ISSN 1063-7729, Astronomy Reports, 2019, Vol. 63, No. 2, pp. 107–111. © Pleiades Publishing, Ltd., 2019.
Russian Text © A. V. Tutukov, 2019, published in Astronomicheskii Zhurnal, 2019, Vol. 96, No. 1, pp. 70–74.

Influence of Digital Noise on Interpretation of a Transit Light Curve

M. K. Abubekerov[1*] and N. Yu. Gostev[1]**

[1] *Sternberg Astronomical Institute, Lomonosov Moscow State University, Universitetskii pr. 13, Moscow, 119234 Russia*

Received June 29, 2018; in final form, September 13, 2018

Abstract—The algorithms often used for the interpretation of light curves, such as the frequently applied JKTEBOP algorithm, have limited accuracy, which causes rounding errors, and hence a non-physical contribution to the residuals (digital noise). The transit light curve of the binary HD 209458 is used as an example to demonstrate the need to take into account this digital noise. Improving the accuracy of light-curve computations enables more reliable determination of whether the observed light curve is adequately described by a particular model, thanks to the elimination of the non-physical contribution to the residuals resulting from computational errors. A website where the algorithm developed can be obtained is given in the Conclusions.

DOI: 10.1134/S106377291902001X

1. INTRODUCTION

Currently, high-precision observations of transit light curves are being obtained on a large scale, making reliable interpretation of these curves important [1, 2]. There is scope to considerably improve the possibilities for fitting (light-curve analysis) compared to other algorithms. This is primarily related to the improved accuracy that can be achieved in light-curve computations thanks to the application of Gaussian-quadrature integration [3, 4].

Most currently used algorithms are based on the JKTEBOP algorithm developed in the beginning of 1980s [5]. The JKTEBOP algorithm computes the integrals in the expressions for the binary's brightness directly, by means of direct summing over subintervals into which the integration region is divided. In this case, the computation error for the integral is inversely proportional to the number of elementary operations (and hence to the computation time), and the accuracy of the result is essentially limited by the time that can be allocated for the computations. This accuracy corresponded to the capacilities of computers available at the time when JKTEBOP was developed and the observational uncertainties of that time.

In many cases, when we fit models to light curves, it is necessary to take into account various non-physical uncertainties, such as computational uncertainties and rounding errors (digital noise). However, the abilities of modern computers and the accuracy of modern brightness observations providing tens of thousands of data points require better accuracy of the computations. Using Gaussian quadrature in the numerical computation of integrals, it is possible to obtain an accuracy corresponding to the modern accuracy of floating-point numbers (about 18 significant digits) within an acceptable time. y

Most importantly, improvement of the computation accuracy enables a more accurate evaluation of whether the observed light curve is adequately described by a model used. The elimination of the non-physical contribution to the residuals related to computational uncertainties makes it possible to draw more reliable conclusions concerning the significance of changes in the residuals in particular cases. For example, it becomes possible to judge the significance of changes in the minimum residuals resulting when additional parameters are used, in particular, how the minimum residual depends on the limb-darkening laws that are applied.

Note that better accuracy of light-curve computations can considerably influence the minimum residuals even without any significant changes in the values of the derived parameters. This is true because digital noise can be reflected in the uncertainties of the parameters derived using Monte Carlo simulations. In this situation, we conclude that, in some cases, the JKTEBOP algorithm can actually make it possible to obtain satisfactory results, namely the parameters of the system estimated using Monte Carlo simulations (assuming a priori that the model is adequate to the observations). At the same time, if we consider fitting without this assumption (that the model is correct), when an empty confidence set is possible or we are

