Straight segments in the galactic discs

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ABSTRACT

We study the properties of the straight segments forming in *N*-body simulations of the galactic discs. The properties of these features are consistent with the observational ones summarized by Chernin et al. Unlike some previous suggestions to explain the straight segments as gas dynamical instabilities, they form in our models in the stellar system. We suggest that the straight segments are forming as a response of the rotating disc to a gravity of the regions of enhanced density (overdensities) corotating with the disc. The kinematics of stars near the prominent overdensities is consistent with this hypothesis.

Key words: galaxies: kinematics and dynamics - galaxies: spiral - galaxies: structure.

1 INTRODUCTION

Straight segments in spiral structure of galactic discs are observed both in real galaxies and in numerical models. The straight segments were first noticed by Vorontsov-Vel'yaminov (1964, 1978), who called them rows. These quite long nearly straight features often outline the grand design spiral structure, as it is, for example, in M101 and in M51, forming ragged but nearly regular spiral arms which are often called polygonal arms. Chernin et al. (2000, 2001) compiled the catalogue of galaxies with rows that includes about 200 objects. They also study the properties of straight segments, which can be briefly formulated as follows.

(i) The length of the straight segment *L* increases with the galactocentric distance *R*, so that $L = (1 \pm 0.13)R$.

(ii) The straight segments can be divided into two types: those that fit well the grand design spiral arms and isolated ones.

(iii) The angle between two neighbouring segments is, on average, $\alpha = 120$, the standard deviation is $\sim 10^{\circ}$.

(iv) The straight segments are observed mostly in late-type galaxies Sbc-Scd.

(v) The straight segments are more frequently observed in interacting galaxies.

(vi) The average number of straight segments in the polygonal spiral arms is n = 3.

(vii) Galaxies with straight segments are quite rare objects, they account for \sim 7 per cent of all spiral galaxies with well-defined spiral arms. Note however that this estimate is based on studying photographic plates and printed images. Our inspection of a small sample of digital galaxy images suggests a considerably larger frequency of straight segments in spiral galaxies.

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Straight segments in numerical models are observed quite frequently in both gaseous and stellar discs, but their appearance is often mentioned only briefly because the emphasis of the authors has been on larger scale structure.

The straight segments in gaseous discs are observed in models by Combes (1994), who studies the gas inflow in barred potentials. She explains the square-like shape of the spiral arms by the resonances and gas viscosity: the periodic orbits must change their orientation with respect to the bar in the inner and outer Lindblad resonances: ILRs and OLR (Buta & Combes 1996).

Khoperskov et al. (2011) and Filistov (2012) study the formation of straight segments in the gaseous discs under the given analytical potential. Their models include the external spiral-like potential perturbation, which rotates with a small angular velocity. As they note, the position of the corotation radius (CR) on the very periphery of the disc is a necessary condition for the appearance of straight segments in their models. Simulations by Khoperskov et al. (2011) reproduce well the main properties of the straight segments: the dependence $L \sim R$ and the average angle between the neighbouring segments $\alpha = 120^{\circ}$. They explain the formation of the straight segments by unstable location of the shock fronts in the spiral potential well.

Chernin (1999) explains the formation of straight segments as the universal stability of a flat shock front against any weak perturbations that disturb its front surfaces. Filistov (2012) also supports this idea. But it is not clear how this mechanism works in the rotating stellar systems (see Khoperskov et al. 2011, for more comments)

Rautiainen, Salo & Laurikainen (2005, 2008), using the potentials extracted from the near-infrared (IR) images of Ohio State University Bright Spiral Galaxy Survey (Eskridge et al. 2002, hereafter OSUBSGS), model the behaviour of the gas subsystem of some disc galaxies. Their models reproduce the straight segments observed in some galaxies, especially well in the case of NGC 4303.

The successful modelling of NGC 4303 is at least partly based on the fact that the straight segments are observed also in the



Figure 1. The *B*- and *H*-band images of NGC 4303. The images are taken from the Ohio State University Sample of Bright Galaxies (OSUBSGS), and have been scaled to enhance the visibility of the straight segments.

near-IR H-band image, which should be dominated by the old stellar population (see Fig. 1), i.e. they were present in the derived gravitational potential. Furthermore, NGC 4303 is not an exceptional case. We found about 40 galaxies with straight segments in the OSUBSGS images: their overall frequency in the sample was \sim 25 per cent, considerably higher than \sim 7 per cent found in earlier studies. This difference was most likely due to advantage of the digital images over the photographic plates and atlases used by Chernin et al. (2001) - the possibility to adjust contrast and other image properties revealed many straight segments that were missed in 'static images'. Our inspection also revealed that in more than half of the cases, the straight segments observed in B band had their counterparts also in H band. In addition to our findings, the straight segments are quite conspicuous in near-IR J- and K-band images of galaxies NGC 3938 and NGC 4254 obtained by Castro-Rodríguez & Garzón (2003).

There are two different approaches to explain the formation of straight segments in the stellar subsystem: one is based on the global modes (Toomre 1981) and the other rests on the chaotically distributed rotating features (Toomre & Kalnajs 1991).

The most popular explanation of the polygonal spiral arms is proposed by Toomre (1981). He explains the square-like shape of the spiral arms by the presence of the leading and trailing spiral waves of very similar wavelengths and amplitudes in the Fourier spectrum of the mode, where the leading wave appears due to reflection of the in-going trailing wave from the centre (see Athanassoula 1984; Binney & Tremaine 2008, for more details).

Salo & Laurikainen (2000b) explain the inner polygonal structure of M51 by the reflection of the trailing wave packets as leading waves from the centre. Their simulations reproduce the polygonal spiral arms in the inner 30 arcsec region of M51 observed in near-IR (Zaritsky, Rix & Rieke 1993).

