe] = 0.3 over solar. The bottom panel shows this prediction for the ZAHB as the solid red line. The effect of evolution, producing the mean position of the horizontal branch (HB) due to the vertical structure of the HB (Sandage 1990a), is the dashed light blue line. Three of the empirical calibrations cited in the text are shown in the bottom panel as the open symbols. They are fainter by ~0.2 mag than the final calibration given later here (Equation 8). The diagram is from figure 20 of VandenBerg et al. (2000).

given by VandenBerg et al. (their figure 20) of a selection of the observational evidence shown here as **Figure 10**.

The theoretical models of VandenBerg et al. (2000) in M_V, B-V in the top panel of **Figure 9** show the sensitivity of the position of the unevolved HB for 17 values of [Fe/H] ranging from −0.3 to −2.31, computed for an alpha element enhancement of a factor of 2 over solar (i.e., [alpha/Fe] = 0.3). The variation of the level of the absolute magnitudes with [Fe/H] is evident in **Figure 9** for the ZABH (i.e., without evolution).

The lower panel shows the effect on the luminosity level for alpha element enhancements of 0.0, 0.3, and 0.6 in the log (i.e., for factors of 1, 2, and 4 in the alpha/Fe numerical ratios) for each of the three values of [Fe/H]. The lowest luminosity level for each [Fe/H] family is for the highest (0.6) alpha element enhancement.

Figure 10 (*top*) compares the predictions from **Figure 9** for the ZAHB using [alpha/Fe] = 0.3 with the absolute magnitudes derived from a number of globular clusters by DeSantis & Cassissi (1999) from semitheoretical considerations from the pulsation equation. The lower panel repeats the solid line from the top panel and also shows three observational calibrations using different empirical methods. The cross from Gratton (1998) is from using *Hipparcos* trigonometric parallaxes giving $M_V = 0.69 \pm 0.1$ at $\langle$[Fe/H]$\rangle = -1.41$. The open square by Fernley et al. (1998a) at $M_V = 0.73 \pm 0.14$ at $\langle$Fe/H$\rangle = -1.53$ is from the BBW method averaged from many pre-1998 investigations. The triangle by Gould & Popowski (1998) at $M_V = 0.77 \pm 0.13$ at $\langle$[Fe/H]$\rangle = -1.68$ is from statistical parallaxes.

Because these determinations are from the observational data, they refer to the mean luminosity after evolution from the ZAHB. The effect of evolution is shown in the bottom panel by the dashed line (Sandage 1990a), which is 0.09 mag brighter than the level of the ZAHB.

Clearly, the three observational determinations in the bottom panel are ~0.2 mag fainter than the level of the dashed line. Either the VandenBerg models are too bright, or these three points have systematic errors of ~0.2 mag too faint. We show later in this section (Equation 8) that we favor the second possibility and therefore that the VandenBerg et al. (2000) models would then be close to reality.

Other observational and/or semitheoretical calibrations also show the nonlinearity, among which are the studies by Caputo (1997); Gratton et al. (1997); DeSantis & Cassisi (1999, their figure 15); Caputo et al. (2000); McNamara et al. (2004); and Sandage (1993b, 2006).

Caputo et al. (2000a) combine a pulsation equation that relates period, luminosity, temperature, and mass with observational data on the periods of RRLs at the blue edge of the instability strip for first overtone variables and the red edge of the fundamental mode in a number of clusters. They determined the distance modulus of each cluster by comparing the observed period-apparent magnitude data with new HB models used by Caputo et al., from which the absolute magnitude of the cluster variables can be determined. **Figure 11** shows the result taken from figure 2 of Caputo et al. (2000a). Five earlier calibrations from the literature are drawn. The upper panel shows the Caputo et al. analysis of their data for an [alpha/Fe] overabundance ratio of $f = 3$. The lower panel is for the solar [alpha/Fe] value.

The calibration is nonlinear, shown as **Figure 12** here as given by Caputo et al. 2000a. Two linear relations are fitted to the data giving $M_V(RR) = 0.71 + 0.17$[Fe/H] $+ 0.03f$ for [Fe/H] < -1.5, and $M_V(RR) = 0.92 + 0.27$[Fe/H] $+ 0.03f$ for [Fe/H] > -1.5, where again $\underline{f}$ is the overabundance ratio ($\log f =$ [alpha/Fe]) of the alpha elements relative to the solar abundance as defined by Salaris, Chieffi & Straniero (1993) and used by Catelan, Pritzl & Smith (2004). These give, respectively, $M_V = 0.55$ and 0.61 at [Fe/H] $= -1.5$ for $\underline{f} = 3$ (i.e., [alpha/Fe] $= 0.3$), but the parabolic

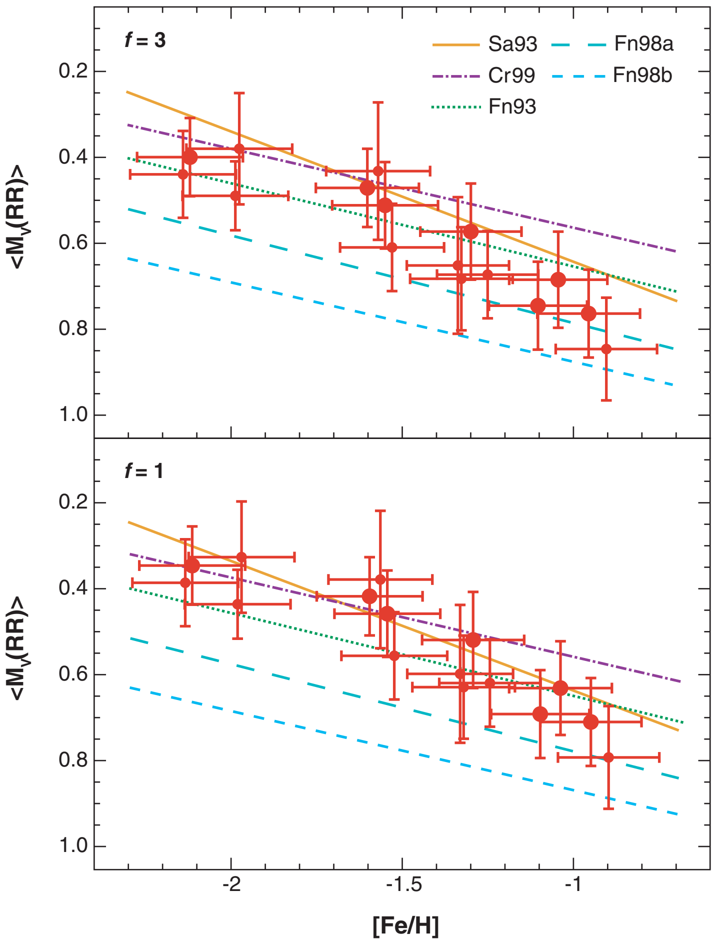


Figure 11

The calibration of M_V(RR) in a number of clusters of different metallicities by Caputo et al. (2000a) using the edges of the theoretical instability strip for the first overtone and the fundamental mode for two different assumptions for the overabundance of the alpha elements as parameterized by *f* as defined by Salaris, Chieffi & Straniero (1993) and by Catelan, Pritzl & Smith (2004). The linear calibrations from five representative results are shown for comparison. Sa93 is Sandage (1993b); Fn93, Fn98a, and Fn98b are Fernley (1993, 1998a,b); Cr99 is Caretta et al. (2000).

