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Current Problems of Dynamics of Moons of Planets and Binary Asteroids Based on Observations

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Abstract—The general approach to studying the dynamics of moons of planets and asteroids consists in developing more and more accurate models of motion based on observational data. Not only the necessary ephemerides, but also some physical parameters of planets and moons are obtained this way. It is demonstrated in the present study that progress in this field is driven not only by the increase in accuracy of observations. The accuracy of ephemerides may be increased by expanding the observation time interval. Several problems arise on the way toward this goal. Some of them become apparent only when the procedure of observational data processing and use is examined in detail. The method used to derive astrometric data by processing the results of photometric observations of mutual occultations and eclipses of planetary moons is explained below. The primary contribution to the error of astrometric results is produced by the unaccounted noise level in photometric readings and the inaccuracy of received values of the albedo of moons. It is demonstrated that the current methods do not allow one to eliminate the noise completely. Extensive additional photometric measurements should be performed at different angles of rotation of moons and in different spectral bands of the visible wavelength range in order to obtain correct values of the albedo of moons. Many new distant moons of the major planets have been discovered in the early 21st century. However, the observations of these moons are scarce and were performed over short time intervals; as a result, some of the moons were lost. The necessity of further observations of these Solar System bodies is pointed out in the present study. Insufficient knowledge of asteroid masses is an obstacle to improving the accuracy of the ephemerides of Mars. The basic method for determining the masses of large asteroids consists in analyzing their influence on the motion of Mars, the Earth, and spacecraft. The masses of more than 100 large asteroids were determined this way. One of the principal techniques for Earth-based measurement of the masses of asteroids involves astrometric observations of binary asteroids. The determination of relative coordinates is made rather difficult by the apparent proximity of components. The success of these efforts depends on the availability of instrumentation and the expertise of observers skilled in adaptive optics and speckle interferometry. Collaboration between different research teams and observers is absolutely necessary.

Keywords: planetary moons, asteroids, theory of motion, observations, astrometry, ephemerides
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GENERAL APPROACH TO STUDYING THE DYNAMICS OF SOLAR SYSTEM BODIES

The primary objective of research into the dynamics of Solar System bodies is the determination of parameters of motion of planets and their moons. This objective is relevant to the perennial challenge of mankind: expanding and exploring our habitat. Planetary moons are the most suitable targets for unmanned and manned landing missions. Research into the structure and dynamics of Solar System bodies is an integral part of astronomy. The methods of celestial mechanics and astrometric observations are used in this research. Interplanetary navigation, which attracted the interest of scientists in the second half of the 20th century, is a new problem of dynamics of Solar System bodies.

The general approach to studying the dynamics of celestial bodies consists in developing models of motion and ephemerides of planets, asteroids, and planetary moons. Such models are built based on the general laws of nature, the physical parameters of celestial bodies, and, most importantly, observations. Advanced mathematical and computational techniques are used in the process. Ephemerides are the end result of this research and incorporate the entire body of knowledge on the dynamics of Solar System bodies.

Ephemerides are used to determine the physical properties of celestial bodies and to study the origins and evolution of the Solar System. They are also needed to prepare and launch space missions to other planets and help discover new celestial bodies. In the

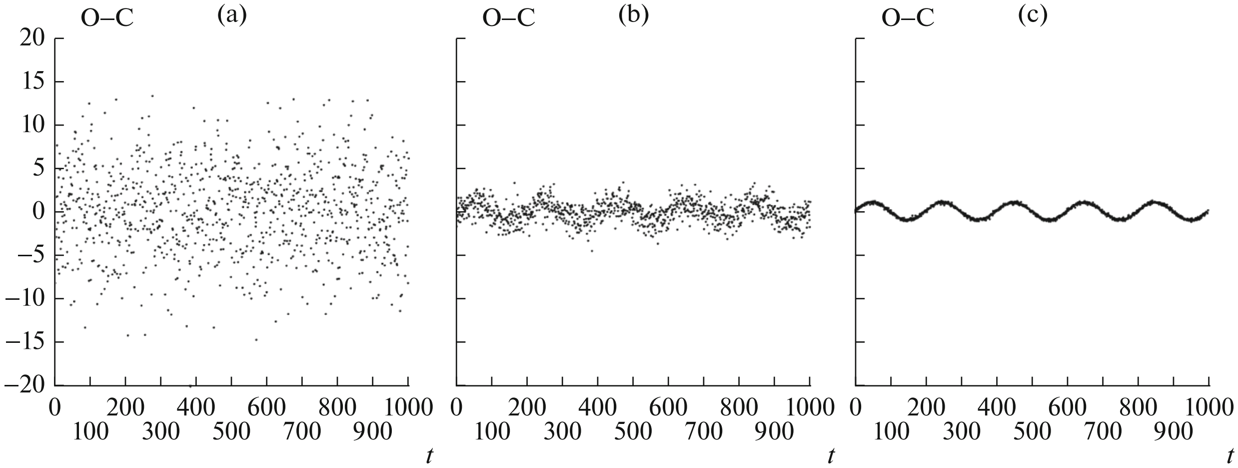

Fig. 1. Examples of O–C residuals of the orbital longitude of a celestial body at different levels of accuracy of observations.

middle of the 19th century, Urbain Le Verrier had used ephemerides to predict the existence of the then unknown planet Neptune, and new planets and moons are still being discovered this way. Therefore, one may conclude that ephemerides also serve as a research tool, since they incorporate all the available data on the motion of planets and moons.

When the accuracy of observations, which increases with time, reaches a certain level, certain new properties of a known celestial body or new planets or moons may be discovered. As an example, consider an accurate motion model, which was used to calculate the so-called O–C differences between the observed and the theoretical orbital longitude values. If measurements are inaccurate, a plot of these differences may look like the one shown in Fig. 1a, where noise is the only apparent component. Let us assume that progress in observational techniques provided an opportunity to improve the accuracy of observations and suppress noise. A certain pattern then emerges (Fig. 1b), and sinusoidal variation of O–C differences becomes clearly visible when observations get even more accurate (Fig. 1c). This "signal" helps determine those factors that were left unaccounted for by the theory.

The orbital motion of celestial bodies is distinctive in that the orbital longitude increases monotonically with time. If one removes the function of theoretical variation of orbital longitude from its observed values, a plot similar to the one in Fig. 2a may be obtained. Again, there is nothing interesting in it. If past and new observations of the celestial body under study are added to the data presented in Fig. 2a, the plot in Fig. 2b is obtained. It can be seen that the longitude varies almost quadratically in time. This effect may be induced by the unaccounted dissipation of the mechanical energy of a celestial body, which, in its turn, may be attributed, for example, to tidal forces. It is now clear that the observation time interval should also be expanded in order to make progress in this field. What is the relation between the observation time interval and the accuracy of ephemerides? Let us take a look at Fig. 3a. Here, the values of orbital longitude of a celestial body, which were derived from observations performed in time interval (t_1, t_2), are shown. "Noise" and linear variation are apparent. Using theory and observational data, we may calculate the probable orbital longitude at time moment t_f of interest to us (vertical lines in Fig. 3). If observations (with the same accuracy) are extended to time point t_3, the ephemeris becomes more accurate, which is seen in Fig. 3b.

The above-mentioned property of orbital motion of celestial bodies provides an opportunity to derive an approximate formula for the dependence of ephemeris accuracy Δ_λ on time intervals between the start (t_1) and the end (t_2) moments of observations and moment t_f for which the ephemeris is calculated. This formula is written as

$$\Delta_\lambda = \sigma_\lambda \frac{t_f - \dfrac{t_1 + t_2}{2}}{t_2 - t_1},$$

where σ_λ is the error of longitude determination from observations.

Naturally, it is not possible to measure the orbital longitude directly in the process of observations. One observes just the projection of orbital motion onto the image plane. However, the above approximate formula written for orbital longitude allows one to draw certain conclusions.

It follows from the above examples and analysis that one needs to raise the accuracy of observations in order to improve ephemerides. Regular observations should be continued (even if their accuracy remains unchanged). Naturally, novel observational techniques, which may provide new data on the motion of celestial bodies, should also be developed.