However, it is not clear how to produce the superposition of the leading and trailing waves on the galactic periphery. In this context, it is worth noting ideas by Sellwood (2012), who supposes that the region of the ILR can acquire ability to reflect the in-going trailing waves into the out-going leading ones.

In the other approach the observed spiral structure is considered as a set of arm features forming due to random density fluctuations in galactic discs (Toomre 1990). Julian & Toomre (1966) consider the response of the stellar disc to a chance overdensity corotating with the disc. The density response can exceed the initial perturbation more than several tens of times (Goldreich & Lynden-Bell 1965; Julian & Toomre 1966; Toomre 1981). This mechanism called swing amplification is based on the concerted action of noise, epicyclic motion and self-gravity (Toomre 1981). Sellwood & Carlberg (1984) study the work of the swing amplification mechanism and show that the maximal amplification is possible on the galactic periphery for the multi-armed spiral patterns.

Recently, many researchers note that the multi-armed spiral structure in their *N*-body simulations does not rotate as a whole, but consists of pieces corotating with the disc at different radii (Wada, Baba & Saitoh 2011; Grand, Kawata & Cropper 2012; Baba, Saitoh & Wada 2013; D'Onghia, Vogelsberger & Hernquist 2013; Roca-Fábrega et al. 2013).

D'Onghia, Vogelsberger & Hernquist (2013) study stellar discs with the TreePM code GADGET-3 using small softening parameter ($\epsilon = 5$ pc). They get very impressive pictures of polygonal spiral arms (or linear segments joined at kinks), which form global multiarmed spiral stricture. In their experiments the system of disturbers ($M \approx 10^6 \text{ M}_{\odot}$) corotating with the disc causes the formation of the multi-armed polygonal spiral arms.

Grand, Kawata & Cropper (2012) study the motions of stars near the spiral arms in *N*-body simulations. Their stellar discs form multi-armed structures, which often exhibit straight segments. They show that particles can join spiral arms from both sides at all radii and migrate radially along the spiral arms.

In the present paper we study properties of the straight segments forming in *N*-body galactic discs. We show that the features of the model straight segments are in a good agreement with the observational ones summarized by Chernin et al. (2000, 2001). We suppose that the straight segments are forming as a response of the rotating disc to a gravity of the regions of enhanced density (overdensities) corotating with the disc. The properties of these respondent perturbations can explain the observational features of the straight segments. The kinematics of stars in the model discs also agrees with this suggestion.

In Section 2 we study kinematical and morphological properties of the respondent perturbation of the disc to the overdensity corotating with it, that was first studied by Julian & Toomre (1966). We show that the respondent perturbation must be nearly straight and its length L is nearly proportional to R. The models and their evolution are considered in Section 3. Here we also demonstrate that the model spiral pattern does not rotate with the same angular velocity, but its different parts nearly corotate with the disc. In Section 4 we compare the characteristics of the model straight segments with the theoretical predictions studied in Section 2. Section 5 is devoted to the kinematics of the straight segments. We compare the stellar motions near the overdensities with the theoretical predictions. Section 6 includes the main conclusions.

2 THE PROPERTIES OF THE DISC RESPONSE TO THE OVERDENSITY

2.1 The shape of the respondent perturbation

Toomre (1964) has shown that the stability of the disc is supported by shared action of the Coriolis forces and the equivalent of pressure, resulting from random motions: the random motions effectively suppress perturbations on the short side of wavelengths, while the Coriolis forces suppress instabilities on the long end. The value of λ_c is the shortest wavelength of axisymmetric perturbations that can be stabilized by epicyclic motions only:

$$\lambda_{\rm c} = \frac{4\pi^2 G \Sigma}{\kappa^2},\tag{1}$$

where Σ is the surface density of the disc and κ is the epicyclic frequency.

Julian & Toomre (1966) consider the response of a thin differentially rotating stellar disc to the presence of a single, particle-like concentration of the interstellar matter (overdensity) corotating with the disc. They have found that overdensity creates quite extended spiral-like response in the disc: the size of the density ridge in the radial direction amounts $\sim \lambda_c/2$.

Toomre (1981) studies the self-gravitating stellar discs with flat rotation curves and shows that the value of amplification of the initial overdensity is very sensitive to the value of the stability parameter Q(Toomre 1964). The other parameter that determines amplification is $X = \lambda_y / \lambda_c$, where λ_y is the length between the neighbouring spirals in the azimuthal direction. Maximum amplification corresponds to $X \approx 1.5$.

Fig. 2(a) shows the trajectory of a star with respect of the initial overdensity (Julian & Toomre 1966). The star in question is located at the larger distance than the disturber, and first has purely circular velocity. In the reference frame corotating with the disturber, the star moves in the direction opposite that of galactic rotation, i.e. clockwise. In the impulse approximation, the star's angular momentum is unchanged and its motion can be thought as a superposition of the purely circular motion and the motion along the epicycle (Binney & Tremaine 2008). Let us suppose that the star gains some impulse and starts its epicyclic motion when the distance between the star and the disturber is minimal, i.e. when the star and the disturber are lying at the same radius vector. The moment of start of the epicyclic motion is denoted by number '1' and corresponds to the maximal additional velocity directed towards the galactic centre. Julian & Toomre (1966) suggest that the resulting stellar density must be the

greatest wherever the individual stars linger longest. That moment denoted by number '2' occurs in nearly one-quarter of the epicyclic period, when the star has the maximal additional velocity directed in the sense of galactic rotation. Note that in the chosen reference frame, the star in question moves in the direction opposite that of galactic rotation, so the moment with the largest velocity in the sense of galactic rotation determines the place where the star lingers most. Moment '3' corresponds to the maximal additional velocity directed away from the galactic centre. The additional velocity in moment '4' is directed in the sense opposite that of galactic rotation. In the absence of occasional perturbations stellar trajectories must repeat their oscillations every epicyclic period.