fit of Equation 7 is a better fit, not shown, as

$$M_V(\mathrm{RR}) = 1.576 + 1.068([\mathrm{Fe/H}]) + 0.242([\mathrm{Fe/H}])^2, \qquad (7)$$

valid for [Fe/H] between −1 and −2.2. The slope, b = 1.068 + 0.482([Fe/H]), varies between 0.59 for [Fe/H] = −1 and 0.10 for [Fe/H] = −2, nicely encompassing all values of the various linear calibrations cited in an earlier section. Equation 7 gives M_V(RR) = 0.52 at [Fe/H] = −1.5.

We must mention that the nonlinearity of the M_V([Fe/H]) correlations predicted by the theoretical models cited earlier refers to the ZAHB, i.e., not what is expected to be observed if evolution away from the ZAHB is severe as is suggested, for example, by Lee, Demarque & Zinn (1990). Hence, the nonlinearity of the Caputo et al. calibration of Equation 7 is important because it is based on the observations at a mean evolutionary state of the HB.

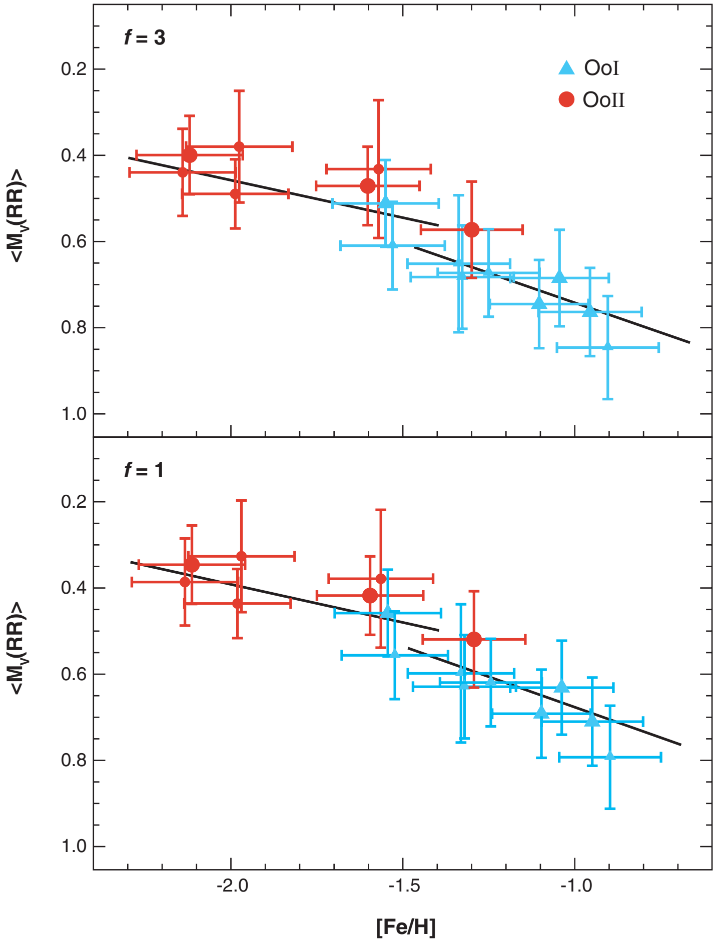


Figure 12

Same as **Figure 11** but with the two linear equations listed in the text. A better fit is the parabola of Equation 7 in the text. Diagram is from figure 3 of Caputo et al. (2000a). The parameter *f* is the measure of the alpha element overabundance (*f* = antilog [alpha/Fe]) compared to solar. The two Oosterhoff period groups are separated by symbols. The two-line fits are those given by Caputo et al. (2000a).

A different approach is by Sandage (2006) using the pulsation equation, upgrading the method used in Sandage (1993b) with improved correlations of the continuum period/metallicity relation, and an improved temperature/metallicity relation at the fundamental blue edge of the instability strip. His nonlinear calibration, zero-pointed from the Clementini et al. (2003) data for RRL for the mean level of the HB in the LMC, is

$$M_V(RR) = 1.109 + 0.600([Fe/H]) + 0.140([Fe/H])^2, \quad (8)$$

valid for [Fe/H] between 0 and −2.0. Equation 8 gives $M_V(RR) = 0.52$ at [Fe/H] = −1.5. The slope, $\underline{b} = 0.60 + 0.28([Fe/H])$, varies from $\underline{b} = 0.6$ to 0.04 between [Fe/H] = 0 and −2.

The nonlinear calibration of McNamara et al. (2004), based on main sequence fittings and on the route through the field SX Phe variables (see below), consists of two linear calibrations below and above [Fe/H] = −1.5 as $M_V(RR) = 0.50$ independent of [Fe/H] for metallicities more metal poor than −1.5, and $M_V(RR) = 1.13 + 0.42([Fe/H])$ for [Fe/H] between −0.5 and −1.5. The calibration is for the mean evolved level of the HB. The assumption by McNamara et al. for a flat M_V-[Fe/H] relation for [Fe/H] more metal poor than −1.5 is consistent with the Caputo et al. data points in **Figure 12**.

Two other nonlinear calibrations from advanced theory of the ZAHB must be mentioned. Catelan, Pritzl & Smith (2004), using oxygen-enhanced opacities, derive

$$M_V = 1.179 + 0.548([Fe/H]) + 0.108([Fe/H])^2, \tag{9}$$

giving $M_V = 0.60$ at $[Fe/H] = -1.5$ for the ZAHB. This must be made 0.09 mag brighter to account for evolution (Sandage 1993b), giving $M_V = 0.51$ for the average RRL state on the HB. The slope of $\underline{b} = 0.548 + 0.216([Fe/H])$ varies between $\underline{b} = 0.44$ and 0.12 for [Fe/H] between −0.5 and −2.0.

VandenBerg et al. (2000) produced ZAHB models with enhanced oxygen abundance. Their nonlinear ZAHB models average 0.05 mag fainter than those of Catelan et al. (2004), but have nearly the same nonlinear behavior as Equation 9. Their predicted $M_V(RR)$ at $[Fe/H] = -1.5$ for the unevolved ZAHB is 0.65, which when corrected for the average evolution of the HB gives $\langle M_V \rangle = 0.56$ at $[Fe/H] = -1.5$ for the average RRL state.