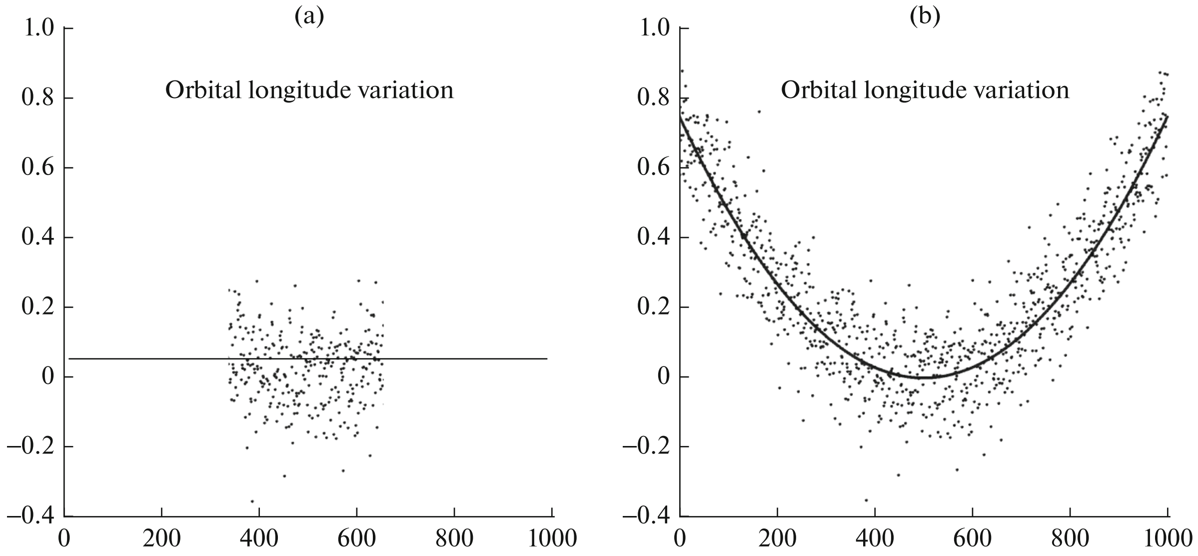

Fig. 2. Examples of O–C residuals of the orbital longitude of a celestial body at different time intervals.

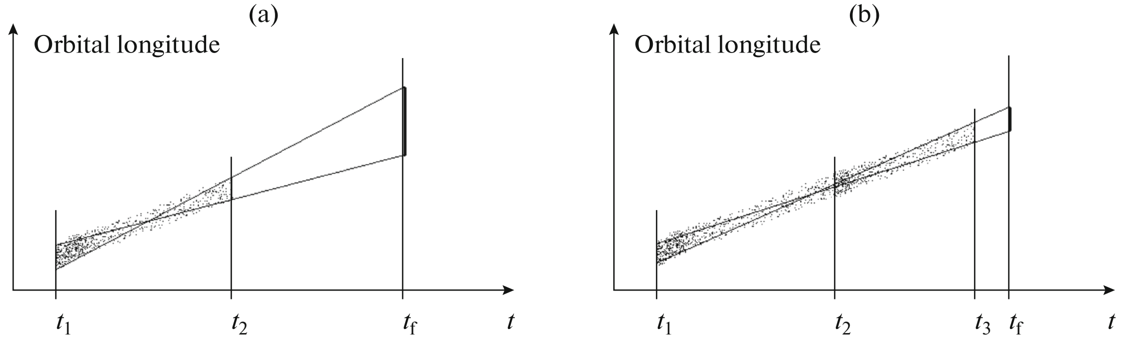

Fig. 3. Illustration of the dependence of accuracy of ephemerides of a celestial body on the observation time interval.

SEVERAL IMPORTANT ACHIEVEMENTS OF THE LAST 20 YEARS

The addition of new astrometric data expands the observation time interval so much that it becomes possible to obtain qualitatively new results. Let us consider several examples.

The Galilean moons of Jupiter have been observed regularly since their discovery by Galileo Galilei in 1610. A great number of astrometric measurements have been performed over a long time interval. The astrometric results of a worldwide campaign focused on photometric observations of Galilean moons of Jupiter during their mutual occultations and eclipses in 2003 have been added to the observation database (Emelyanov, 2009). A baffling acceleration of orbital motion of Io has been discovered in the end of the 20th century. The problem is that Jupiter rotates faster than the moon along its orbit. Planets and moons are viscoelastic bodies. The common model of tidal forces suggests that the angular velocity of Io should decrease, and the moon itself should move away from the planet. New high-accuracy astrometric data revealed the opposite effect in the orbital motion of Io: it moves toward Jupiter. The first explanation for this was proposed by Aksnes and Franklin (2001). It was hypothesized that Io moves in a spiral toward Jupiter, and the energy lost due to internal dissipation is higher than that gained from Jupiter tides. Using the observations of Galilean moons performed through to 2007, Lainey et al. (2009) have proven that the orbital motion of Io accelerates, while that of Europa and Ganymede decelerates. It has also been proven that the orbital energy lost by Io due to tides in its body, which are caused by the gravitation of Jupiter and the resonance interaction of moons (Laplace resonance), is higher than the energy gained from the tidal influence of rotating Jupiter. The parameters of viscosity of Jupiter and Io have also been determined by Lainey et al. (2009).

A new model of motion of the major moons of Saturn was developed in (Lainey et al., 2012). The observation database supplemented with the astrometric results of a worldwide campaign focused on photometric observations of moons of Saturn during their mutual occultations and eclipses in 2007 (Arlot et al., 2012) was used. As a result, a new value of the viscosity parameter of Saturn was found. This value is approximately ten times larger than the common one that was determined based one the physical model of the planet. In addition, an unexpectedly high secular acceleration of Mimas was found.

Models of motion of the major moons of Uranus have been developed since the time of their discovery (i.e., since 1787). The most comprehensive analysis was undertaken by Laskar and Jacobson (1987). All observations of moons performed from 1911 to 1986 were used. This study is unique in that an analytical theory was developed using the classical Laplace–Lagrange secular perturbation method. This model has remained the most accurate one for 20 years; a more precise model of motion of the major moons of Uranus has been developed only in 2007 (Rush and Jacobson, 2007) based on numerical integration of the equations of motion. The observation time interval was extended to 1911–2006. Another, more recent model of motion of the major moons of Uranus (Emelyanov and Nikonchuk, 2013) makes use of all observations performed from 1787 to 2008 and the astrometric results of a worldwide campaign focused on photometric observations of moons of Uranus during their mutual occultations and eclipses in 2007 (Arlot et al., 2013). The extension of the observation time interval has resulted in the determination of coefficients of the quadratic (in time) perturbations in the orbital longitudes of moons. These coefficients turned out to be negative. This suggests that the mechanical energy of orbital motion is gained under the tidal influence of Uranus, which rotates faster than the moons along their orbits. The theory developed earlier in (Rush and Jacobson, 2007) was refined further in (Jacobson, 2014) by adding a considerable number of new observations and extending the observation time interval to 2013. Quite a few researchers have made attempts at constructing a model of the motion of Triton (the largest moon of Neptune). The authors of the most recent study on that subject (Emelyanov and Samorodov, 2015) have also tried to determine the coefficient of the quadratic (in time) perturbations in the longitude of Triton. The obtained value was negative, and its modulus was just a little larger than its error. This negative value may indicate the presence of the tidal influence of Neptune, which rotates faster than Triton along its orbit.

The above examples suggest that progress in the development of models of motion of planetary moons was always made by adding new observations and extending the observation time interval. These advancements resulted not only in the enhancement of accuracy of ephemerides, but also in the acquisition of new data on the physical properties of planets and moons.

EPHEMERIS SERVICES FOR PLANETS, ASTEROIDS, AND MOONS

The availability of high-performance computers and access to the Internet have transformed the development and use of planetary and moon ephemerides. The very concept of ephemerides has changed: it is now equivalent to a model or a theory of motion of a celestial body. When developing a theory or a model of motion and the ephemerides themselves, one has to decide where to publish them and how to provide access to them. Previously, ephemerides were published in astronomical almanacs, and the researchers using them had to look through the almanac to find the needed data. It is evident that these methods have now become obsolete and are ineffective in current research.

The development of theories of motion and ephemerides of celestial bodies based on observations is a very labor-intensive process and may thus be carried out only at specialized research centers. The end product (the means to calculate and access ephemerides) now has two components: a large numerical data file and a computational program, which reads out the data, calculates ephemerides requested by the user, and returns the result. The process may be organized in several ways. The user may upload the data file to the work computer and insert the corresponding computational program modules into the program used to solve the current problem. While running, the latter program requests the needed data from the ephemeris calculation program, acquires them, and proceeds with calculations. Alternatively, the user may operate the ephemeris calculation program, which runs on a separate computer, via the Internet using a browser application. The user sends a request via the Internet to the computer hosting the ephemeris calculation program, and the calculation results (ephemerides) are displayed on the monitor of the work computer of the user. Advanced users may modify problem-solving programs so that they send requests to the ephemeris calculation program automatically and receive the needed data via the Internet in the process of calculations.

Such research tools are called ephemeris services. Let us name the major ones that are the most accurate and are provided by those research centers where models of motion of celestial bodies are constructed based directly on observations.