[*] E-mail: marat@sai.msu.ru
[**] E-mail: ngostev@mail.ru

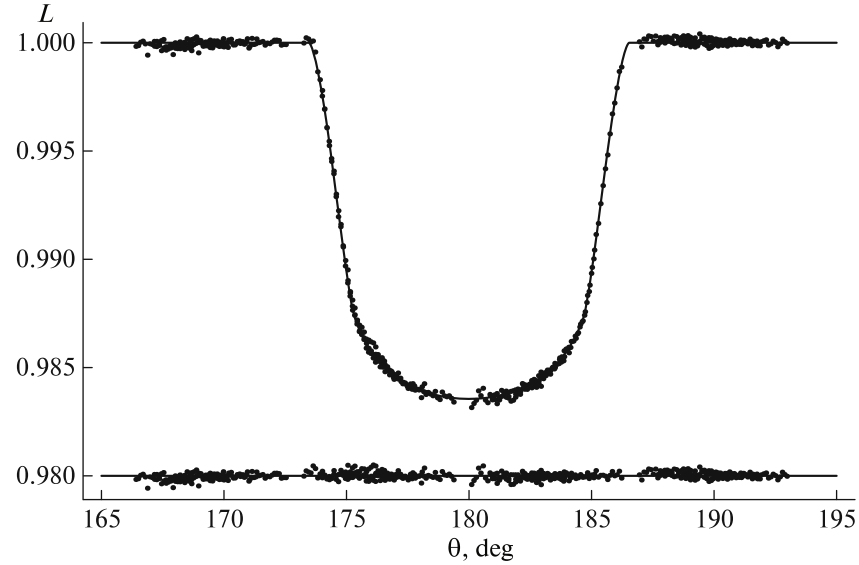

Fig. 1. The observed (points) and theoretical (solid curve) light curves of the exoplanet binary HD 209458 from [6]. Deviations of the observed brightness from the theoretical light curve calculated for a model with a non-linear (quadratic) limb-darkening law are shown in the bottom.

determining the statistical significance of a change in the residuals, more accurate computation of the residuals is needed.

2. OBSERVATIONAL DATA

We will demonstrate the operation of the algorithm we have developed using the observed light curve of the binary with an exoplanet HD 209458 [6]. The observed light curve presented in [6] was obtained with the Hubble Space Telescope in April–May 2000. The spectra were taken with the STIS spectrometer and the G750M spectral grating. The observations at 5813–6382 Å had a resolving power $R = \lambda/\Delta\lambda = 5440$ (see [6] for details). The normalized light curve of the exoplanet's transit across the stellar disk is presented in Fig. 1. This light curve is based on 556 individual brightness measurements of the binary. The rms uncertainty of an individual measurement, $\sigma_i^{\rm obs}$, is between 1.13×10^{-4} and 2.47×10^{-4} (in units of the out-of-eclipse intensity) for various points in the light curve. The relative uncertainties (in units of the eclipse depth) are between $\sim 7 \times 10^{-3}$ and $\sim 1.5 \times 10^{-2}$. We fitted this light curve using linear and quadratic limb-darkening laws.

The light curve of the binary Kepler-15b, which we also used to test the algorithm, was taken from the NASA Exoplanet Archive. We used the short-cadence data of the 18th set of observations, obtained between September 18 and October 18, 2009.

3. DESCRIPTION OF THE MODEL

We used a model with a spherical star and a spherical planet in a circular orbit, assuming no reflection effect and neglecting their ellipsoidal shapes. The geometry of the model is shown in Fig. 2, where R_* is the radius of the star, R_0 the radius of the planet, D the distance between the centers of the component disks, and ρ and Ψ are the polar coordinates of a point on the stellar disk. The coordinate origin is at the center of the star, and the polar angle is measured counter-clockwise.

For a circular orbit, the distance between the centers of the disks, Δ, depends on the phase θ and orbital inclination i as

$$\Delta(\theta, i) = \sqrt{\cos^2 i + \sin^2 i \sin^2 \theta}. \tag{1}$$

When calculating the light curve, we used a linear limb-darkening law for the brightness distribution on the stellar disk, with the linear limb-darkening coefficient x,

$$I(\rho) = I_0 \left(1 - x + x\sqrt{1 - \frac{\rho^2}{R_*^2}}\right), \tag{2}$$

and also a quadratic limb-darkening law that has an additional term containing the quadratic limb-darkening coefficient y,

$$I(\rho) = I_0 \left(1 - x\left(1 - \sqrt{1 - \frac{\rho^2}{R_*^2}}\right) - y\left(1 - \sqrt{1 - \frac{\rho^2}{R_*^2}}\right)^2\right). \tag{3}$$

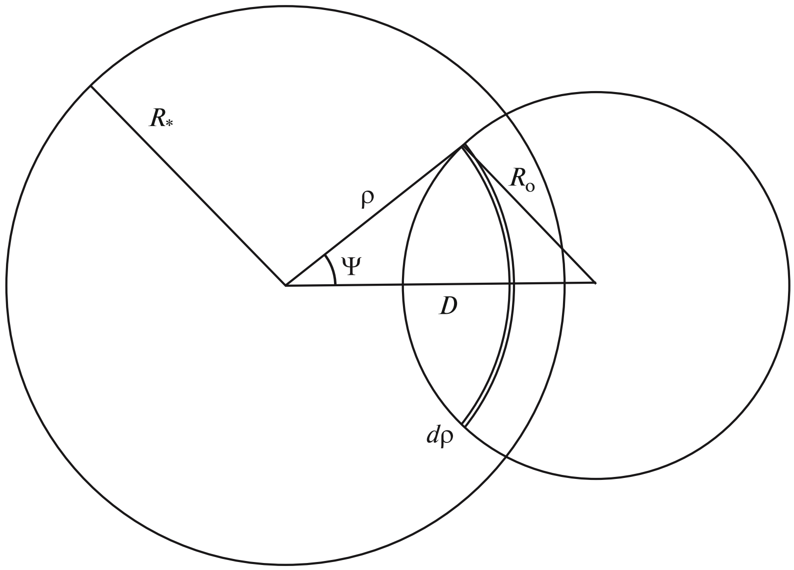

Fig. 2. Model of an eclipsing binary projected onto the plane of the sky.