All of the calibrations discussed here are ∼0.15 mag brighter than the statistical parallaxes of Layden et al. (1996) and Gould & Popowski (1998) cited earlier, suggesting systematic errors in one or the other of the methods. The main sequence fitting method and the route through SX Phe variables favor the long distance scale, whereas the BBW moving atmospheres methods in the hands of the Carney and the Fernley groups sometimes favor the short distance scale. Reviews of the status up to 2000 are by Gratton (1998), Popowski & Gould (1999), and VandenBerg et al. (2000).

5.3. Main Sequence Fitting

In principal, the cleanest method of HB calibration might seem to be the main sequence fitting method. However, the observational realization that the position of the main sequence is a strong function of metallicity (Sandage & Eggen 1959, Eggen & Sandage 1962), within nearly the same time frame as the theoretical predictions by Stromgren (1952), Reiz (1954), Demarque (1960), and others showing the same thing, greatly complicated the method (Sandage 1970; Sandage & Cacciari 1990).

The method became competitive when the handful of trigonometrically calibrated subdwarf distances available in the 1970s was increased to a useful number with marginally adequate accuracy by the *Hipparcos* parallax satellite. Based on this fundamental advance, a large literature has arisen to solve the intricate problem of correcting the fitting problem to a variable main sequence position due to variations of the chemical composition.

Modern applications of the method are by Reid (1997, 1999), Carretta et al. (2000), Gratton et al. (1997), and VandenBerg et al. (2000), among others, using the *Hipparcos* subdwarf data in concert with the theoretical models of main sequence position together with various applications of the Lutz-Kelker bias (Lutz & Kelker 1973; Hanson 1979; Lutz 1983; Smith 1987).

Much literature exists for the position of the main sequence as a function of metallicity from calculated models. Entrance to this literature can be gained from VandenBerg (1992), Gratton et al. (1997), D'Antona, Caloi & Mazzitelli (1997), Carretta et al. (2000), Straniero, Chieffi, & Limongi (1997) updating Straniero & Chieffi (1991), and VandenBerg et al. (2000) as representative. Extensive references are given to other literature in these citations.

Most of the main sequence fitting calibrations, some of which are discussed above, have as representatives $M_V(\mathrm{ZAHB}) = 0.82 + 0.22([\mathrm{Fe/H}])$ from Gratton et al. (1997), $M_V(\mathrm{ZAHB}) = 0.80 + 0.18\ ([\mathrm{Fe/H}])$ from Carretta et al. (2000), and $M_V(\mathrm{evolved}) = 1.00 + .31([\mathrm{Fe/H}])$ by McNamara mentioned earlier. All are consistent with $M_V(\mathrm{evolved}) = 0.52$ at $[\mathrm{Fe/H}] = -1.5$ to better than 0.05 mag. A summary to 1998 is by Gratton (1998).

A problem with main sequence fitting using *Hipparcos* trigonometric parallaxes of subdwarfs of different metallicities is the application of the Lutz-Kelker bias to the subdwarf ensemble calibration, depending at the 0.1-mag level in a complicated way on the relative parallax error (delta pi/pi). The correction is controversial (see the next section) and can only be avoided at the 0.01-mag level when trigonometric parallaxes of subdwarfs can be known at about five times the accuracy of the *Hipparcos* data (see Sandage & Saha 2002 for a history of the bias and a modern simulation of it).

5.4. The Route Through the SX Phoenicis Stars

The initial discovery of the large class of small amplitude ($A_V < 0.3$ mag), very short period pulsators (periods shorter than 0.25 days to as small as 80 minutes) in the field, that have near main sequence luminosities, was made by Harlan Smith (1955; also unpublished dissertation) and Eggen (1956a,b). Smith called these stars dwarf Cepheids; others called them AI Vel stars (Bessell 1969 in his catalog), or RRL stars (Kukarkin et al. 1969). Eggen (1970, 1979 in his catalog) proposed the term USPC for ultra short period Cepheids, a designation gaining favor in recent years. The class as a whole is known as Delta Scuti stars. A review of what was known by 1967 about these stars that occur in both populations was made by Danziger & Dickens (1967).

A most promising method of RRL calibration in globular clusters is by using the SX Phoenicis USPC, which are the population II subclass of Delta Scuti stars. They are used as templates to step from their well-determined, near main sequence luminosities (from trigonometric parallaxes), to the RRL HB variables that are three magnitudes brighter. The method became possible with the discovery of USPC in globular clusters beginning in the 1990s (Nemec & Mateo 1990a,b; McNamara 1997b for reviews).

Most of the Delta Scuti class are population I, high metal abundance, low space motion stars. However, a small subgroup have high velocity and low metal abundance. They are the population II subtypes, whose prototype examples are SX Phoenicis, CY Aqr, and DY Peg (McNamara & Feltz 1978; Balgin et al. 1973; Breger 1979, 1980, for reviews). This population II subgroup is now universally named SX Phe stars.

An early listing of the known SX Phe stars in globular clusters is by Nemec (1989) and Nemec & Mateo (1990b, their table 2). The position of the dwarf Cepheid instability strip is discussed by McNamara & Powell (1990). A listing complete to 1997 is McNamara (1997b), which also gives a definitive calibration, also of RRL stars.

McNamara (1997b) uses the apparent magnitude difference between the HB and RRL variables in the clusters that contain SX Phe stars. The HB absolute magnitudes are zero-pointed by five SX Phe field stars that have adequate *Hipparcos* trigonometric parallaxes. From these parallaxes he obtained a calibration of the cluster SX Phe stars of

$$M_V(\text{SX Phe}) = -3.725 \log P - 1.933, \tag{10}$$

with a statistical error of 0.05 mag. He later revised that calibration (McNamara 2000) on the basis of a suggestion by Peterson (1999) that a Lutz-Kelker type correction should be applied to the five *Hipparcos* calibrators to give a zero point to Equation 10 of −1.969. However, as mentioned earlier, Lutz-Kelker corrections are controversial, depending crucially on the statistical nature of the sample (Sandage & Saha 2002), which is generally unknown.

Using Equation 10 for each of the USPC in each of the globular clusters in which they occur, the distance modulus of the clusters is determined with high statistical accuracy, from which the absolute magnitude of its RRL variables is determined. In that way McNamara (1997b) derives

$$M_V(\text{evolved HB}) = 0.29[\text{Fe/H}] + 0.90, \tag{11}$$

which gives $M_V(\text{RR}) = 0.52$ at $[\text{Fe/H}] = -1.5$. Equation 11 can be compared with the BBW moving envelope method used by McNamara (1997b), which gave him

$$M_V(\text{BBW, RRL}) = 0.31[\text{Fe/H}] + 0.96 \tag{12}$$

with one method of weighting, and identical with Equation 11 with another. Equation 12 gives $M_V(\text{RR}) = 0.50$ at $[\text{Fe/H}] = -1.5$. Both Equations 11 and 12 refer to the position of the mean evolved HB rather than to the ZAHB.