The most flexible and well-developed service is the one provided by the US-based Jet Propulsion Laboratory (JPL; Acton et al., 2015). Two research tools are offered by JPL. The first one is the SPICE system: a large set of data and computational programs for calculating the ephemerides of almost all known Solar System bodies and the coordinates of spacecraft at a given time point. Different versions of planetary ephemerides (DE, EPM, INPOP) may be used. The data are output as binary or text files in the ASCII format. Programs are provided both in the form of libraries and in the form of source codes written in the most common programming languages (C, Fortran, etc.). SPICE may be accessed at http://naif.jpl.nasa.gov/naif/. The second JPL research tool is the HORIZONS ephemeris server (Giorgini et al., 1996), which provides web access to ephemerides. The address of the ephemeris request page is as follows: http://ssd.jpl.nasa.gov/horizons.cgi.

The Institut de Mécanique Céleste et de Calcul des Ephémérides (IMCCE; Paris, France) is another research center that provides ephemeris services and is involved in constructing new models of celestial bodies based on observations. Models of planetary motion are being developed at IMCCE by a specialized research team (Fienga et al., 2011), and new models of motion of planetary moons are being developed by V. Lainey in collaboration with his colleagues (Lainey et al., 2007, 2009, 2012; Lainey, 2008). The MULTI-SAT planetary moon ephemerides server (Emel'yanov and Arlot, 2008) may now be accessed at the IMCCE site (http://www.imcce.fr/sat/). This server is supported by IMCCE and Sternberg Astronomical Institute (SAI) of Moscow State University. A copy of the MULTI-SAT server is found at the SAI site (http://www.sai.msu.ru/neb/nss/index.htm). The MULTI-SAT ephemeris server has several specific features. In addition to the ephemerides of all known moons of planets (Pluto included), one may calculate the ephemerides of Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto.

Several versions of planetary ephemerides developed at JPL, IMCCE, and the Institute of Applied Astronomy (IPA; St. Petersburg, Russia) are provided. In addition to calculating the coordinates of moons and planets at a given time point, the user may upload observational data to the server and determine the differences between observed and ephemeris coordinate values. The MULTI-SAT server pages are translated into Russian, English, and French. The models of motion and ephemerides of the majority of planetary moons were developed at SAI (Emelyanov, 2005; Emel'yanov and Kanter, 2005; Emelyanov and Nikonchuk, 2013; Emelyanov and Samorodov, 2015).

Models of motion of all asteroids and comets are developed based on observations at the Minor Planet Center (MPC, United States). The MPC site provides access not only to the ephemerides of all known asteroids and comets, but also to the complete database of observations and orbital parameters. The ephemerides of distant moons of major planets are also available. The MPC site is located at http://minorplanetcenter.net/iau/mpc.html.

Models of motion of major planets and the Moon were developed at IPA independently from other research groups. The work on accurate ephemerides of planets and the Moon (EPM), which were intended to be a support for Russian deep-space experiments, commenced in the 1970s (first at the Institute of Theoretical Astronomy, and then at IPA). High-accuracy series of ephemerides have been developed (see, for example, Pitjeva, 2013). These ephemerides may be accessed at ftp://quasar.ipa.nw.ru/incoming/EPM/. Chebyshev expansions of ephemerides in text, binary, and SPICE (JPL) formats are provided. The researchers at IPA have also developed their own models of the motion of the major moons of planets (Kosmodamianskii, 2009; Poroshina, 2013). The ephemerides calculated based on these models are available at http://ephemeris.ipa.nw.ru/.

Several other services for calculating the ephemerides of Solar System bodies based on the above-mentioned theories and models are also available on the Internet. We mention here only the so-called virtual observatories, a number of which are now online. They are generally used to visualize certain areas of the starry sky and see how they should appear in observations. Some planetary moons and asteroids are also visualized in certain virtual observatories. One such observatory is found at http://vo.imcce.fr/ (Erard et al., 2015).

RETRIEVING ASTROMETRIC DATA ON THE MOTION OF MAJOR MOONS OF JUPITER, SATURN, AND URANUS FROM PHOTOMETRIC OBSERVATIONS PERFORMED DURING THEIR MUTUAL OCCULTATIONS AND ECLIPSES

Description of Phenomena

The technique of observation of celestial bodies is being improved constantly. Much depends on the current technology level. At the same time, astronomers, with their unique expertise, search for new and improved observation methods. It was found at some point of research into planetary moons that the Galilean moons of Jupiter move along orbits located approximately in a single plane, which is aligned with the equatorial plane of Jupiter, but is inclined to its orbital plane. The Jupiter–Earth line falls within this plane twice in a single period of revolution of the planet about the Sun. At approximately the same epochs, the Jupiter–Sun line goes through the plane of orbits. At certain time points, the images of disks of some pairs of moons, which are observed from the Earth, overlap. At the same epochs, the shadow of one moon occasionally falls onto the other one, and this shadow is visible from the Earth. In both cases, the total brightness of the moons decreases temporarily. This reduction in luminous flux may be measured even if we do not distinguish the images of the disks of

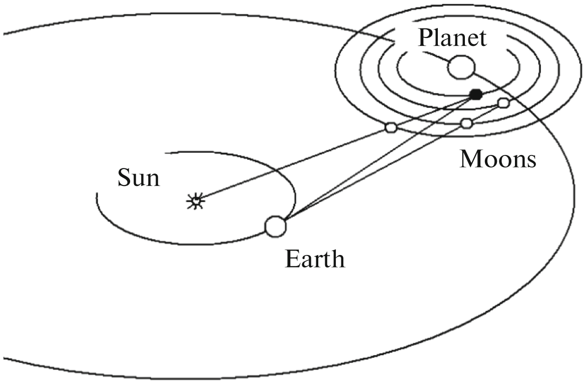

Fig. 4. Heliocentric orbits of the Earth and a planet and the orbits of its moons. This configuration enables mutual occultations and eclipses of moons.

moons. The so-called mutual occultations and eclipses of planetary moons thus occur. These phenomena may also be observed for the major moons of Saturn and Uranus. The Sun–planet–moons configuration established in such mutual phenomena is shown in Fig. 4. The length of the interval of reduced brightness of moons generally varies from 4 to 15 min. The 6- to 9-month-long periods of these phenomena are repeated in half a revolution of the planet about the Sun. Mutual occultations and eclipses of the major moons have already occurred and will occur in 1997, 2003, 2009, 2015, 2021, (Jupiter); in 1995, 2009, 2023 (Saturn); and in 1965, 2007, 2049, (Uranus). A total of 1–10 events are observed in a week. Each event may be observed simultaneously by only 30% of observatories on the Earth.

It is evident that the luminous flux from moons during the event depends on their apparent relative positioning, which is defined by relative coordinates X, Y in the plane of the event. This is demonstrated in Figs. 5 and 6, where normalized luminous flux S is plotted on the ordinate. The event plane is the plane passing through the occulted or eclipsed moon perpendicularly to the line that goes through the moon and the observation point (in the case of mutual occultations) or through the moon and the center of the Sun (in the case of mutual eclipses). The Y axis is directed toward the celestial north pole, and the X axis is directed to the east. The luminous flux of moons is measured at certain time points in the process of their photometric observation. The measured light curve is thus obtained (see Fig. 7).

Since the reduction in luminous flux depends on the coordinates of moons, one may also solve the reverse problem: retrieve astrometric data from the light curves of moons measured during their mutual occultations and eclipses.

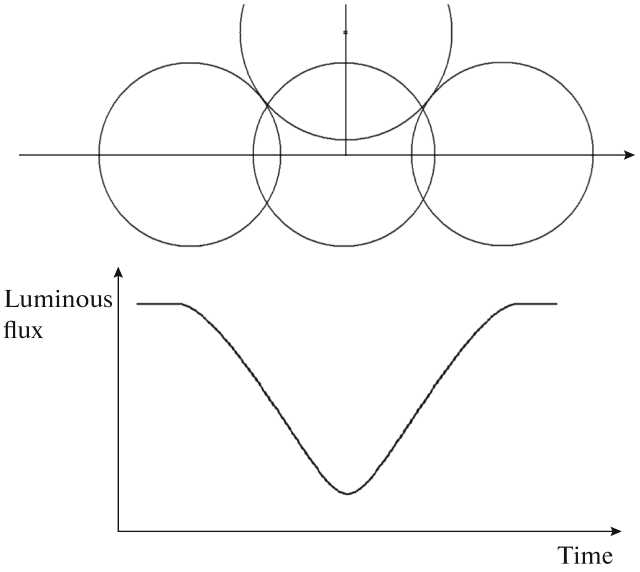

Fig. 5. Three different apparent positions of disks of moons during mutual occultation and the corresponding light curve.