Let us calculate an angle β , which determines the position of the respondent perturbation with respect to the azimuthal direction and corresponds to the pitch angle of the spiral arms. Julian & Toomre (1966) suppose that without taking into account the self-gravity the angle β must be ~45° for flat rotation curve. Generally, the fact that the angle β is independent from the coordinates ΔR and Δy suggests that the stellar response has the shape of a straight line. We will show that the distance Δy to the point, where the star lingers most, is nearly proportional to ΔR . The angle β can be determined by the ratio

$$\tan \beta = \frac{\Delta R}{\Delta y}.$$
(2)

In the first approximation, we neglect the additional velocities due to the epicyclic motions. Then the distance Δy , which is passed by the star with respect to the initial disturber during one-quarter of the epicyclic period $\pi/(2\kappa)$, is determined by the relation

$$\Delta y = |\Omega(R_1) - \Omega(R_0)| \frac{\pi R_0}{2\kappa_1},\tag{3}$$

where $\Omega(R)$ is the angular velocity of rotation curve. Subscripts '1' and '0' are related to the star considered and to the initial disturber, respectively. For flat rotation curve ($\Omega(R) = V_0/R$, $\kappa = \sqrt{2}\Omega$) we can express the distance Δy in the following form:

$$\Delta y = \frac{\pi}{2\sqrt{2}} \Delta R. \tag{4}$$

And the value of the angle β is determined by the expression

$$\beta = \arctan \frac{2\sqrt{2}}{\pi} = 42^{\circ},\tag{5}$$

which is very close to the value suggested by Julian & Toomre (1966). Thus, in the impulse approximation the value of β is independent from ΔR , and the respondent perturbation must have the shape of the straight segment. However, the impulse approximation is not accurate, especially in the very vicinity of the initial disturber, because any star changes its angular momentum during the approach phase of the encounter and passes the disturber with a slower relative velocity than it would be without interaction (Julian & Toomre 1966).

Note that near the disturber, stars oscillate conspicuously in the radial direction, moving first towards the disturber and then away from it. And the star can continue its radial oscillations as it moves in the azimuthal direction.

2.2 Influence of self-gravity

In the cold discs (but Q > 1) the self-gravity plays important role, so after some moment, the stellar trajectories are rather determined by the gravity of the straight segment itself than by the initial disturber.



Figure 2. The trajectory of the star (black curve) perturbed by the initial overdensity (Julian & Toomre 1966). The motion is considered in the reference frame, corotating with the initial disturber, which lies at the origin and rotates with the circular velocity. The star in question is lying at the distance larger than that of the disturber and initially moves with purely circular velocity. In the chosen reference frame, it moves in the sense opposite that of galactic rotation. The numbers 1–4 denote positions of the star at moments separated by 1/4 of the epicyclic period. The upper row shows the position of the star in the epicyclic orbit at moments 1–4. (a) Position of the straight segment without taking its self-gravity into account. Here the greatest stellar density corresponds to the point '2'. (b) Position of the straight segment with self-gravity. In this case the highest density corresponds to the point '3', where stars are moving nearly along the straight segment.

Let us again consider the motion of the star initially moving on the circular orbit (Fig. 2b). The self-gravity effects are maximal at the time interval, when the star is leaving the straight segment and is moving nearly parallel to it. In Fig. 2(b) it is a path between points '2' and '3'. Because of the gravity of the straight segment, the position of the density maximum is shifting in the direction of the point '3', because here the star has maximal value of the radial velocity, which allows it to move along the straight segment during the longest period of time.

Using the approach described above, we can calculate the pitch angle of the self-gravitating straight segment, which must be nearly two times less than the angle calculated without self-gravity, because the time interval to reach the point '3' is nearly two times larger (π/κ) than that needed to reach the point '2' from the start

of the epicyclic motion at the point '1'. In the first approximation, the angle β of the self-gravitating straight segment equals

$$\beta = \arctan \frac{\sqrt{2}}{\pi} = 24^{\circ}.$$
 (6)

This result agrees with the estimate by Toomre (1981), who thinks that self-gravity must decrease the value of the pitch angle by two times at least. Moreover, he supposes that due to the distortion of epicycle motions the pitch angle can drop to 15° .

Note that the direction of the radial velocity V_R inside the selfgravitating straight segments coincides with that in the density-wave spiral arms (Lin, Yuan & Shu 1969): at the larger *R* from the initial disturber (outside the CR) stars located inside the straight segment (inside the spiral arm) have the radial velocity V_R directed away from the galactic centre, while stars located at the smaller *R* than the initial disturber (inside the CR) have the velocity V_R directed towards the galactic centre. Generally, on the edges of the straight segment stars must move in the opposite directions away from each other (away from the initial disturber).

2.3 The length of the straight segments

The most interesting parameter is the linear size of the respondent density perturbation. Julian & Toomre (1966) show that the size of the density ridge in radial direction is $\Delta R \approx \lambda_c/2$.