5.5. Exceptions to the Calibration: The Second Parameter Clusters NGC 6388 and NGC 6441

The results of the preceding four subsections are consistent in the $M_V(\text{RR})$ calibration of the correlation with metallicity that almost certainly is nonlinear, with a stable zero point at $M_V(\text{RR}) = 0.52$ for $[\text{Fe/H}] = -1.5$. However, there are blatant exceptions that warrant caution in assuming that the equations set out in the previous sections are unique and universal.

The exceptions were discovered in the globular clusters NGC 6388 and NGC 6441 (Rich et al. 1997; Pritzl et al. 2000, 2001), which have high metal abundances ($[\text{Fe/H}] \sim -0.5$) yet very long mean periods for their RRL stars, and furthermore, have large star-by-star period shifts relative to the period-amplitude diagrams of "normal" Galactic globulars such as M3 and M15. Furthermore, the morphologies of their HBs are highly abnormal for their metallicities, being nonhorizontal in V and extending across the HR diagram on either side of the RRL instability strip instead of being confined to the red side as in the high metallicity clusters in the Galaxy such as 47 Tuc.

From the abnormal period-amplitude shifts it is certain that the HBs of these clusters are more luminous by about 0.5 mag than the level of the HB in 47 Tuc and about 0.25 mag brighter than for the Oosterhoff II clusters such as M15.

Not only do these clusters violate the metallicity-luminosity equations set out in earlier sections, but they also violate the normal correlations that relate the Fourier component $(\phi)_{31}$ with period and metallicity, discovered by Simon & Clement (1993) and developed by Jurcsik & Kovacs (1996) and Kovacs & Jurcsik (1996, 1997); see also Sandage (2004).

The reasons are not yet understood, but explanations have begun (Sweigart & Catelan 1998; Bono et al. 1997a,b) on several fronts.

NGC 6388 and NGC 6441 may be the extreme examples known so far of the second parameter clusters that to a lesser degree have abnormal period-amplitude correlations and anomalous period shifts. Others with smaller period-amplitude-metallicity anomalies and abnormal HB morphologies include M2, M13, NGC 5986, and NGC 7006. This entire group may not follow the equations of the last sections, with NGC 6388 and NGC 6441 being the most extreme.

It seems likely that we are only near the beginning of an understanding of these and similar aspects of the second parameter problem. Clearly, the next step is to determine the luminosity anomalies of the second parameter clusters at the level of 0.1 mag, and to relate them to the abnormal morphologies of the HBs (Sandage 2006).

5.6. Other Methods and Controversies

A comprehensive review of the status of the RRL absolute magnitude calibration up to 1999 is by Popowski & Gould (1999). In addition to the methods discussed earlier here, they analyze two additional methods, which are (*a*) globular cluster kinematics and (*b*) white dwarf sequence fittings in the HR diagram.

In the cluster kinematic method, used first by Cudworth (1979), the dispersion in radial velocities of individual cluster stars is compared with the dispersion in the proper motions, which require a distance to convert from arcsec per year in the plane of the sky to km s^{-1}. Simple as this appears, it is model-dependent because the three-dimensional velocity distribution of the cluster stars must be known. Any anisotropy must be allowed for. This is usually taken from a fit of Mitchie (1963)-type models (cf. Lupton, Gunn & Griffin 1987) to the cluster light profile, but the method then becomes a combination of observations and theory, giving possible systematic uncertainties.

In the white dwarf (WD) cooling sequence method, the position of the observed WD sequence in a globular cluster is compared with the calibrated sequence of local

WDs whose distances are from trigonometric parallaxes. Entrance to the considerable literature on this method can be gained from Richer et al. (1995, 1997) and Renzini & Fusi Pecci (1988).

The conclusion reached by Popowski & Gould (1999) in their discussion of all seven methods they review, keeping the three methods they consider the most robust (statistical parallax, trigonometric parallax, and internal cluster kinematics), is $\langle M_V(RR)\rangle = 0.71 \pm 0.07$ at [Fe/H] $= -1.6$. This is 0.20 mag fainter than the calibration in Equation 8 here that requires $\langle M_V\rangle = 0.51$ at [Fe/H] $= -1.6$.

A decision between the two calibrations is possible by considering the resulting distance to the LMC based on RRL stars. Popowski & Gould derive $(m\text{-}M)^o_{LMC} = 18.33 \pm 0.08$ from their calibration. Our Equation 8 here, together with the observation by Clementini et al. (2003) that the mean level of the RRL is $\langle V^0\rangle = 19.06$ in the LMC, gives $(m\text{-}M)^o_{LMC} = 18.55$. This agrees to within 0.01 mag with the mean of 13 determinations of the LMC modulus, each of which are independent of the long period Cepheids (Tammann, Sandage & Reindl 2003, their table 6). This result favors Equation 8 for the long distance scale.

6. THE "ABOVE HORIZONTAL BRANCH" POPULATION II (AHB1) VARIABLES IN THE PERIOD RANGE OF 0.8 TO 3 DAYS

In his Mount Wilson survey of the spectra of variable stars in globular clusters, Joy (1949) divided his spectroscopic data into five groups. In addition to the RRL stars, he defined the other four groups on the basis of the light curves as RRL-like variables but with periods of 1–2 days, type II Cepheids (periods of 13 to 19 days) named after the W Vir and RV Tauri field variables (25–90 days), and irregular or semiregular red, longer period variables (60–110 days). These five groups still define the menagerie of Baade's population II variables, both in clusters and in the field (with the addition of the Anomalous Cepheids; see below). Except for the longer period red irregular and semiregular variables that are at the top of the red giant branch, all are pulsators in the Cepheid-RRL instability strip.

The RRL-like variables with periods between 0.8 and 3 days are of particular interest because their existence clarifies the evolutionary state that is the transition from the post-HB to the base of the asymptotic giant branch. These variables provide a test of the models for this very rapid transition phase of evolution. Joy showed that these "short period type II Cepheids" (using an old name) are brighter than the normal RRL stars by up to one mag. They lie above the HB. Field variables of the same type were also known, such as XX Vir (P = 1.3d), SW Tau (P = 1.6d), and, incorrectly, BL Her (P = 1.3d); the latter is not of population II (see below) but is the population I analog.