Here, ρ is the polar distance from the center of the stellar disk, I_0 the brightness at the disk center, and R_* the radius of the stellar disk. We will designate the brightness at the disk center of component 1 (the star) I_0. The brightness at the center of component 2 (the planet) and the brightness at any point of its disk were assumed to be zero. Component 2 eclipses component 1 at orbital phase $\theta = \pi$. The unit of length in our models is the distance a between the centers of the two bodies, $a = 1$. There is no "third light" in the model. The unknown parameters in the model are the radii of the star, R_*, and the planet, R_o, the orbital inclination i, the limb-darkening coefficient x, and, in the case of quadratic limb-darkening law, the limb-darkening coefficient y.

The total brightness of the star, that is, the total out-of-eclipse brightness of the system, is

$$L^{\text{full}} = 2\pi \int\limits_0^{R_*} I(\rho)\rho d\rho = \pi R_*^2 I_0 \left(1 - \frac{x}{3}\right) \quad (4)$$

in the model with the linear limb-darkening law and

$$L^{\text{full}} = 2\pi \int\limits_0^{R_*} I(\rho)\rho d\rho = \pi R_*^2 I_0 \left(1 - \frac{x}{3} - \frac{y}{6}\right) \quad (5)$$

in the model with the quadratic limb-darkening law.

The brightness at the center of the stellar disk, I_0, was selected so that the total brightness of the system is unity (as a normalization condition).

The brightness decrease during the eclipse is

$$L^{\text{dec}}(\Delta, R_*, R_o, x, y) = \iint\limits_{S(\Delta)} I(S) dS, \quad (6)$$

where $S(\Delta)$ is the overlapping area of the disks (which depends on the distance between the disk centers). The main task when calculating the light curve is to calculate the brightness decrease in the eclipse.

A model of two spherical bodies for an optical transit light curve is physically justified. The star and the planet have sharp edges in the optical. Effects due to deformation of the exoplanet's atmosphere by the stellar wind (the cometary-tail effect) are not appreciable or are negligible in the optical. Thus, approximating the star and exoplanet's disks as circles is satisfactory in the optical.

4. DESCRIPTION OF THE ALGORITHM

When fitting the data, we used the algorithm for calculating the brightness decrease in an eclipse in a binary system presented in detail in [3, 4]. In this algorithm, the integration in the region of overlapping disks is performed by analytically computing the integrals using elliptical functions (efficient algorithms for computing these integrals with any given accuracy are available) or by applying Gaussian-quadrature integration after some analytical transformations of the integrand.

5. COMPUTATIONS

According to Southworth [5], who used a JKTEBOP-based algorithm for his light-curve fitting, the residual in the linear limb-darkening law for HD 209458 is $\chi^2 = 1.1457$, corresponding to a maximum significance level of 1%. Note that the parameters in the fitting in [5] were the radii of the star

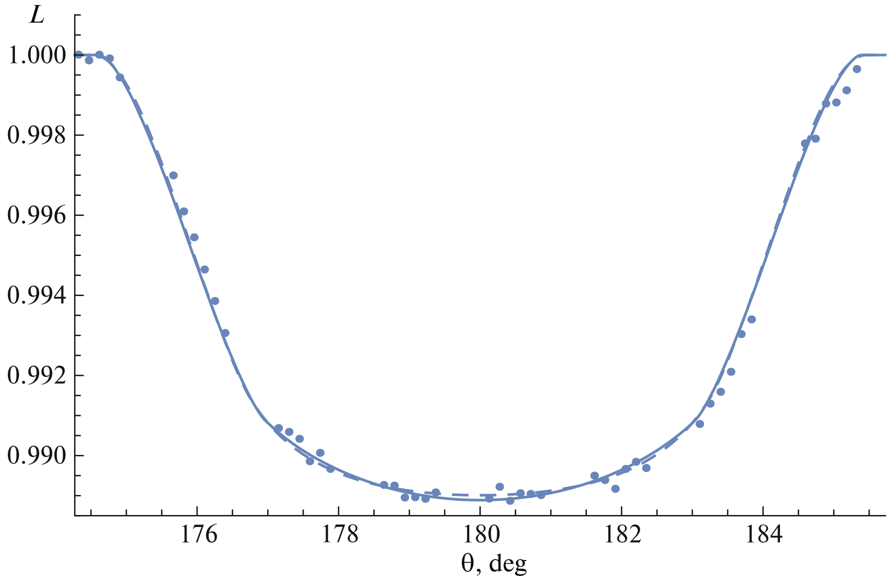

Fig. 3. Observed (points) and theoretical light curves of Kepler-15b, a binary with an exoplanet. The light curves calculated for the linear and quadratic limb-darkening laws are plotted as solid and dashed curves, respectively,

and planet, the orbital inclination, the eccentricity, and four limb-darkening coefficients.