Let us compare the value of λ_c with the radius *R*, at which it is calculated (see also fig. 5a in Toomre 1977). We can approximate the distribution of the disc density using the relation

$$\Sigma \approx \frac{f_{\rm d} V_0^2}{2\pi G R},\tag{7}$$

where V_0 is the velocity of rotation curve and f_d is the contribution of the disc to the total rotation curve. This formula is absolutely true for Mestel's discs, but for exponential discs it is true within several per cent (Binney & Tremaine 2008). Substituting Σ in equation (1) and using the relations $\Omega(R) = V_0/R$ and $\kappa = \sqrt{2}\Omega$ for flat rotation curves we obtain

$$\lambda_{\rm c} \approx \pi f_{\rm d} R. \tag{8}$$

And the maximal size of the straight segment in the radial direction ΔR is defined by the expression

$$\Delta R \approx \pi f_{\rm d} R/2. \tag{9}$$

Then the full size of the straight segment L, under the angle $\beta \approx 42^{\circ}$, must be following:

$$L \approx 2.4 f_{\rm d} R. \tag{10}$$

Generally, the self-gravity effects cannot increase the length of the straight segments.

In the distance range considered in our models, the value of f_d varies in the limits $f_d = 0.2-0.5$. So the maximal possible length of the straight segment must lie in the range L = (0.5-1.2)R. On the whole, this result is consistent with the observations (Chernin et al. 2001).

2.4 Amplitude of the velocity perturbation

Binney & Tremaine (2008), using the impulse approximation, estimate the value of the radial velocity V_R acquired by a star after the encounter with a molecular cloud:

$$V_R = -\frac{Gm}{A_0 b^2},\tag{11}$$

where *m* is the mass of molecular cloud, which corotates with the disc at the radius R_c and A_0 is Oort constant at this radius. The star considered is initially moving on the circular orbit of radius *R*, so $b = R - R_c$ is the impact parameter.

We can see that the value of acquired velocity V_R depends on the current value of Oort constant *A*, which varies with radius. For flat rotation curve, *A* is inversely proportional to R, $A = \Omega/2 \sim 1/R$. So the overdensity of the same mass can create the larger velocity perturbation on the galactic periphery than in the intermediate regions. And the physics of this dependence is clear: the smaller *A*, the weaker differential rotation, the smaller relative velocity of passage, the more time of gravitational interaction. So on the galactic periphery stars can acquire larger radial velocities after encounters with the same overdensities.

3 MODELS

The *N*-body simulations used in this paper were done by P. Rautiainen during year 2012 by applying the code written by H. Salo. In these 2D models we use a logarithmic polar grid with 216 azimuthal and 288 radial cells to calculate the gravitational forces and motions with leap-frog integration. The stellar disc consists of 4×10^6 self-gravitating particles, but the bulge and halo are analytical. The gas component is omitted in this paper, in models we do not show here, the gas component was modelled as inelastically colliding massless test particles. For more details on the code, see Salo (1991) and Salo & Laurikainen (2000a).

We have made a large set of models to study the formation and evolution of straight segments in the galactic discs. In these models we varied several parameters such as the mass fractions of different components, the value of the initial Toomre parameter of the disc, the extent of the disc and the value of the gravitational softening parameter (Plummer softening). For the purposes of this paper, we have selected two models, hereafter Model 1 and Model 2, which show the characteristics of the straight segments most clearly, and discuss the other models only briefly.

The rotation curves of Models 1 and 2 are shown in Fig. 3, with the adopted physical scaling of the simulation units. In both models, the disc particles were originally distributed as an exponential disc with scale length $R_e = 3.0$ kpc. The bulge component was modelled as an analytical Plummer sphere, and the analytical halo was of the same form as in Rautiainen & Mel'nik (2010). In both cases the asymptotic rotation velocity of the halo is 189 km s^{-1} , but the core radius is different. As the rotation curves show, both models are mostly dominated by the spherical (analytical) component (bulge and halo); the reason for this choice of parameters was to delay the bar formation, but the initial value of the Toomre parameter is low enough ($Q_T = 1.2$ in Model 1 and $Q_T = 1.1$ in Model 2) that still allows the discs to develop well-defined spiral arms. The essential model parameters are given in Table 1.

Model 1 first develops a multi-armed structure. In the outer parts of the disc there are m = 10-20 short arms. The structure becomes more regular in the inner parts. Even there the number of arms is varying (m = 2-5). A large scale bar forms at $T \approx 800$ Myr. After its formation, the inner spiral structure becomes effectively two-armed and the number of spiral arms diminishes also in the outer parts, although the outer spiral structure still remains multi-armed.

Fig. 4 demonstrates the evolution of two straight segments (in one spiral arm), whose locations at $R \approx 7-9.5$ and $\approx 9.5-13$ kpc are indicated with arrows. To make following the evolution easier, we have used a rotating coordinate system with angular velocity



Figure 3. The rotation curves of Models 1 and 2. The continuous lines show the total rotation curves, the bulge contribution is drawn with a dotted line, the disc contribution with a dashed line and the halo contribution with a dash-dotted line.

Table 1. Essential parameters in Models 1 and 2. $M_{\rm disc}$ is the disc mass, $M_{\rm bulge}$ is the bulge mass, $M_{\rm halo}$ is the mass of the halo inside $R = 15 \,\rm kpc$, $b_{\rm bulge}$ is the bulge scale radius and $R_{\rm c}$ is the halo core radius. The initial value of the Toomre parameter $Q_{\rm T}$ and the gravitational softening ϵ are also indicated.

	Model 1	Model 2
$\overline{M_{\rm disc}({ m M}_{\bigodot})}$	2.9×10^{10}	3.0×10^{10}
$M_{\text{bulge}}(\widetilde{M_{\odot}})$	9.2×10^{9}	1.5×10^{10}
$M_{\text{halo}}(M_{\bigodot}), R < 15 \text{ kpc}$	9.9×10^{10}	1.1×10^{11}
b_{bulge} (kpc)	0.6	1.1
$R_{\rm c}$ (kpc)	7.5	5.3
Q_{T}	1.2	1.1
ϵ (pc)	75.0	225.0

that keeps the two straight segments in nearly the same place in our frames. This corresponds to pattern speed of about 18 km s⁻¹ kpc⁻¹. In the beginning of the shown time sequence, T = 509 Myr, the region R = 7-13 kpc has $m \approx 10$ spiral arms. However, 10–30 Myr later, the particles form $m \approx 4$ longer spiral arms. One of them clearly has two straight segments, forming around T = 550 Myr and being strongest at about T = 591 Myr. After that, the straight segments, and also the associated spiral arm itself, become weaker. In the last couple of frames, there is no sign of the two straight segments, but there is a new one in the opposite side of the galaxy.