The first interpretation, subsequently proved to be correct, of the evolutionary state of these stars was by Strom et al. (1970) as post–helium shell burners after the helium core is exhausted. They named such stars AHB, meaning above horizontal branch, because, by then, Joy's bright luminosities of up to one mag brighter than the RRL luminosities (although still in the instability strip) had been confirmed both in the clusters and in the field. Abt & Hardy (1960) derived the higher luminosity for BL Her, as would Wallerstein & Brugel (1979) later for XX Vir. However, Abt & Hardy did not find metal weakening in BL Her, but rather a solar metal abundance, confusing the classification as a population II variable. Because of this, confusion as to population types existed for several years on the status of the "population II BL Her stars" as they were then called (Smith et al. 1978). However, later, Wallerstein & Brugel (1979) found profound metal weakening in XX Vir, and the picture finally became clear.

Earlier, Kwee (1967) had done photometry on a number of field examples of these short period Cepheids (or long period RRL stars) and had described their light curves, from which a start could be made on a morphological separation according to light curve shape between the XX Vir real population II variables and the population I BL Her stars. Their light curves are fundamentally different, permitting separation into separate population classes.

The separation on the basis of light curve shape was also argued, using new observations, by Diethelm (1983), who could define four separate groups among the 1–3 day Cepheids. In a comprehensive study, following the summary paper by Kwee & Diethelm (1984), Diethelm (1990) proposed the division into, and the names of, AHB1 for the XX Vir RRL-like population II brighter than HB stars, AHB2 for the light curves similar to HQ CrA ($P = 1.4$d) and RT TrA ($P = 1.9$d) shown in figure 2 of Diethelm (1983), and AHB3 for the population I prototype of BL Her ($P = 1.3$d) as in figure 3 of Diethelm (1983). The AHB1 are the true population II variables similar in all respects to those in globular clusters. That name is now coming into general use, and is adopted here. As mentioned earlier, the name was invented by Strom et al. (1970). It was first used after that by Kraft (1972). The reader is referred to the atlas of light curves for the four types isolated by Diethelm (1983).

An analysis for luminosities of the data available to 1994 was made by Sandage, Diethelm & Tammann (1994), both for AHB1 field variables and those in globular clusters. Comparison with the post-ZAHB models of Dorman (1992) of evolution toward the AGB was made there.

Dorman's calculated tracks that greatly clarified the evolution status of the AHB1 stars are shown in **Figure 13** for $[Fe/H] = -1.66$. The blue fundamental edge of the instability strip is drawn as $\log T_e(FBE) = 0.035\, M_{bol} + 3.832$ from Iben & Tuggle (1972). The ZAHB is the starting point for the tracks of different mass, marked in solar units along the branch. Tracks of ever decreasing mass cross the blue fundamental edge of the instability strip at ever increasing luminosities. This produces the ABH1 stars.

A detail of the evolution through the strip is shown in **Figure 14** for the Dorman tracks for $[Fe/H] = -2.26$. Shown are the blue and red edges of the instability strip and tracks for masses from 0.74 to 0.54 threading the strip. Lines of constant period (in days) are shown, based on the pulsation equation of van Albada & Baker (1973).

Comparison of **Figure 14** with similar diagrams that can be made from the Dorman tracks for $[Fe/H] = -1.03$ and -1.66 shows that the resulting P-L relations (i.e., made by reading the M_{bol} at each period) from the three diagrams are

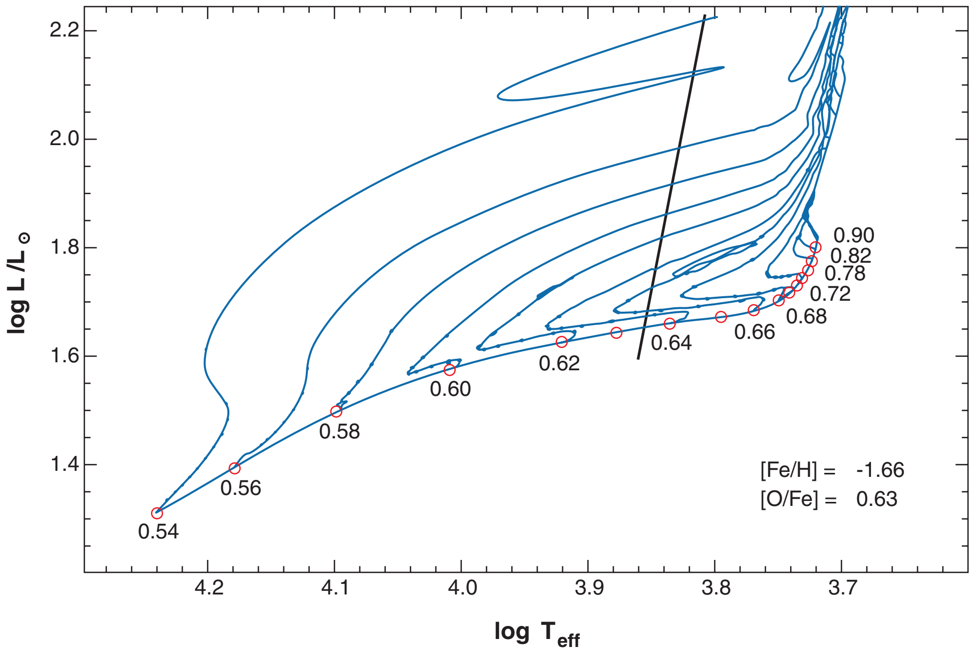


Figure 13

Dorman's (1992) post-zero age horizontal branch (ZAHB) tracks for [Fe/H] $= -1.66$ for different masses, showing the position of the fundamental blue edge of the instability strip. The mass values are marked for each track as it begins from the ZAHB. Time ticks on the tracks are for every 10^7 years. Diagram is from figure 13a of Dorman (1992) and figure 1 of Sandage, Diethelm & Tammann (1994).

nearly identical. There is no metallicity dependence of M_V at a given period for those luminosities that are brighter than the ZAHB. This is contrary to the strong dependence for the near ZAHB RRL variables one mag fainter that is manifest in a comparison (not shown) of Dorman's other two diagrams for [Fe/H] $= -1.03$ and -1.66 that are similar to **Figures 13** and **14**. Said differently, Dorman's models give a variation of M_V with [Fe/H] for the ZAHB, which, however, disappears for brighter luminosities in the strip.

An empirical calibration of the P-L relation for the ABH1 population II variables was made by Sandage, Diethelm & Tammann (1994). It was derived by comparison with the RRL (mean evolved state on the HB) luminosities from the Sandage (1993) calibration of $M_V(RR) = 0.94 + 0.30[Fe/H]$. The result in **Figure 15** gives the calibration of

$$M_V = -2.00 \log P - 0.10, \quad (13)$$

with an rms scatter of 0.29 mag at a given period.