When we used our algorithm [3, 4] assuming a linear limb-darkening law, we obtained the residual $\chi^2 = 1.103$, corresponding to a maximum significance level of 6%. Though the derived parameters agree within the uncertainties, we have a considerable disagreement in the values of the minimum residuals.

In the case of the quadratic limb-darkening law, our algorithm gives the residual $\chi^2 = 1.0134$, corresponding to a maximum significance level of 46%. Thus, using a more accurate algorithm that suppresses the digital noise, i.e., eliminating the contribution from computational errors, we were able to draw a firm conclusion about the statistically significant decrease in the residuals when using the quadratic limb-darkening law. In addition, eliminating the non-physical contribution to the residuals opens the possibility of working with a non-empty confidence set at a higher significance level (lower confidence level). Note that it was also concluded in [7] that the quadratic limb-darkening law was preferable for the HD 209458 system, but this conclusion was based more on the non-physical correlation of the orbital inclination and wavelength indicated by the linear limb-darkening law. The decrease of the residuals when using the quadratic limb-darkening law rather than the linear law was considered to be insignificant. Applying our algorithm that makes it possible to neglect rounding errors, we find that the decrease in the residuals changes the maximum significance level from 6 to 46%, an obviously substantial difference. Thus, eliminating the digital noise has enabled us to detect the advantage of the quadratic limb darkening law for the HD 209458 system solely from the resulting decrease in the residuals.

An opposite example is provided by the light curve of Kepler-15b (see Fig. 3), where we did not find any considerable decrease in the residuals when we applied the quadratic limb-darkening law, despite the presence of visible differences between the corresponding light curves.

6. DISCUSSION

Our results clearly demonstrate the need to apply algorithms enabling very accurate computations of light curves if we wish to ensure sound judgments about whether a model fully adequately represents the observations when fitting transit light curves. The algorithm we developed based on Gaussian-quadrature integration makes it possible to work with an accuracy corresponding to the modern number of digits that can be used for floating-point numbers (19 significant digits). This accuracy applied to light-curve calclations enables more reliable conclusions about the adequacy of a given model in describing the observed light curve.

Applying our algorithm virtually eliminates the non-physical contribution to the residuals due to computational errors, making it possible to draw more reliable conclusions concerning the significance level of changes in the residuals for different fits. For example, it is possible to understand the significance of a change in the minimum residuals that comes about when parameters are added to the fit; we have considered here the example of how the minimum residuals depend on the applied limb-darkening law. In addition, the possibility of working at a higher significance level (lower confidence level) is of interest by itself.

The quality of the description to the observations provided by a theoretical light curve is closely related to the detection of physical phenomena described by the model applied. The high sensitivity of the χ^2 criterion to deviations of the observations from a model can enable the detection of fairly fine physical effects, but this also requires sufficiently accurate calculation of the χ^2 statistics.

We plan to apply our algorithm to large-scale fitting of transit light curves for binaries with exoplanets in order to determine the empirical limb-darkening coefficients for stars of different spectral types [2].

7. CONCLUSIONS

Our algorithm for fitting of transit light curves is freely available. Application of the algorithm will enable better-quality fitting for the rich observing material supplied by both ground- and space-based observatories. The algorithm can be accessed at `http://lnfm1.sai.msu.su/~ngostev/algorithm.html`.

ACKNOWLEDGMENTS

The authors are grateful to A.M. Cherepashchuk for fruitful discussion of this study.

REFERENCES

1. M. K. Abubekerov, N. Yu. Gostev, and A. M. Cherepashchuk, Astron. Rep. **52**, 99 (2008).
2. M. K. Abubekerov, N. Yu. Gostev, and A. M. Cherepashchuk, Astron. Rep. **54**, 1105 (2010).
3. M. K. Abubekerov and N. Yu. Gostev, Mon. Not. R. Astron. Soc. **432**, 2216 (2013).
4. M. K. Abubekerov and N. Yu. Gostev, Mon. Not. R. Astron. Soc. **459**, 2078 (2016).
5. J. Southworth, Mon. Not. R. Astron. Soc. **379**, L11 (2007).
6. T. M. Brown, D. Charbonneau, R. L. Gilliland, R. W. Noyes, and A. Burrows, Astrophys. J. **552**, 699 (2001).
7. J. Southworth, Mon. Not. R. Astron. Soc. **386**, 1644 (2008).

Translated by N. Samus'