These two segments in Model 1 were selected because they show exceptionally well the formation and destruction of these features. Also, they are exceptionally long-lived, lasting about 80 Myr, which corresponds to about fourth of the circular rotation period at the radial distance of the segments. Most straight segments seen in our models have shorter lifespans, corresponding to 10–30 Myr.

More insight to the evolution of Model 1 can be obtained by Fourier analysis of its surface density. Fig. 5 shows the amplitude spectra (Masset & Tagger 1997; Salo & Laurikainen 2000b) for the m = 2-5 components during the epochs when the two straight segments appear. Also shown are the frequency curves Ω and $\Omega \pm \kappa/m$. In the vicinity of the two segments, i.e. R = 7-13 kpc, the strongest feature can be seen in the m = 4 and 5 amplitude spectra. This is not surprising, although the number of spiral arms in this region is a bit varying, a four- or five-armed structure is the most prevalent case.

The feature seen in the R = 7-13 kpc region both in m = 4 and 5 spectra has a pattern speed $\Omega_p \approx 18 \text{ km s}^{-1} \text{ kpc}^{-1}$, which corresponds to the corotation resonance radius of ~ 11 kpc, coinciding with outer of the two segments. In the m = 5 spectrum, there is also a clear feature with $\Omega_p \approx 24 \text{ km s}^{-1} \text{ kpc}^{-1}$, which probably has an effect on the inner segment. There are also features in the spectra of higher values of m (6–12), but these are limited to immediate vicinity of the Ω curve.

In Model 2 the disc does not form a large-scale bar during the simulation time, which corresponds to about 5 Gyr. The disc shows mostly multi-armed (m = 5-10) morphology, which occasionally develops straight segments. In many time-steps these arms appear to be long, extending throughout most of the disc, but a closer look at their evolution and the amplitude spectra demonstrates that they actually consist of a large number of short features, whose pattern speeds are close to the local circular velocity. This kind of behaviour resembles the recent models by Grand et al. (2012) and D'Onghia et al. (2013). In the later phase of the simulation, the number of arms in the inner parts of the disc diminishes to m = 2-4, and the innermost part resembles a small bar or oval.

In other models, which are not shown or analysed in this paper, we made further experiments with model parameters, such as the value of the gravitational softening and the initial extent of the stellar disc. There is already a quite large difference in softening parameters between Models 1 and 2, and tests with other values show that it is not critical for the formation of straight segments, as long as its value is not so high to suppress the formation of all the sharp features on the disc. The situation is quite similar with the disc extent; there are more straight segments in larger discs, but even models, where the initial stellar particle distribution reaches only 6 kpc or two disc scale lengths, can have them.

4 GENERAL CHARACTERISTICS OF STRAIGHT SEGMENTS IN MODEL DISCS

We identified straight segments in stellar discs of Models 1 and 2. For their identification we have used the images of model discs processed by the masking program, which leaves only regions of enhanced density, i.e. regions where the density exceeds its average level at the same radius. This procedure increases the contrast between the arms and the interarm space and facilitates the study of the galactic morphology. In identification of straight segments we adhered to the following principles.

(i) The line, connecting the ends of a straight segment, must lie entirely in the region of the enhanced density.

(ii) The ends of straight segments must have some specific features: either the density dropping below the average level or the significant increase in the pitch angle of a spiral arm.

(iii) In all cases we try to identify straight segments so that their length L would be maximal.



Figure 4. The formation and evolution of straight segments in one spiral arm (Model 1). The frames have a width of 18 kpc and they show the density enhancement above the average density at the same radius. The time moment (in Myr) is exhibited at each frame. The two arrows shown in the frames indicate the locations of two straight segments. The densities are shown in a rotating coordinate system (see details in text).

(iv) The straight segment must be quite elongated: the ratio of its length to the width must exceed 4.

Table 2 exhibits the average characteristics of the straight segments identified in Models 1 and 2. It shows the total number *N* of moments considered, the number of the selected straight segments n_s , the coefficient *k* in the dependence L = kR and its error. It also includes the standard deviations σ_0 and σ_1 calculated for linear relation L = kR and for non-linear law $L = 2.4 f_d(R)R$, respectively. We also present the average value of the angle $\overline{\beta}$ between the straight segment and the azimuthal direction, its standard deviation (in brackets), the average value of the angle $\overline{\alpha}$ between two neighbouring straight segments, its standard deviation (in brackets) and the number n_{α} of measurements of α .

4.1 L-R dependence

The variations in the length of the straight segments *L* along radius *R* in Models 1 and 2 are shown in Fig. 6. The galactocentic distance *R* for the straight segment is determined as the distance to its median point. The thick grey curve shows the value of *L* calculated from the formula $L = 2.4 f_d R$, where $f_d(R)$ is the relative contribution of the disc to the total rotation curve at each radius. The value of $f_d(R)$ achieves the maximum at $R \approx 5$ kpc and then gradually decreases with increasing *R* (Fig. 7).