From Dorman's models (and all the subsequent models on the same problem) it can be seen that AHB1 stars cross the instability strip in less than 10^6 years. This is

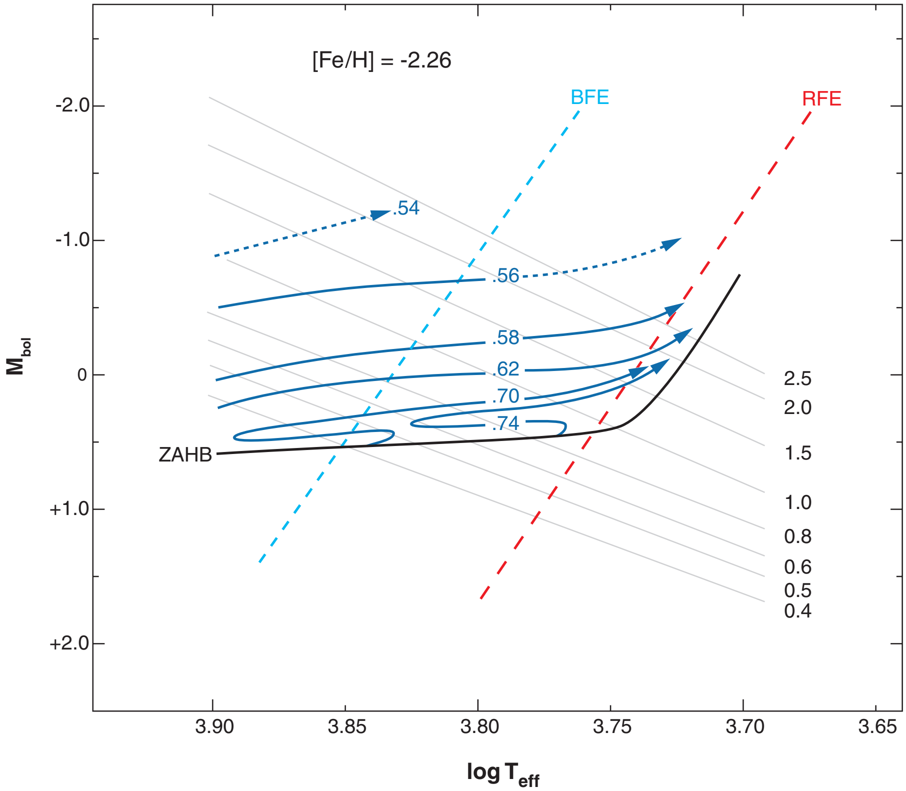


Figure 14

Detail of the evolution tracks through the instability strip as read from a diagram similar to **Figure 13**, but for [Fe/H] = −2.26. The lines of constant period are drawn. The periods are marked in days. The blue and red edges of the strip are drawn for the fundamental mode pulsators. Diagram is from figure 4 of Sandage, Diethelm & Tammann (1994).

100 times faster than the time for crossing the strip for ordinary RRLs on or near the ZAHB. Hence, the AHB1 stars offer the best opportunity to test this prediction for secular period changes due to evolution.

Such period changes at the predicted rate have, in fact, now been observed by Wehlau & Bohlender (1982) for nine AHB1 variables in globular clusters, and by Diethelm (1996) for three field AHB1 variables, confirming the prediction in a most satisfactory way.

7. THE POPULATION II CEPHEIDS

Many summaries of the observational aspects of the population II Cepheids have been made in the past two decades, following the pioneering work by Arp (1955) that was his PhD subject. These later summaries include the reviews and models by

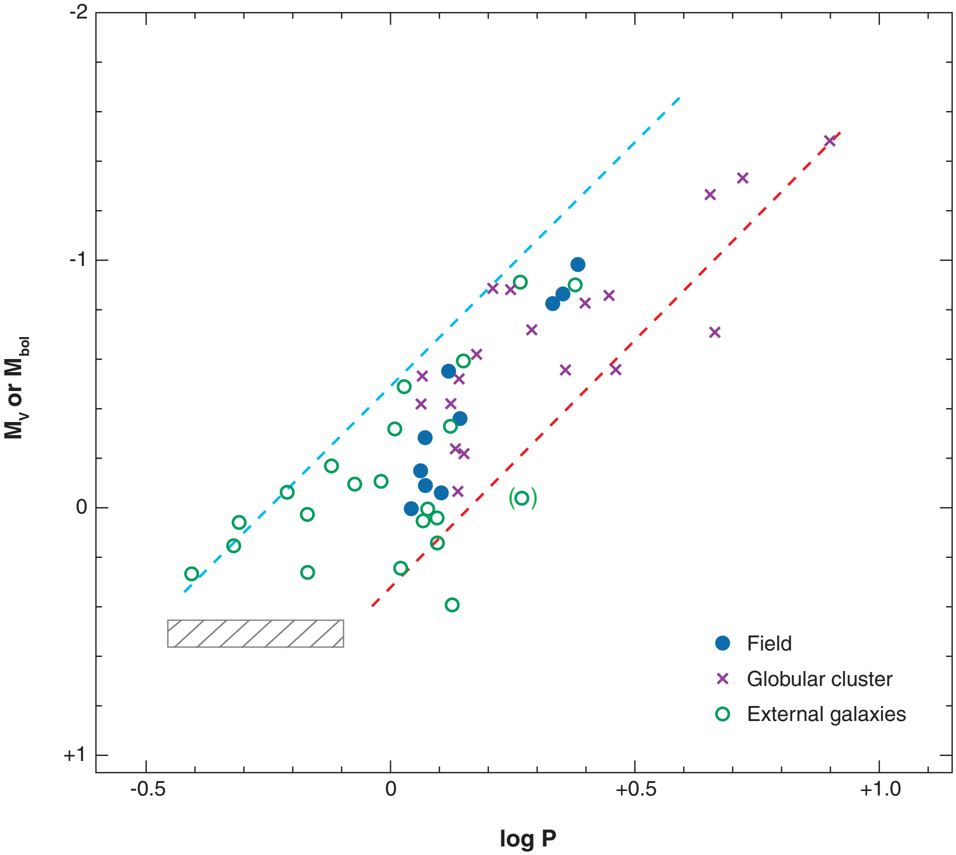


Figure 15

The P-L relation for the above horizontal branch (AHB1) variables in the field, in globular clusters, and in external galaxies known in 1994. The position of the RR Lyrae variables is shown by the hatched rectangle. The fundamental blue and red edge boundaries of the instability strip are estimated from the data. The diagram is from figure 5 of Sandage, Diethelm & Tammann (1994).

Bohm-Vitense et al. (1974), Gingold (1976, 1985), Wallerstein & Cox (1984), Harris (1984, 1985), Nemec, Wehlau & Mendes de Oliveira (1988), Wallerstein (1990, 2002), Whitlock, Feast & Catchpole (1991), Nemec, Nemec & Lutz (1994), McNamara (1995), and the indispensable catalog of Clement et al. (2001). A modern calculation of advanced models of both the population II Cepheids and the anomalous Cepheids (see next section) is by Bono, Caputo & Santolamazza (1997a,b).