Astrometric Data Acquisition Method

The method used to derive astrometric data from the results of photometry of mutual occultations and eclipses of planetary moons had been proposed in the 1970s in (Aksnes and Franklin, 1976; Aksnes et al., 1984) and developed further in (Vasundhara, 1994; Noyelles et al., 2003). The model of events was simplified somewhat in this method. Another method for processing photometric observational data and retrieving astrometric results was developed in (Emelianov, 2003; Emelyanov and Gilbert, 2006). Let us discuss the basic principles of this method.

A measured value of luminous flux E on a certain scale, which is fixed for each individual event, is obtained in the process of photometry of moons. This scale is not known before the measurement, nor do we need to know the absolute flux value. This value may be measured up to any undetermined multiplier. Let us denote a certain normalized luminous flux of moons as S. We assume that the value of S equals unity before the event starts and after it ends. During a mutual occultation or eclipse, the luminous flux value decreases; i.e., $S < 1$. Then,

$$E = KS, \tag{1}$$

where K is a certain indefinite coefficient, which is assumed to remain constant during the event. It is evident that S depends on relative coordinates X, Y in the system described above, and we define function $S(X, Y)$. Coordinates X, Y may be calculated at any time point t using the ephemerides of a planet and its moons. Let us denote these ephemeris values as $X_{\text{th}}(t)$, $Y_{\text{th}}(t)$. It is not possible to obtain the actual value of luminous flux E by inserting $X_{\text{th}}(t)$, $Y_{\text{th}}(t)$ into function $S(X, Y)$

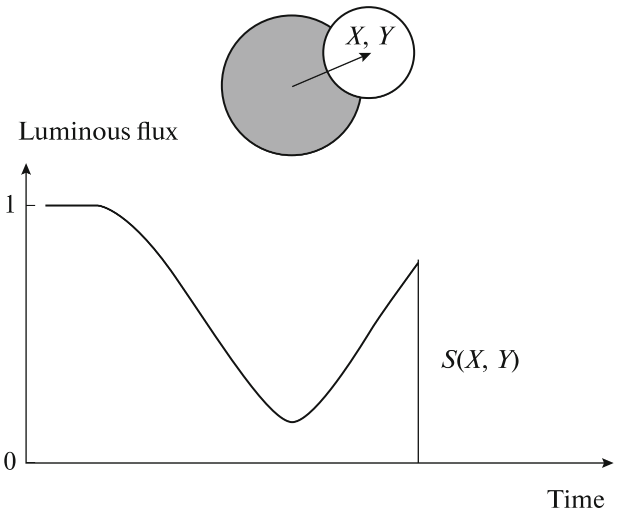

Fig. 6. Dependence of the normalized luminous flux of a pair of moons during mutual occultation on the apparent distance between the centers of disks.

and inserting this function into relation (1), since the ephemerides have a certain error. Let us assume that the actual coordinates during the event differ from the ephemeris ones by certain constants D_x, D_y. Thus, the actual luminous flux is defined as

$$E = KS\left(X_{th}(t) + D_x,\ Y_{th}(t) + D_y\right).$$

Let us assume that photometric observations have been performed (i.e., measured values E_i have been obtained at time points t_i ($i = 1, 2, ..., m$)). The following system of conditional equations in unknown parameters K, D_x, D_y may then be written:

$$E_i = KS\left(X_{th}(t_i) + D_x,\ Y_{th}(t_i) + D_y\right),\ i = 1, 2, ..., m.$$

We linearize function S with respect to its arguments and solve the system of linear conditional equations using the least squares method. After the solution is found, the astrometric result is expressed as coordinates $X(t^*) = X_{\rm th}(t^*) + D_x$, $Y(t^*) = Y_{\rm th}(t^*) + D_y$, where t^* is an arbitrary time point within the event time interval. For definiteness, we choose the time point when $X^2 + Y^2$ is minimized (i.e., when the apparent distance between moons is minimized).

It is natural to assume that when light from the moons is lacking, the measured value of E should equal zero. To this end, the sky background and all instrumental light fluxes are removed in photometric processing of observations. Naturally, it is not possible to remove them completely in actual studies, and a certain background level P remains. Therefore, one should solve conditional equations

$$E_i = KS\left(X_{th}(t_i) + D_x,\ Y_{th}(t_i) + D_y\right) + P$$
$$i = 1, 2, ..., m,$$

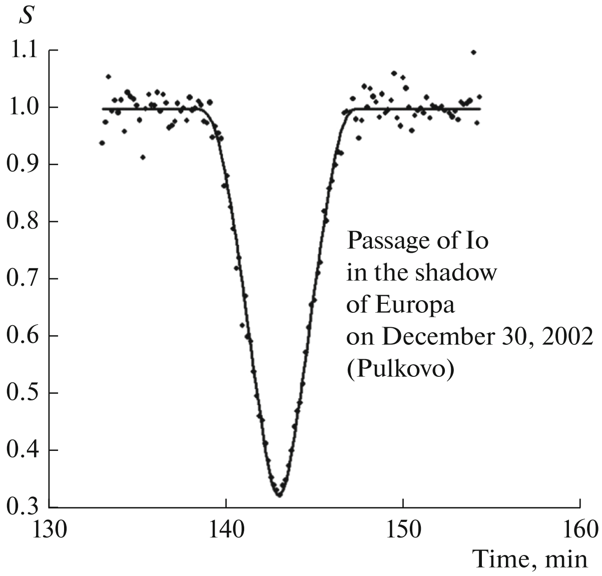

Fig. 7. Variation of the measured normalized luminous flux of Io eclipsed by another moon and the corresponding model curve after correction of the model parameters.

and include P among the parameters to be defined. However, such extended equations are solvable only in rare special cases of mutual apparent motion of moons.

In order to implement this method, one should know how to calculate $S(X, Y)$ as a function of its arguments. Luminous fluxes from each point of the moon are summed in the photoreceiver. Each point on the surface of the moon has its own scattering properties, and the direction of incidence of solar light and the direction of propagation of reflected light toward the observer (relative to the surface) differ from one point to the other. Naturally, no light comes from points covered by the occulted moon. The mutual positioning of moons and the observer, which is determined from the model of motion, defines whether a certain point is occulted. In the case of mutual eclipses of moons, the luminous flux falling on each point of the eclipsed moon is the sum of fluxes from each point of the solar disk section that is not covered by the eclipsing moon. One should also take into account the effect of solar limb darkening. If a telescope could resolve the disks of moons, the observer would see a partially occulted or darkened disk of the moon with a nonuniform brightness and an unilluminated limb, since the Sun illuminates the moon somewhat from the side. In actual observations, the occulted moon and the occulting one produce a single spot at the photoreceiver. Their combined luminous flux is measured. The light sensitivity of any photoreceiver varies with wavelength. Therefore, one should take into account the dependence of light scattering on the wavelength and the properties of the optical filter.

In practice, we split the hemisphere of the moon facing the Earth into finite elements, calculate the incident flux from each element individually, and integrate all these fluxes. A certain law of light scattering by a point on the moon surface may be used at this stage. A number of parameters governing the reflective properties of the surface of a specific moon should be determined. One of these parameters is the albedo that is distributed over the moon surface and is sensitive to surface features.

Several parameters are considered here. Taken together, these factors form a photometric model of the event. One of the most accurate photometric models of mutual occultations and eclipses of the Galilean moons of Jupiter was characterized in (Emelyanov, 2003; Emelyanov and Gilbert, 2006). Similar models for the major moons of Saturn and Uranus were presented in (Arlot et al., 2012, 2013). A simplified version of such a model is discussed below.

Hindrances to Raising the Accuracy of Astrometric Results

Several major issues regarding the processing of photometric observations of mutual phenomena with the object of retrieving accurate astrometric data still remain unsolved.

Two sources of errors of the resulting astrometric coordinates of moons are present: random photometry errors and inaccuracies of the photometric model. Error analysis reveals that the errors caused by inaccuracies of the model are 3–4 times larger than those attributable to random photometry inaccuracies.

In order to clarify the reasons behind this, we consider a simplified photometric model of mutual occultation of moons. Let us assume that uniform disks of moons are observed from the Earth, and their integral albedos are known. The disk of moon no. 1 occults completely or partially the disk of moon no. 2. Let us denote the unocculted fraction of the disk of the occulted moon as k_2. Naturally, k_2 depends on distance d between the disk centers. This is illustrated by Fig. 8. Function $k_2(d)$, the process of calculation of which is detailed in (Emel'yanov, 1995) is considered below.