We can see that both dependencies $L = (0.86 \pm 0.02)R$ (Model 1) and $L = (0.88 \pm 0.01)R$ (Model 2) derived for the model straight segments are consistent with observations, $L = (1.0 \pm 0.13)R$. However, the connection between L and R is conspicuously non-linear in both models: there are a lot of relatively short straight segments at large radii. So L-R relation is better described by formula $L = 2.4 f_d R$. The standard deviation σ_1 is less than σ_0 derived for the linear law by 27 per cent in Model 1 and by 16 per cent in Model 2. This difference can be related to the fact that the amplitude of variations of f_d is larger in Model 1 than in Model 2 (Fig. 7).

4.2 The angle between the neighbouring segments

Since the straight segments rotate in the disc with the angular velocity of their parent overdensities, they can never form stationary polygonal structure. Moreover, straight segments must destroy each other during their merging. The only possible way for their contact without destruction is a touch with their edges. In this case they can even increase each other, because the appearance of an extra density at their endpoints gives both of them an extra ability to hold stars inside them.

Table 2 indicates that the average value of the angle α between two neighbouring straight segments is $\overline{\alpha} = 127^{\circ}$ and 125° in Models 1 and 2, respectively. The maximum in distribution of α lies near



Figure 5. The amplitude spectra of Model 1 based on the Fourier decomposition components m = 2-5. The continuous lines show Ω and $\Omega \pm \kappa/m$ in each frame. The contour levels are 0.025, 0.04, 0.1 and 0.2 above the azimuthal average density at each radius. The sampling period (in Myr) is indicated in each frame.

 Table 2. Characteristics of the model straight segments.

	Model 1	Model 2
N moments	36	53
ns	238	273
k in L = kR	0.86 ± 0.02	0.88 ± 0.01
σ_0 in $L = kR$	2.10 kpc	1.80 kpc
σ_1 in $L = 2.4 f_d R$	1.54 kpc	1.51 kpc
$\overline{\beta}$	28° (9°)	25° (9°)
$\overline{\alpha}$	127° (13°)	125° (11°)
n _α	101	129

 $\alpha = 130^{\circ}$ in both cases (Fig. 6). All these values are consistent with observations.

Chernin (1999) gives an explanation of the value of $\overline{\alpha} = 120^{\circ}$, which is based on the relation L = R. We present here a bit different explanation, which also invokes the correlation between L and R.

Let us consider two straight segments at the moment, when they touch each other with their edges (Fig. 8). They, together with the radius vectors, form the quadrangle OCMD. We are looking for the angle α between two straight segments. As the sum of the angles in a quadrangle is 360°, we can find the angle α by subtracting the other angles from this value. The galactocentric angle is denoted by θ , two other angles have values 90° + β and 90° - β . So it is the angle θ that determines the value of α : $\alpha = 180^{\circ} - \theta$. We can find θ from the triangle COD. Because of the relation L = Rthe side CD in the triangle COD has the value $\sim (R_1 + R_2)/2$, what correspond to $\theta \approx 60^{\circ}$ [use the law of cosines and neglect terms of $(R_1 - R_2)^2/R_1R_2$]. Thus, the angle α has the value of $\alpha = 120^{\circ}$ and is practically independent of β .

We have found that in Models 1 and 2 the coefficient k in the relation L = kR is less than unity, k = 0.86-0.88, so the angle θ must be less than $\theta < 60^\circ$ here. Under k = 0.87 it must have the value of $\theta = 52^\circ$ and, consequently, α must be $\alpha = 180^\circ - \theta = 128^\circ$. The last value is in good agreement with corresponding estimates in our models.

4.3 The angle β between the straight segments and the azimuthal direction

We measured the angle φ between the straight segment and the radius vector drawn from the galactic centre to the median point of the straight segment and calculated the angle β , supplementing it up to 90°, $\beta = 90 - \varphi$. Generally, angle β is an analogue of the pitch angle for the spiral arms. Its average value equals 28° and 25° in Models 1 and 2, respectively (Table 2).



Figure 6. Left-hand panel: the dependence between the length L of a straight segment and its galactocentric distance R in Models 1 and 2. The thick grey curve shows the value of L calculated from the formula $L = 2.4 f_d(R)R$. The bisectrix is also drawn. Right-hand panel: the histograms of the distribution of the angle α between two neighbouring straight segments.



Figure 7. Variations in f_d , the relative contribution of the disc to the total rotation curve (v_c^2) , in Models 1 (solid line) and 2 (dashed line).

Fig. 9 shows the variations of β along radius and the histograms of the distribution of β . We can see that the angle β , on average, decreases with radius. The approximation of these variations by the linear law gives the following parameters: $\beta = (-1.63 \pm 0.17)R + 41.7 \pm 1.5$ for Model 1 and $\beta = (-1.68 \pm 0.17)R + 37.6 \pm 1.3$ for Model 2, where β is in degrees and *R* in kpc.

The variations in β along *R* can be partly (within 10°) explained by the deviations of the model rotation curves from flat one. For nonflat rotation curve the angle β can be estimated from the relation

$$\beta \approx \arctan \frac{\kappa}{\pi R |\Omega'(R)|},$$
(12)

where $\Omega'(R)$ is the first derivative of $\Omega(R)$ with respect to *R*. This expression is combination of equations (2) and (3), but obtained for the case 'with self-gravity', in which the maximal density



Figure 8. Two straight segments (grey lines) at the moment of contact. The angle α between them can be found from the quadrangle OCMD: $\alpha = 360^{\circ} - \theta - (90^{\circ} + \beta) - (90^{\circ} - \beta) = 180^{\circ} - \theta$. As the triangle COD has sides R_1, R_2 and $\sim (R_1 + R_2)/2$, the angle θ equals $\theta \approx 60^{\circ}$. Consequently, angle α is practically independent of β and equals $\alpha \approx 120^{\circ}$.

corresponds to the point '3' situated one-half of the epicyclic period (π/κ) downstream the initial disturber (Fig. 2). In the case of flat rotation curve, equation (12) transforms to equation (6). Generally, the rising rotation curve increases β , while the descending one decreases it.