A recent census of the population II Cepheids (including the AHB1 stars) in clusters and dwarf spheroidal galaxies is by Pritzl et al. (2003), where the new ABH1 and W Vir Cepheids found in the globular clusters NGC 6388 and NGC 6441 (four in NHC 6388 and six in NGC 6441) are included. (Note that Pritzl et al.

incorrectly call the AHB1 variables in NGC 6388 and NGC 6441 BL Her variables, which, as discussed above, should be named either AHB1 or XX Vir variables. BL Her is the population I analog of the AHB1 population II variables as argued earlier.)

These authors have calibrated the absolute magnitudes of the AHB1, the W Vir, and the RV Tau variables in their list of all such variables in clusters as based on RRL luminosities of $M_V(RR) = 0.17[Fe/H] + 0.82$ from Lee, Demarque & Zinn (1990, their equation 7). This gives $M_V(RR) = 0.57$ at $[Fe/H] = -1.5$, whereas the calibration used by Sandage, Diethelm & Tammann (1994), in **Figure 15** here, uses $M_V(RR) = 0.30\,[Fe/H] + 0.94$ from Sandage (1993), which gives $M_V(RR) = 0.49$ at $[Fe/H] = -1.5$. Hence, the calibration by Pritzl et al. (2003, their equation 3), on their scale, is

$$M_V(\text{pop II}) = -1.64 \log P - 0.05, \tag{14}$$

which is 0.08 mag fainter than Equation 13 for the AHB1 stars according to Sandage, Diethelm & Tammann (1994). On the zero point scale of Equation 13, Equation 14 would be $M_V = -1.64 \log P - 0.14$. Therefore, at $\log P = 0.3$ ($P = 2$ days), the Sandage, Diethelm & Tammann calibration in Equation 13 gives $M_V(2\text{ d}) = -0.56$, whereas the calibration by Pritzl et al. from the rezero pointed Equation 14 is $M_V(2\text{ d}) = -0.63$.

The difference in slope between Equations 13 and 14 is due to the different period range over which each equation has been derived. Equation 14 is preferred for the long period population II Cepheids (P between 13 and 30 days) because this is the range covered by table 8 of Pritzl et al. (2003), far beyond the period range for the AHB1 stars studied by Sandage, Diethelm & Tammann (1994).

It remains only to remark on the difference in slope of the P-L relations for the population II Cepheids, in Equation 14, as −1.64 compared with that of population I Cepheids, as in Equation 4, as −3.09 of Section 3 for the Galactic Cepheids. Hence, the P-L relations of population I and II Cepheids are not parallel, as was assumed when the first distinctions between them were made by Baade and by Swope (circa 1950–1960). This, of course, is due to the different mass-luminosity relations for the type I and II variables. Population II Cepheids are stars in the post-HB phase of evolution on the blue loops (Hoffmeister 1967) on the second giant branch (the AGB) with nearly constant masses of less than 0.8 solar (e.g., Gingold 1976, 1985). In contrast, population I Cepheids are stars with masses that parallel the mass-luminosity relation of their main sequence progenitors that range from about 2 to 30 solar mass, depending on the luminosity.

8. THE ANOMALOUS CEPHEIDS

Up to this point the picture seemed very clean, where the differences in the periods, luminosities, and shapes of the light curves of the population I and II Cepheids, the AHB1 variables, and the RRL stars along the HB of globular clusters were understood as the result of well-known differences in evolution stages and of the relevant stellar masses, which were themselves understood.

Then a complication arose with the discovery of a new class of Cepheid-like variables with periods between 0.8 and 2 days in the dwarf spheroidal galaxies that are companions to the Galaxy (Baade & Swope 1961, Swope 1968, Zinn & Searle 1976 for a review). The shapes of their light curves are distinctly different from either the population I BL Her stars or the population II AHB1 XX Vir stars in the same period range in globular clusters. The light curves of stars of this new class are of small amplitude and are much more symmetrical than the RRL-like asymmetrical rapid rise AHB1 variables of the same period.

At first, no such star was known in globular clusters, then one was discovered in NGC 5466 (Zinn & Dahn 1976). The high velocity, low metal abundance field star XZ Cet (P = 0.82 d) was also discovered with its small amplitude and symmetrical light curve (Dean et al. 1977). The light curves of these stars are so distinctive that they are easily seen to differ from AHB1 stars. Hence such stars can be classified by the morphology of their light curves alone (Diethelm (1983), 1990), but clearly their evolutionary origins are different from the AHB1 stars, understood via **Figures 13** and **14**.

In the dwarf spheroids, these stars, early-on called anomalous Cepheids (AC), are brighter than the HB RRL stars, and are also brighter than the AHB1 stars of the same period. But they are fainter than the classical type I Cepheids (Zinn & Searle 1976, their figure 1). They have been found in every dwarf spheroidal surveyed (Swope 1968; Nemec, Nemec & Lutz 1994 for a comprehensive review), but in only the globular cluster 5466, and two suspected variables in Omega Cen. **Figure 16** here, taken from Nemec, Whelau & Mendes de Oliveira (1988, their figure 13), is a summary of the situation known to 1987 in the B photometric band.

The distinctive place of the ACs in the P-L plane seen in **Figure 16** shows that they must be in a different evolutionary state than either the RRL stars or the population II Cepheids. It was early suggested (Bohm-Vitense et al. 1974, Demarque & Hirshfeld 1975, Norris & Zinn 1975, Zinn & Searle 1976, Zinn & King 1982, Hirshfeld 1980, and undoubtedly others) that they are pulsators with higher mass than either the RRL stars or the AHB1 pulsators with periods of 0.8 to 3 days (the same range as the ACs) and for the blue loop W Vir and RV Tau variables of longer period.

Because such masses are impossible for single stars in the normal evolutionary state of old globular clusters (they have all evolved off the main sequence at this epoch), their high mass must be due to coalescence of a binary star into a single star that then pulsates when it is in the strip, or alternatively, to mass transfer in a contact binary before coalescence, or as a third possibility, to recent star formation. Masses of about 1.5 solar masses are necessary to explain their position in **Figure 16**.

A review of the status of such explanations is by Bono (2003), which, when used together with the earlier definitive review by Nemec, Whelau & Mendes de Oliveira (1988), brings the story up to date. The modern data for the P-L relation discussed by Bono et al. show wide scatter within the boundary lines drawn in **Figure 16**, suggesting a random distribution of mass values and, hence, probably several different evolutionary channels by which the single old stars gain their increased mass.