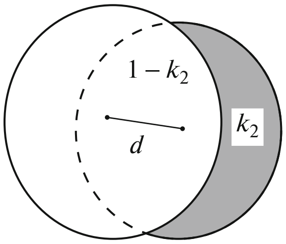

Fig. 8. Mutual occultation of moons. The unocculted section of the occulted moon is grayed out. The ratio of the area of this section to the entire area of the disk is k_2.

Luminous fluxes $Rr_1^2p_1$ and $Rr_2^2p_2k_2$ correspond to the occulting moon and the occulted one, respectively. Here, r_1 and r_2 are the disk radii, p_1 and p_2 are the albedos of moons, and R is an undetermined coefficient. The total luminous flux of both moons is measured in the case of mutual occultation. Under these conditions, normalized flux S may be written as

$$S = \frac{p_1r_1^2 + p_2r_2^2k_2(d)}{p_1r_1^2 + p_2r_2^2} = \frac{1 + \dfrac{p_2r_2^2k_2(d)}{p_1r_1^2}}{1 + \dfrac{p_2r_2^2}{p_1r_1^2}}.$$

If there is no occultation, the complete occulted moon is visible, and we have $k_2 = 1$, $S = 1$, and $E = K$.

Total occultations occur sometimes. Figure 9 shows the configuration of moons in this case. It is evident that the occulted moon is completely invisible in time interval (t_1, t_2), and $k_2 = 0$. In this interval,

$$S = \frac{1}{1 + \dfrac{p_2r_2^2}{p_1r_1^2}};$$

i.e., the luminous flux does not depend on distance d.

Problems arise due to the fact that the observed luminous flux during total occultation often differs from the calculated one; i.e.,

$$E_{\text{observed}} \neq K\frac{1}{1 + \dfrac{p_2r_2^2}{p_1r_1^2}},$$

where K is the flux in the case of no occultation. Figures 10–12 present three such examples (the database identifiers of observations are indicated below). Here, the values of E_{observed}/K, which were derived from the measured flux values (dots), and model variations of S are shown. It can be seen that a certain negative additional flux is present in the values measured during total occultation.

The model may be corrected in two ways. One may set

$$E_{\text{observed}} = K\frac{1}{1 + \dfrac{p_2r_2^2}{p_1r_1^2}} + P,$$

where P is the parasitic luminous flux from the unaccounted background. Alternatively, one may set

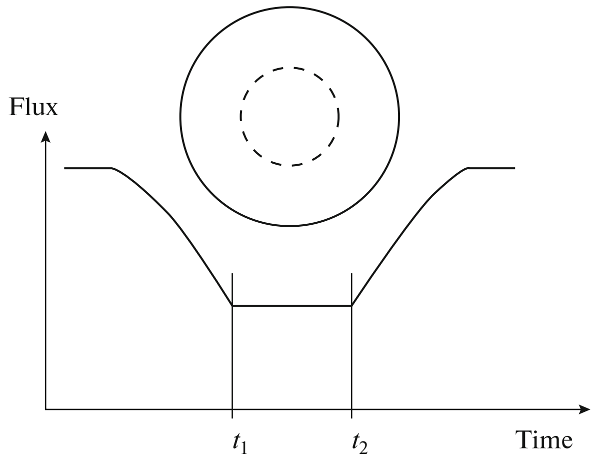

Fig. 9. Total occultation occurs in time interval (t_1, t_2). The corresponding section of a curve representing the combined normalized flux of a pair of moons is shown.

$$E_{\text{observed}} = K\frac{1}{1 + m\frac{p_2r_2^2}{p_1r_1^2}},$$

where *m* is a certain additional factor. This factor corrects the inaccuracy arising due to the fact that the exact ratio of albedos of the moons is not known. It becomes unclear which of the methods to choose. The equivalence of these methods leads to the following equality:

$$K\frac{1}{1 + \frac{p_2r_2^2}{p_1r_1^2}} + P = K\frac{1}{1 + m\frac{p_2r_2^2}{p_1r_1^2}}.$$

Since the parasitic flux in observations turns out to be negative in most cases, it is fair to assume that it is actually present and is not associated with inaccuracies in the values of the albedo of moons.

When processing the results of observations of partial mutual occultations of moons, one is unaware of the presence of a parasitic background in observations and does not know of the inaccuracy of adopted albedo values. As a result, an artificial correction Δ to the apparent distance between moons needs to be introduced in order to bring the model into agreement with observations. This leads to systematic errors in astrometric results and is illustrated by the following relation:

$$E_{\text{observed}} = K\frac{1 + \frac{p_2r_2^2k_2(d+\Delta)}{p_1r_1^2}}{1 + \frac{p_2r_2^2}{p_1r_1^2}}.$$

The photometric model of the event was simplified for clarity in the present analysis. However, the problem persists when observations are processed using the complete model characterized in (Emelyanov, 2003; Emelyanov and Gilbert, 2006). The same problem arises when the observations of mutual eclipses of moons are processed.

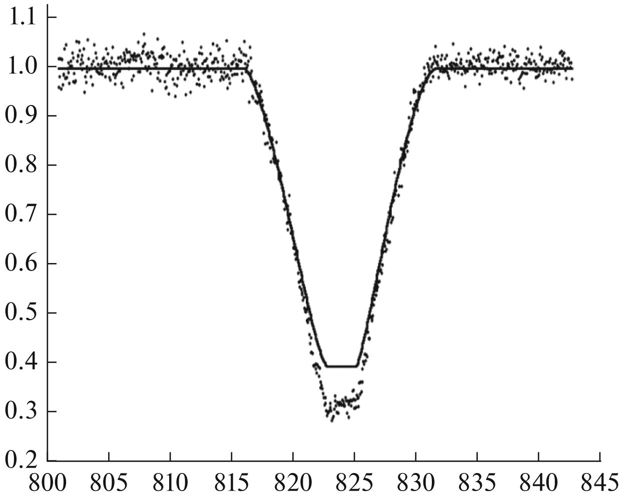

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Fig. 10. Measured normalized luminous flux of Io during its total occultation by another moon and the corresponding model curve after correction of the model parameters. The event was observed on November 2, 2014. Time (min) elapsed from the start of the day is plotted on the abscissa. The negative background level in the measured flux is apparent.

We are forced to look for the sources of the above-mentioned errors. Parasitic background at the photoreceiver may be produced by the background sky, the scattering of light in the telescope and in the camera, or by the photoreceiver itself. A certain specialized photometric processing method is used to calculate luminous fluxes of moons based on their CCD images. The error of this method may give rise to a certain background. Since this is the only source able to produce a negative background level, it is the most probable cause: the source of the parasitic background should be concealed within the photometric processing method. We are not going to review the available methods for photometric image processing in the present study. Instead, we list only the designations of methods found in the support documentation for IMCCE data: Source Extractor, DAOPHOT(IDL), Audela, Tangra, and LiMovie. Different proprietary specialized methods are used in different observatories. It is evident that these methods need to be examined closely in order to reveal the sources of systematic errors.

It can be seen from the above simplified photometric model that the normalized luminous flux of moons depends on the ratio of integral albedos. This dependence is retained in more accurate photometric models. In our studies involving the processing of observa-

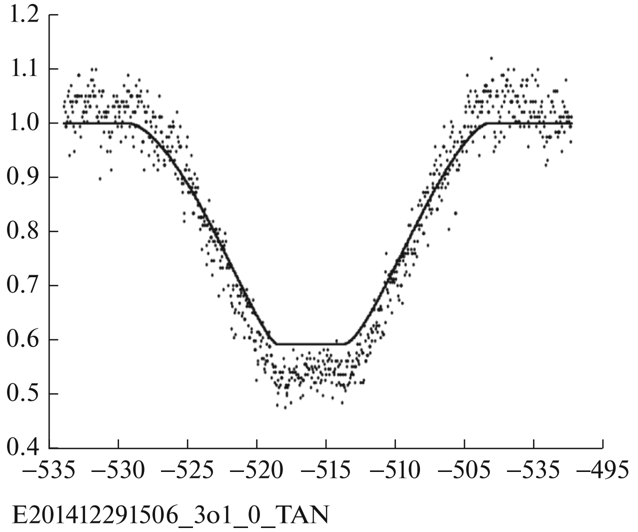

Fig. 11. Measured normalized luminous flux of Io during its total occultation by another moon and the corresponding model curve after correction of the model parameters. The event was observed on December 28, 2014. Time (min) counted from the start of the following day is plotted on the abscissa. The negative background level in the measured flux is apparent.