5 KINEMATICAL FEATURES OF THE STRAIGHT SEGMENTS

The role of the initial overdensities in production of the straight segments is to adjust the epicyclic motions of stars passing by. So the overdensities must create the specific velocity field in their neighbourhood. To study the kinematics of stars in our models we calculated the residual velocities of stellar particles in the radial and azimuthal directions, V_R and V_T .

In our previous paper (Rautiainen & Mel'nik 2010) we determined V_R and V_T as differences between the model velocities and the velocity due to rotation curve, but there we considered the gas subsystem, which rotated practically with the velocity of rotation curve. However, it is not true for the stellar discs. Because of the conspicuous velocity dispersion, the stellar discs rotate, on average, with the smaller velocity than that of rotation curve. It is so-called asymmetric drift (Binney & Tremaine 2008). In the present paper we calculate the azimuthal residual velocity V_T with respect to the average azimuthal velocity of stellar particles at the same radius, but not with respect to the rotation curve. Nothing have changed for the radial residual velocity V_R , which coincides with the radial velocity with respect to the origin.

Fig. 10 demonstrates the distribution of the average azimuthal velocity of stars \overline{v}_{θ} and that of the rotation curve v_c along radius in Models 1 and 2. It also shows the velocity dispersion in radial direction σ_R at different radii. For example, at R = 7 kpc, the asymmetric drift amounts $v_c - \overline{v}_{\theta} = 9$ and 4 km s⁻¹ in Models 1 and 2, respectively. And the velocity dispersion σ_R at the same distance has the value of 26 and 20 km s⁻¹ in Models 1 and 2, respectively. Generally, the asymmetric drift and the velocity dispersion are larger in Model 1.

Let us consider the velocity field created by the overdensities, corotating with the disc, in two cases: without self-gravity and with it.

Without self-gravity, the maximal density of the straight segment must correspond to the maximal absolute value of the azimuthal residual velocity V_T . At the radii larger than that of the initial disturber, V_T must be directed in the sense of the galactic rotation (Fig. 2a), so it must have positive value ($V_T > 0$ under $R > R_0$), while at the smaller radii it must be directed in the opposite sense ($V_T < 0$ under $R < R_0$). As for the radial velocity, stars achieve their maximal absolute value of V_R , when they are leaving the straight segment. Thus, without self-gravity, stars in regions of enhanced density must have conspicuous velocity V_T and nearly zero V_R .

When self-gravity is important, the maximal density of the straight segment must correspond to the maximal radial velocity V_R : at radii larger than that of the initial disturber, V_R must be directed away from the galactic centre and be positive ($V_R > 0$ under $R > R_0$) (Fig. 2b), while at the smaller radii it must be directed towards the galactic centre ($V_R < 0$ under $R < R_0$). The azimuthal velocities, on the contrary, achieve their extremal values, when stars leave the straight segment. So with self-gravity, regions of enhanced density must exhibit conspicuous velocity V_R and nearly zero V_T .

It is possible a mixed case, when both the gravity of an initial disturber and self-gravity of a straight segment are important. In this case, we can observe the conspicuous gradient of the radial and azimuthal velocities in the straight segments near overdensities. But both gradients must have definite direction: the larger (smaller) R, the more positive (negative) values of V_R or of V_T .

To study the kinematics in the model discs, we divided them into small squares with the size of $150 \times 150 \text{ pc}^2$ and calculated the average radial and azimuthal residual velocities, V_R and V_T , for stars located inside them at different moments. We divided the velocities into tree groups: negative, positive and close to zero, the latter were those, which did not exceed 3 km s⁻¹ in absolute value, $|V_R| < 3$ or $|V_T| < 3 \text{ km s}^{-1}$.

Fig. 11 exhibits the distribution of the radial V_R and azimuthal V_T velocities averaged in squares in Model 1 at three moments T = 618.75, 632.50 and 646.25 Myr (three rows). Positive velocities



Figure 9. Left-hand panel: variations in the angle β (one between the straight segment and the azimuthal direction) along the radius *R* in Models 1 and 2. Right-hand panel: the histograms of distribution of β .



Figure 10. The average azimuthal velocity of stars \overline{v}_{θ} (solid line) and the velocity of rotation curve v_c (dashed line) in Models 1 and 2. The bars represent the velocity dispersion σ_R . Calculations are made for moments T = 632.5 and 1402.5 Myr in Models 1 and 2, respectively.



Figure 11. Distribution of the radial V_R and azimuthal V_T residual velocities averaged in squares $150 \times 150 \text{ pc}^2$ throughout the galactic disc in Model 1 at three moments T = 618.75, 632.50 and 646.25 Myr (three rows). The average velocities are divided into three groups: negative (light grey squares), close to zero (dark grey squares) and positive ones (black squares). The first column shows the distribution of the relative density n/n_0 in the galactic disc, where *n* is the number of particles in a square and n_0 is the average number of particles in squares at the same radius. The greater the density, the darker the colour of the square. Two overdensities are designated by letters 'A' and 'B'. Near them the velocities V_R and V_T demonstrate the following gradients: the larger (smaller) *R*, the more positive (negative) velocity.



Figure 12. Distribution of the residual velocities V_R and V_T averaged in squares throughout the galactic disc in Model 2 at T = 1402.50 Myr. The left-hand image shows the distribution of the relative density. For more details see caption to Fig. 11. Two overdensities are designated by letters 'C' and 'D'. Near them the velocities increase (decrease) with increasing (decreasing) *R*. Model 2 exhibits density and velocity perturbations of less amplitude in comparison with Model 1.