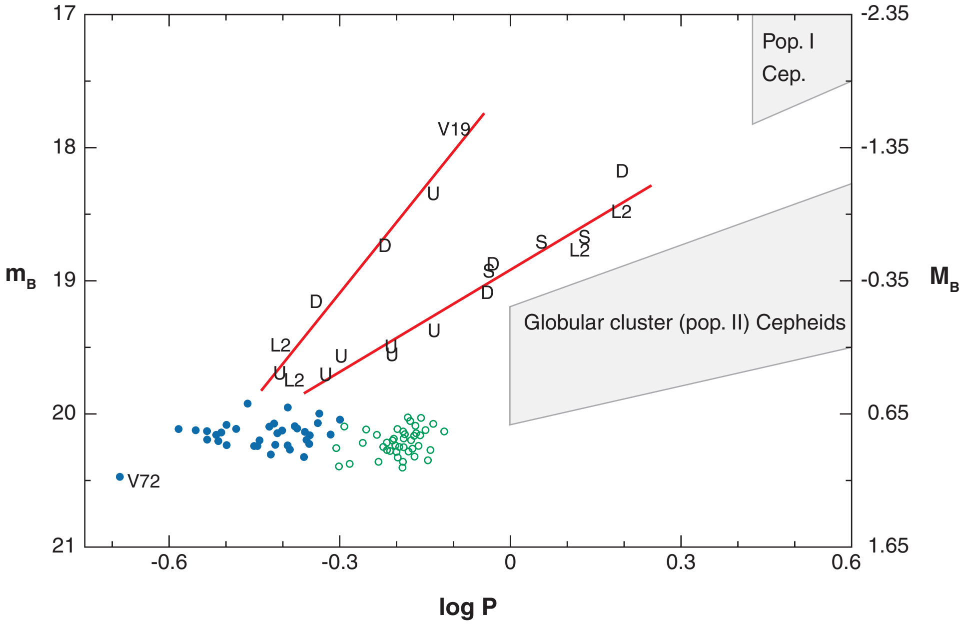


Figure 16

Summary of the B-band P-L relation for anomalous Cepheids (the AC stars) compared with the position of the RRL variables in the Ursa Minor dwarf spheroidal, and compared with the boundary positions of the P-L relations for population I and II variables. The symbols on the lines are for the dwarf spheroidals of U = Ursa Minor, L_2 = Leo II, D = Draco, and S = Sculptor. The mean apparent magnitude of the Ursa Minor RRLs is assumed to be 20.16, with $<M_B(RR) = 0.8$ to set the zero point in M_B on the right-hand ordinate. The lines through the dwarf spheroidal points are for the fundamental and first overtones. More recent data (Bono et al. 1997, their figure 6) show a wider scatter of the AC stars throughout the region between the two lines, presumably caused by a stochastic distribution of masses. Diagram from Nemec, Whelau & Mendes de Oliveira (1988).

An interesting suggestion is offered by Wallerstein (2002) for the use of the AC to study the origin of at least part of the Galactic halo by accretion of dwarf spheroidals. This is to exploit the absence of AC in globular clusters and their abundance in dwarf spheroidals, and conversely, to make use of the fact that long period type II Cepheids (W Vir and RV Tau) are absent in the dwarf spheroidals but present in globular clusters. Systematic searches for the percentages of each of these types of variables in the distant Galactic halo with the data from surveys such as the current Sloan can be expected to shed light on whether or not an appreciable part of the halo has been formed by such accretions.

But even more basic will be a search for the explanation for the presence and absence of these types of variables in their respective systems. This is perhaps the single most pressing problem remaining for a complete understanding of the astronomy of the population II variables.

FUTURE ISSUES

1. Progress in understanding the reason for the slope and zero point differences in the period-luminosity (P-L) relations between the Galaxy and the LMC can be expected by analysis of the Cepheid data in the SMC, NGC 6822, IC 1613, Wolf-Lundmark-Melotte, and other low-metallicity local galaxies in the same way that the data for the Galaxy have been analyzed (Tammann, Sandage & Reindl 2003; Sandage, Tammann & Reindl 2004). It is yet to be proved or rejected that differences in metallicity are the cause.
2. Clues for or against a metallicity effect on the Cepheid P-L relation can be expected by relating slope and zero point differences to metallicity differences across the disks of galaxies such as M33 and NGC 300 in the presence of metallicity gradients and where the internal absorption is small, in addition to M31 and M101 where the test has already been made.
3. The well-known problem of the origin and evolutionary status of high metal abundance ($-0.5 < [Fe/H] < +0.1$) RR Lyrae (RRL) variables in the field with disk kinematics is still unsolved. Identification of such stars in high metallicity open clusters would provide an opening toward a solution, but no such clusters, nor any other site of origin, has been identified, and the mystery remains. Population I analogues of the AHB1 population II XX Vir exist with the prototype BL Her (0.8 to 3 day) Cepheids, but their evolutionary origin is also a mystery. The two problems are undoubtedly related.
4. Absolute magnitude calibrations are needed of the second parameter effect for RRL stars where the morphology of the horizontal branch (HB) for different metallicity does not follow the pattern for the majority of the Galactic globular clusters. The most blatant cases are NGC 6388 and NGC 6441, where the RRL stars are believed to be 0.25 mag brighter than is predicted by the M_V(RR)-[Fe/H] equations given in the text. Hence, although these calibrations are valid for the "normal" RRLs, they are expected to fail at the level of 0.2 mag for second parameter cluster variables. Precision main sequence photometry or any other method accurate at the 0.1-mag level is required to measure M_V(RR) for all second parameter clusters.
5. The HB morphology-metallicity correlations in the dwarf spheroidal companion satellites of the Galaxy, combined with period-amplitude and period-rise-time studies of their RRL stars, can be expected to clarify the morphology-metallicity anomaly present in the dwarf spheroidals. Such studies relate directly to the second parameter problem.
6. The problem remains why Galactic globular clusters contain W Vir population II Cepheids and RV Tauri variables while the dwarf spheroidals do not. And, visa versa, why the dwarf spheroidals contain the anomalous Cepheids (AC; 0.8 to 3 days) that differ from the above horizontal branch (AHB1)

XX Vir variables (the population II dwarf Cepheids), while Galactic globular clusters do not. Whatever the reason, presently it is not understood—although perhaps because the AC stars are young, whereas the AHB1 stars are old (Caputo, private communication)—that difference can be exploited in surveys of variables in the Galactic halo, such as the Sloan, to test if part of the remote halo is formed from accreted dwarf spheriodals. The problem may also be related to the absence of the population II W Vir Cepheids in the high Galactic halo where such stars have always been expected, but whose absence presents a problem; unexpectedly, the field W Vir variables have thick-disk kinematics and thick-disk scale heights in the Galaxy, rather than remote halo values.

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