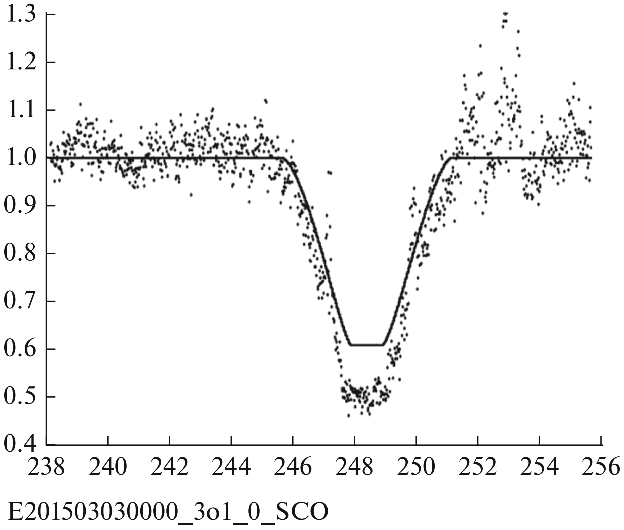

Fig. 12. Measured normalized luminous flux of Io during its total occultation by another moon and the corresponding model curve after correction of the model parameters. The event was observed on March 3, 2015. Time (min) elapsed from the start of the day is plotted on the abscissa. The negative background level in the measured flux is apparent.

tions of the Galilean moons of Jupiter (Emelyanov, 2009; Arlot et al., 2012), the photometric properties of moons were averaged over the visible disk, but variations of the integral albedo with the angle of rotation of the moons were considered. In simple words, the luminous flux of the moon depended on which side was facing the observer. The data on integral albedo variations were taken from (Morrison, D and Morrison, N.D., 1977; Prokof'eva-Mikhailovskaya et al., 2010; Abramenko et al., 2011). The accuracy of these data is not sufficient, and they may serve as the source of errors in the photometric model of mutual occultations and eclipses of the Galilean moons of Jupiter. The need to conduct extensive photometric observations of the Galilean moons of Jupiter at various angles of rotation of moons (from 0° to 360°) in different spectral bands is evident.

Worldwide Campaigns Focused on Observations of Planetary Moons during Their Mutual Occultations and Eclipses

Approximately 400 events occur over 9–14 months in each epoch of mutual occultations and eclipses of moons of Jupiter, Saturn, and Uranus. The duration of each event, which may be observed only from a small part of the Earth facing the corresponding planet, is 5–15 min. Worldwide photometric campaigns are organized in order to observe as many events as possible. Beginning in 1985, these campaigns have been coordinated by Jean-Eudes Arlot (IMCCE, Paris). All the photometric results obtained in a campaign are uploaded to a single database and, after a while, are subjected to astrometric processing. The relative coordinates of moons constitute a database of astrometric results of an observational campaign. Studies focused on characterizing the light curves of moons and detailing the final astrometric results are published 2–3 years after observations. All the observers involved are listed as coauthors of these papers. In certain cases, astrometric results have been published separately. The astrometric processing is usually performed by one of the researchers, who uses his/her own techniques. Table 1 lists the characteristics of worldwide campaigns focused on observations of mutual occultations and eclipses of moons of Jupiter, Saturn, and Uranus.

CHARACTERISTICS OF ASTROMETRIC OBSERVATIONS AND ASTROMETRIC RESULTS OF PHOTOMETRY OF MUTUAL EVENTS OF PLANETARY MOONS

The technique of astrometric observations of planetary moons has been perfected over centuries. Prior to the 20th century, the observers looking in the eyepiece of a telescope saw, in addition to the images of stars, crosshairs and a thread that could be moved using a micrometer screw mechanism. The observer had to rotate the crosshairs and align one celestial body with the center of the reticle, while the other body was to be located at the point of intersection between the thread and the reticle. Angular distance s between two celestial bodies and position angle P with its vertex at one of the bodies and its arms directed at

Table 1. Characteristics of worldwide campaigns focused on observations of mutual occultations and eclipses of moons of Jupiter, Saturn, and Uranus

Planet with a moon system and years of observations	Number of obtained light curves of moons	Number of participating observatories	Author of the method and astrometric processing, citation (in brackets)
Jupiter, 1973	46	18	K. Aksnes (Aksnes et al., 1984)
Jupiter, 1979	19	11	K. Aksnes (Aksnes et al., 1984)
Saturn, 1979–1980	14	6	K. Aksnes (Aksnes et al., 1984)
Jupiter, 1985	166	28	J.-E. Arlot (Arlot et al., 1992)
Jupiter, 1991	374	56	J.-E. Arlot (Arlot et al., 1997)
Saturn, 1995	66	16	B. Noyelles (Noyelles et al., 2003)
Jupiter, 1997	292	42	R. Vasundhara (Vasundhara et al., 2003; Arlot et al., 2006) N.V. Emel'yanov and S.N. Vashkov'yak (Emel'yanov and Vashkov'yak, 2009)
Jupiter, 2002–2003	377	42	N.V. Emel'yanov (Emelyanov, 2009; Arlot et al., 2009)
Uranus, 2007	41	19	N.V. Emel'yanov (Arlot et al., 2013)
Jupiter, 2009	457	74	N.V. Emel'yanov and M.I. Varfolomeev (Arlot et al., 2014)
Saturn, 2009	33	17	N.V. Emel'yanov (Arlot et al., 2012)
Jupiter, 2015	607	75	N.V. Emel'yanov (publication is being prepared)

the other body and the celestial north pole were then measured. In most cases, the values of s and P were measured at different time points. Such observations are called micrometric ones in the literature.

In the 20th century, astronomers spent much of their time looking into the eyepiece of a microscope and studying the images of celestial bodies on photographic plates. These are photographic observations. Relative coordinates were first measured in millimeters using a microscope and were then converted into angular values. The coordinates of planets and moons were measured relative to the stars, the equatorial coordinates of which were taken from star catalogues. These data were used to derive absolute equatorial coordinates of planets and moons. The errors in stellar coordinates from star catalogues were translated directly into absolute coordinates of the observed celestial bodies. Relative angular coordinates (differences in angular coordinates of two moons) may be used to refine the models of motion of moons. Based on these data, the parameters of orbits of both bodies are refined simultaneously in a single system of equations. Relative measurements are free from errors inherent in star catalogues; however, these data are also inaccurate due to the uncertainty in the scale and orientation of images.

At the end of the 20th century, photographic plates were replaced by CCD image sensors. These photoreceivers turned out to be more sensitive and provided better image resolution than photographic plates, but the factors limiting the accuracy of observations remained the same. High-performance computers have simplified the processing of results. Thus, astronomers of the 21st century spend more time with a computer than with a telescope. These are CCD observations.

Semiconductor CCD sensors provided an opportunity to implement a special observation technique called speckle interferometry. Each incoming photon is detected separately by a photoreceiver. If a single moon or a single asteroid is observed, there is not much use in this detection. However, a close pair of bodies produces a wealth of pairs of dots from incident photons, and each pair is shifted due to deflection in constantly moving layers of the terrestrial atmosphere. Using a special mathematical technique, one may find a matching dot (from the other body) for each dot and align the images. The coordinates of the first body relative to the second one derived this way are much more accurate than those found in common observations. One drawback of this method is that celestial bodies need to be very close to each other and should have approximately equal brightness values. Speckle-interferometric measurements are fairly efficient in observations of binary asteroids.

Naturally, spacecraft-based observations of moons provide much more accurate positioning data than Earth-based observations, since the observer–target

Table 2. Estimates of the internal astrometric accuracy of different types of observations

Observation type	Internal accuracy, arcsec
Micrometric	0.3–3.0
Photographic	0.08–0.8
CCD	0.04–0.4
Speckle-interferometric	0.01–0.05
Photometry of mutual events, Galilean moons of Jupiter	0.005–0.05
Photometry of mutual events, major moons of Saturn and Uranus	0.003–0.006

distance is shorter. However, when spacecraft-based and terrestrial observations are used in combination, the contribution of the former observations to the increase in ephemeris accuracy does not always turn out to be significant. Spacecraft-based observations are especially ineffective if the motion of moons needs to be tracked over long time intervals in order to reveal the dissipation of mechanical orbital energy.

The exceptionally high accuracy of spacecraft-based radio observations of moons should be noted. In such observations, the distance and the radial velocity of a celestial body are measured relative to the spacecraft, while the coordinates of this spacecraft are determined with a high accuracy relative to the Earth using trajectory data provided by NASA Deep Space Network (DSN) radio telescopes and very-long-baseline interferometry (VLBI).

Note that it is not possible to conduct regular spacecraft-based observations of planetary moons over long time intervals. The necessity of such observations was demonstrated above.

Earth-based astrometric observations of celestial bodies have recently progressed in two promising directions. First, new and considerably more accurate star catalogues, which are based on the data of extensive observations of stars made by orbital telescopes, have been compiled. Second, special electronic devices designed to scan old photographic plates and create their digital images have been constructed. As a result, several major research centers have initiated projects focused on scanning photographic plates with images of planets and moons in order to process these images once again using new star catalogues. Consequently, old results of absolute observations are replaced by new and much more accurate versions. Examples of this may be found in (Lainey et al., 2014; Kiseleva et al., 2015).