 $(V_R \text{ or } V_T)$ are shown in black and the negative ones in light grey, the velocities close to zero are denoted in dark grey. The first column shows the distribution of the relative density n/n_0 in the galactic disc, where *n* is the number of particles in a square and n_0 the average number of particles in squares at the same radius. The greater the density, the darker the colour of the square.

We can follow the formation of the straight segments near two overdensities designated by letters 'A' and 'B'. There are conspicuous gradients of velocities V_R and V_T near them at all three moments. And the directions of these gradients coincide with the expected ones.

We also study the velocity field in Model 2. Fig. 12 shows the distribution of the relative density and residual velocities, V_R and V_T , averaged in squares throughout the galactic disc in Model 2 at T = 1402.50 Myr. Two overdensities are designated by letters 'C' and 'D'. We can see the expected velocity gradients near them as well.

However, Figs 11 and 12 demonstrate the direction of the velocity gradients but not the amount of velocity changes. To illustrate them we selected stars inside detail 'B'. Here we study stars located inside the ellipse shown in Fig. 11. The stars were divided into sectors of width $\Delta \theta = 2^{\circ}$ 5 along the galactocentric angle θ . In each sector we calculated the average radial V_R and azimuthal V_T residual velocities, which are shown in Fig. 13. We can see that the range of changes of the velocity V_R is ± 10 km s⁻¹ and that of V_T is ± 5 km s⁻¹. Note that the geometry of pieces of trailing spiral arms is such that the increase in θ corresponds to the decrease in R. The range of changes in R is shown at the upper boundary of Fig. 13. For comparison, the range of changes the velocities V_R in detail A (T = 646.25 Myr) is ± 7 km s⁻¹, but that in details C and D is from -1 to +5 km s⁻¹.

On the whole, the distribution of the negative and positive residual velocities agrees with hypothesis that the straight segments are forming as the response of the disc to the overdensity corotating with



Figure 13. The radial V_R (black circles) and azimuthal V_T (crosses) residual velocities of stars located inside detail 'B'. The velocities were calculated in sectors of width $\Delta \theta = 2^\circ.5$ along galactocentric angle θ . The range of changes in *R* is shown at the upper boundary.

it. The amplitude of velocity changes varies from a few to 10 km s^{-1} . Generally, Model 2 exhibits density and velocity perturbations of less amplitude in comparison with Model 1.

6 DISCUSSION AND CONCLUSIONS

We consider the formation of the straight segments in the stellar galactic discs. For this purpose we constructed two *N*-body simulations, which differs in concentration of mass to the galactic centre. In Model 1 the stellar disc forms the bar in the central region, while in Model 2 the central part of the disc is occupied by the multi-armed spiral pattern.

We identified more than 500 straight segments in the two models. The straight segments are temporal features, which rotate with the average velocity of the disc. The relation between the length L of the model straight segment and its galactocentric distance R can be approximated by the linear law L = kR with the coefficients lying in the range k = 0.86-0.88. The average angle between two neighbouring straight segments in our models appears to be $\overline{\alpha} = 125^{\circ}-127^{\circ}$. All these values are consistent with the observational estimates, $L = (1.0 \pm 0.13)R$ and $\alpha = 120^{\circ}$, derived by Chernin et al. (2001).

We suggest that the formation of the straight segments in stellar discs is connected with the appearance of overdensities corotating with the disc. The great role of such overdensities is revealed in the numerical experiments by D'Onghia et al. (2013). In the first approximation, the response of the stellar disc to such overdensity must have the shape of a straight segment with the length determined by the formula $L = 2.4 f_d R$.

Comparison of the average characteristics of the model straight segments with the parameters of the respondent perturbations shows that the non-linear law $L = 2.4 f_d R$ describes better the connection between L and R than the linear one L = kR (Fig. 6, Table 2).

We suppose that the straight segments can form the polygonal structures only when they touch each other by their edges. In other cases they must destroy each other. Using this hypothesis, we can explain, why the average value of the angle α between two neighbouring segments appears to be $\overline{\alpha} = 125^{\circ} - 127^{\circ}$ in our models.

The angle β between the straight segment and the azimuthal direction has the average value of $\overline{\beta} = 25^{\circ}-28^{\circ}$ in our models. We found the conspicuous decrease in β with radius that can be only partly (within 10°) related to the deviations of the model rotation curves from flat one. Fig. 9 exhibits relatively large values of β in the central and intermediate regions (R < 6 kpc) in both models. One possible explanation of these departures is that the bar or oval modes can interfere directly in the formation of the straight segments here.

We study the kinematics of stars near the overdensities forming in the stellar discs. For this aim we divided model discs into small squares, $150 \times 150 \text{ pc}^2$, and calculated average residual velocities in the radial and azimuthal directions, V_R and V_T . We found specific velocity gradients in the straight segments near the overdensities: at the radii larger than that of the overdensity, the velocities V_R and V_T are positive, while at the smaller radii they are negative. Such velocity field agrees with the hypothesis that the straight segments are forming due to the tuning of the epicyclic motions near the initial disturbers. The amplitude of velocity changes inside straight segments can achieve 10 km s^{-1} .

The most interesting question is the nature of the overdensities bringing the formation of the straight segments. We suppose that the appearance of such overdensities in our models is connected with the interaction of different modes or waves, forming on the galactic periphery and in more central region of the disc. This suggestion has some kinematical foundation. The stars located in the spiral arms inside and outside the CR have opposite phase of the epicyclic motions, and consequently, the opposite residual velocities. Probably, the superposition of such waves destroys the adjusted epicyclic motions of both waves and create overdensities, which have no systematic residual velocities and nearly corotate with the disc.

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