Observations used to construct planetary ephemerides include different kinds of positional measurements ranging from classical meridian observations to modern radio observations. A revolution in dynamical astronomy began in 1961, when the first radar observations of Venus were performed in the United States, USSR, and England. The importance of radar observations rests on two factors. First, two new types of measurements were added: the measurement of delay time, which is related to distance by the speed of light, and the measurement of Doppler frequency shift, which gives the relative radial velocity of the reflection surface. Second, the accuracy of radar measurements is five orders of magnitude higher than that of classical optical measurements. However, only the terrestrial planets and Saturn (Cassini spacecraft) are fully covered by radio observations. The amount of such data for Jupiter is much lower, and only a single position vector value (provided by Voyager-2) is available for Uranus and Neptune. Therefore, terrestrial observations retain their significance in the case of outer planets. It should be noted that ranging observations of surfaces of planets have already been superseded by more accurate trajectory observations.

Astrometric results of observations of the major moons of giant planets made during their mutual occultations and eclipses produce a substantial contribution to the aggregate observational database and help maintain the regularity of observations over long time intervals.

Let us perform a comparative analysis of accuracy of various types of positional observations of planetary moons. First, it should be noted that the accuracy is estimated in two ways. The so-called normal places are created while processing positional observations. The internal observation error, which is induced primarily by random measurement errors, manifests itself in the process. In certain cases, the least squares method, which provides an estimate of the accuracy of the astrometric result, is used in preprocessing. This applies fully to the astrometric processing of photometric observations of planetary moons during their mutual occultations and eclipses. An entirely different error estimate is obtained by comparing the measured coordinates of a celestial body with the values calculated based on the available model of motion or theory. The differences between them are denoted as O–C. These differences incorporate both observational errors and errors in theory. Observational errors generally dominate the O–C differences. Systematic errors, which arise while processing observations, may manifest themselves here. This is the so-called external estimate of observational error. The external estimate is often several times larger than the internal one. Approximate estimates of the internal accuracy of dif-

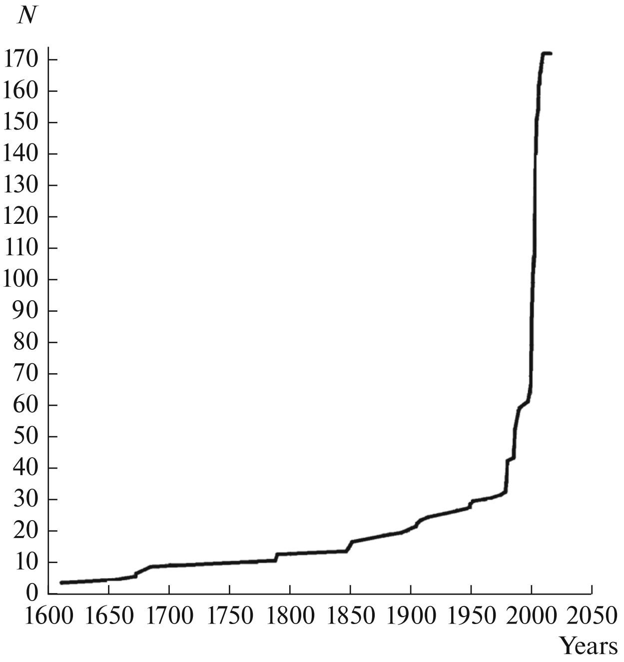

Fig. 13. Variation of number *N* of known planetary moons with time.

ferent types of terrestrial observations are listed in Table 2. The intervals given in this table are quite wide due to the presence of various sets of observations, which differ in accuracy, but remain useful nonetheless. The errors of micrometric measurements of s and P are converted in Table 2 into the errors of rectangular celestial coordinates.

When one tries to refine the model of motion of a planet or a moon, the first question is where to get the needed observations. Originally, the results of observations of planets and moons were published in academic journals. The data from these old journals are now hard to use, since certain parameters (depending on the standards of presentation of results that were adopted at the time of publication) of observations were omitted. In certain cases, even the observatory in which the observations were performed remains unknown. It is sometimes unclear in which scale the observation times are given and what methods of preliminary reduction of data were applied. Now, data may be stored on hard drives and accessed via the Internet. Thus, only the descriptions of observations and sample data are now published in academic journals, while the complete obtained dataset is made available via the Internet.

However, the acquisition of data from published papers and files found on websites is a rather time-consuming process. Naturally, databases containing all observations of certain groups of celestial bodies are created to simplify this task. Several major observatories have compiled their own observational databases (see, for example, the database of the Central Astronomical Observatory of the Russian Academy of Sciences at Pulkovo). Two databases deserve to be called the most extensive and important. MPC provides access to all observations of asteroids, comets, and distant moons of major planets. Search and sampling aids are also provided. Natural Satellites Data Center (NSDC) is another major observation data center. This is not just a database, but an Internet service that hosts all published results of observations of planetary moons, lists various physical parameters of moons, and allows the user to access a specialized reference database (Arlot and Emelyanov, 2009). This database was compiled and is maintained by IMCCE and SAI. The website pages are translated into Russian, English, and French. The basic principle of this database is that the data are presented as is (with no modifications or conversion). Therefore, each block of data corresponding to a certain publication or a certain source has its own format. Complete accuracy of data presentation is thus achieved. The NSDC pages may be accessed at http://www.imcce.fr/nsdc/ and http://www.sai.msu.ru/neb/nss/index.htm.

It was demonstrated already that regular observations are needed in order to enhance the accuracy of ephemerides and obtain new results regarding the dynamics of planets and moons. Some observatories have regularly performed high-precision observations over several decades. Let us note some of the recent publications of these observatories. Regular observations of planetary moons (Camargo et al., 2015) and Pluto (Benedetti-Rossi et al., 2014) are conducted in Brazil. High-accuracy observational data on planetary moons are published regularly by Chinese astronomers (Qiao et al., 2013). The history of regular observations of planets and moons at the Central Astronomical Observatory of the Russian Academy of Sciences at Pulkovo dates back more than a century; the paper authored by Roshchina et al. (2015) is one of the notable recent publications.

DISCOVERY AND LOSS OF DISTANT PLANETARY MOONS

Distant moons of giant planets are located much farther from their planets than the major moons. Distant moons are minor Solar System bodies. Their orbits are specific in that their inclinations and eccentricities vary widely. The eccentricity may be as large as 0.75. The inclination of orbits of distant moons to the planes of planetary orbits may be even larger than 90° (these are the so-called retrograde moons that move in the direction opposite to the direction of orbital motion of their planet). Prior to 1997, only eight distant moons of Jupiter, one distant moon of Saturn (Phoebe), and one distant moon of Neptune (Nereid) were known. The commissioning of new large telescopes initiated a series of discoveries of distant moons of Jupiter, Saturn, Uranus, Neptune, and Pluto. These moons were discovered "accidentally" in the process of searching for asteroids: after the determination of orbital parameters, some of the asteroids were found to be planetary moons. Observations performed by spacecraft moving close to planets have also contributed to the discovery. The number of new distant moons has increased rapidly in the early 21st century. Some new close planetary moons have also been discovered. Figure 13 presents the variation of the number of known planetary moons with time.

Newly discovered distant planetary moons are rather small (2–100 km). Their magnitudes vary from 20 to 26. Owing to their small size and the lack of dedicated observation programs, the observation time intervals for these moons are still fairly short. In certain cases, the time interval covered by observations is only 30 days (0.03 of the orbital period of the corresponding moons). When the observation time interval is that short, it is hard to expect the orbit parameters and ephemerides to be accurate.

As for the accuracy of ephemerides, the first studies of this subject have been performed only recently (in the early 21st century). It turned out that the methods providing more or less reliable and accurate estimates are fairly sophisticated. Certain results concerning two major moons of Saturn were obtained in (Desmars, 2009) based on an artificial set of observational data and ephemerides. The accuracy of ephemerides of all distant moons of major planets has been analyzed (Emelyanov, 2010) around the same time. Three different statistical methods for evaluating the accuracy of ephemerides were used. The first one (RO, random errors) is based on forming a large number of observational datasets that differ from the actual one in their sets of errors, which were generated using the Monte Carlo method. The orbit was determined for each dataset, and this orbit was then used to calculate the ephemeris at a given time point. Statistical evaluation of variation of ephemerides corresponding to different datasets produced an estimate of accuracy. In the second method (RP), the covariance matrix of parameter errors, which is obtained in the process of determination of an orbit from actual observations, is used to generate sets of ephemerides. Orbit parameters were varied using the Monte Carlo method. An ephemeris was calculated for each version. Just as in the first method, an estimate of accuracy was derived from the statistical properties of ephemeris variations. The third method (BS, bootstrap samples) involves the formation of datasets by random (bootstrap) sampling from actual observations. This sampling method is specific in that each observation, after being chosen at random from a set of actual observations, is returned to the initial set. The number of sampling steps equals the number of actual observations. As a result, some observations may be sampled several times, and some may not be chosen at all. The orbit, which was determined for each set of observations, was used to calculate the ephemeris. The accuracy was again estimated based on ephemeris variations. It was demonstrated in (Emelyanov, 2010) that the mentioned methods yield the same result when a sufficient number of observations distributed over a sufficiently long time interval are available. If observations are scarce and the observation time interval is very short, the BS method produces unrealistic estimates, which differ strongly from the ones obtained with the other two methods.

The cause of this discrepancy is now clear. When applying the BS method, one should sample not the observations themselves, but their errors, leaving the actual observation time points unchanged. Experiments showed that this correct algorithm of application of the BS method allows one to obtain such accuracy estimates that match the ones produced by the other two methods.

The estimates of accuracy of ephemerides for all 107 distant moons of Jupiter, Saturn, Uranus, and Neptune known at the time were obtained in (Emelyanov, 2010). It was demonstrated that the error of ephemerides for certain moons is now as large as half a revolution of these moons about their planets. Such moons may be considered lost. According to the results presented in (Emelyanov, 2010), 21 moons were lost by 2015. They need to be rediscovered.

Jacobson et al. (2012) have conducted a similar research into the accuracy of ephemerides of distant planetary moons. The authors of this study have not only verified the loss of certain moons, but have also performed new observations of distant moons. Several moons were rediscovered, while the ephemerides for certain other moons were corrected by adding new data. The authors have concluded that 16 of the known distant moons of Jupiter and Saturn still remain lost.

It is thus evident that the observation of distant moons of major planets and correction of their ephemerides is a relevant objective of modern observational astronomy and celestial mechanics, and this issue remains open.

DETERMINATION OF MASSES OF BINARY ASTEROIDS

Those (few) researchers who are concerned with the development of planetary ephemerides are well aware of a serious problem. The gravitation of asteroids should be taken into account in the calculation of the motion of Mars. In order to do that, one needs to know their masses, which remain undetermined. Approximately 300 asteroids are treated as gravitating points with their own models of motion. The models of motion of these asteroids are reliable, while their masses are largely unknown. The other 300–400 thousand asteroids are treated as a continuous medium, the masses of elements of which are also poorly defined. All this constitutes an obstacle to correcting the ephemerides of Mars and other planets. The paper authored by Standish and Fienga (2002) is aptly titled

"Accuracy limit of modern ephemerides imposed by the uncertainties in asteroid masses."

Several methods for the determination of asteroid masses are known. Spacecraft flybys may help calculate the masses of asteroids by revealing the influence of their gravity on the motion of spacecraft. Evidently, this method is much too costly, since asteroids are large in number. Since asteroids attract each other, one may also try to determine masses by observing this effect in their motion. However, the mutual attraction of these small Solar System bodies is too weak to produce such perturbations of their apparent motion that could at least be comparable to observation errors. Therefore, close approaches of asteroids, during which the mutual attraction gets stronger, should be tracked. Such cases were identified by Fienga et al. (2003), and a catalogue of close approaches of the most massive asteroids was compiled. Using this catalogue, one may decide when to perform the second most useful type of asteroid observations (after the observations of asteroids approaching Earth).

Anyway, close approaches of asteroids are rare events, and the accuracy of observations is not always sufficiently high to determine their masses. Other methods are needed.

Let us turn our attention to the Central Bureau for Astronomical Telegrams International Astronomical Union (CBATIAU). This Bureau has long been receiving regular reports concerning different known asteroids that were found to be binary. Unfortunately, the majority of these reports simply state that a certain asteroid is a binary one. The results of astrometric observations of binary asteroids are rarely received, although these data are the most useful. If a sufficient number of astrometric observations yielding the coordinates of the companion asteroid relative to the primary component are available, one may determine the orbit along which the companion moves. Ideally, such observations provide an opportunity to determine the revolution period T and the semimajor axis of the orbit a independently. The sum of the masses of components $m_1 + m_2$ (the mass of a binary asteroid) is then determined from the following relation:

$$\left(\frac{2\pi}{T}\right)^2 a^3 = G\left(m_1 + m_2\right),$$

where G is the universal gravitational constant. It is now clear why observations of binary asteroids are needed so badly.

In order to organize observations of binary asteroids, one first has to determine which asteroids are binary ones and the observational data for which targets are needed the most. Let us mention some of the available data sources. A catalogue of binary asteroids is published regularly. Its seventh version is currently available (Johnston, 2014). One convenient way to locate this publication and the catalogue itself is to start from its reference in the SAO/NASA Astrophysics Data System at http://adsabs.harvard.edu/abstract_service.html. The site of the author of this publication may be accessed at http://www.johnstonsarchive.net/astro/asteroidmoons.html. The catalogue already contains more than 280 binary asteroids.

It is important to keep in mind that the distances between components of binary asteroids observed from Earth do not exceed 1.4″. Therefore, they should be observed with instruments that produce high-resolution images. The best results are provided by telescopes with adaptive optics and cameras for speckle interferometry.

As for the determination of orbits of components of binary asteroids from observations of their relative motion, this issue was addressed in quite a number of papers. Let us mention some studies highlighting different trends in research into this subject. The mass and the density of asteroid 121 Hermione were determined in (Marchis et al., 2005) by analyzing the orbit of its "companion." Sokova et al. (2014) have reported the results of speckle-interferometric observations of binary asteroid 22 Kalliope-Linus and have used an original method to determine the parameters of its orbit. Another original method of statistical inversion for the determination of orbits of binary asteroids was proposed and studied in (Kovalenko et al., 2015).

Note that the determination of orbit of a component of a binary asteroid based on astrometric observations is similar to the determination of the orbits of planetary moons. The same methods are applicable to planetary moons and to binary asteroids.

CONCLUSIONS: WHICH TERRESTRIAL OBSERVATIONS OF SOLAR SYSTEM BODIES ARE NEEDED THE MOST

The above problems of dynamics of planetary moons and moons of asteroids are solved with the object of constructing motion theories and ephemerides. This is done on the basis of observations. Let us determine which observations are now needed the most. These types of observations are listed below in an order that does not reflect their importance (they are equally important).

Regular astrometric observations of the major moons of Jupiter, Saturn, Uranus, and Neptune. Absolute coordinates of moons should be obtained using new star catalogues. Much is expected of the GAIA catalogue that is now being compiled.

Photometric observations of the Galilean moons of Jupiter during their mutual occultations and eclipses in 2020. Even small telescopes may be used to conduct these observations. Amateur astronomers are welcome. The number of observatories involved should be as high as possible so as to document all the events. Simultaneous observations at several observatories will provide an opportunity to exclude systematic errors. Ephemerides for these observations may be found at http://www.sai.msu.ru/neb/nss/nssephmr.htm.

The methods for photometric processing of images of moons obtained during their mutual occultations and eclipses must be analyzed. The sources of systematic errors should be revealed, and a new technique free from these errors should be developed.

A campaign focused on photometric observations of Galilean moons of Jupiter aimed at compiling a table of integral albedos as functions of the rotation angle in different spectral bands of the visible region is much called for. These observations should be performed outside of any mutual events, but at epochs close to them. It should be taken into account that absolute magnitudes are not needed here: the end results are ratios of luminous fluxes of two moons. Powerful telescopes are also not required, since the moons have a magnitude of 4. Fine photoreceivers and processing methods are necessary in order to obtain such ratios of luminous fluxes of moons that are free from systematic errors. After a sufficient number of these ratios for all combinations of moons and all angles of rotation of moons are obtained, one may solve the equations that yield the sought variations of albedo.

Powerful telescopes are essential for astrometric observations of distant moons of major planets. The observations of new distant moons with a magnitude of 20 (and above) are especially valuable. Lost and completely unknown new moons may be found near Jupiter and Saturn. There is good reason to believe that a great number of moons are present in this region.

Binary asteroids are a major target for observers. Astrometric observations are much in demand. These objects are hard to observe, since the distance between their components is 1″ or less. Speckle interferometry and adaptive optics could prove useful here. Telescopes used in such observations should be fairly powerful, since binary asteroids have a magnitude of 12 at best. The majority of them are even fainter.

Observers should work in close collaboration with those researchers who develop models and theories of motion of planets and moons. The occasional situations in which observers are forced to search for theorists willing to use the data should be excluded.

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