Сагитовские чтения, ГАИШ МГУ
4 -5 февраля 2008

Ю.В. Баркин

Проблемы небесной механики, астрометрии и гравиметрии в современных проектах исследования Луны и Меркурия

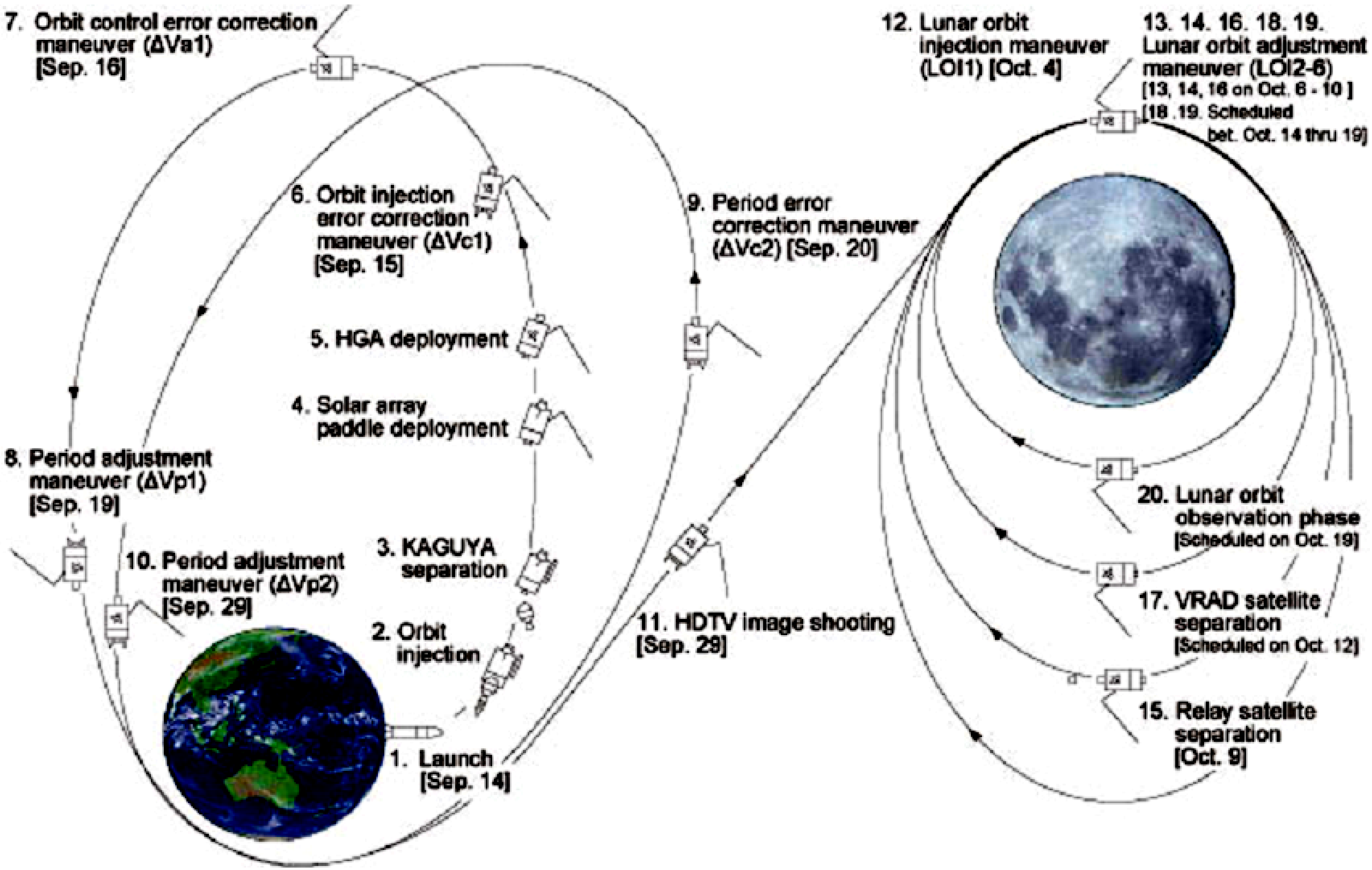
The Moon

Lunar projects

Реализация современных долгосрочных **программ исследования Луны** в рамках проектов:

ЛЛЛ,
SELENE, ILOM (JAXA, Japan),
LRO (NASA),
Chang'e (China),
Chandrayaan (India),
Луна Глоб (РКА, Россия)

направлена как на создание долговременных лунных баз, так и на получение широкого спектра информации о лунном гравитационном поле, о точном положении Луны в инерциальной системе координат, динамической и геометрической фигуре Луны и ее внутреннем строении.



VRAD Satellite (Vstar) separation: 17 in the above figure.

*VRAD Satellite (Vstar): an onboard baby satellite of the KAGUYA equipped with a radiation source for relative VLBI (Very Long Baseline Interferometer)

Table1. SELENE Science Instruments and Experiments

Elemental distribution measurements	
X-ray Spectrometer (XRS)	**Global mapping of Al, Si, Mg, Fe distribution using CCD, spatial resolution 20 km**
Gamma-ray Spectrometer (GRS)	**Global mapping of U, Th, K, major elements, distribution using large pure Ge crystal, Spatial resolution 160 km**
Mineralogical distribution measurements	
Multi-band Imager (MI)	**UV-VIS-NIR imager, spectral bandwidth from0.4 to 1.6 microns, 9 bands filters, spectral resolution 20-30 nm, spatial resolution 20 m**
Spectral profiler (SP)	**Continuous spectral profile ranging from 0.5 to 2.6 microns, spectral resolution 6-8 nm, spatial resolution 500 m**
Topographic measurements of lunar surface and subsurface	
Terrain Camera (TC)	**High resolution stereo camera, spatial resolution 10 m**
Lunar Radar Sounder (LRS)	**Mapping of subsurface structure using active sounding, frequency 5 MHz, echo observation range 5 km, resolution 75 m. Detection of radio waves from the Sun, the Earth, Jupiter, and other planets**
Laser Altimeter (LALT)	**Nd:YAG laser altimeter, 100 mJ output power, height resolution 5 m, spatial resolution 800 m with pulse rate 2 Hz**
Precise gravity field measurements	
Differential VLBI Radio Source (VRAD)	**Differential VLBI observation from ground stations, selenodesy and gravitational field, onboard two sub-satellites**
Relay Satellite Transponder (RSAT)	**Far-side gravimetry using 4 way range rate measurement from ground station to orbiter via relay satellite, perilune 100 km, apolune 2400 km in altitude**
Plasma environment study	
Lunar Magnetometer (LMAG)	**Magnetic field measurement using flux-gate type magnetometer, accuracy 0.5 nT**
Charged Particle Spectrometer (CPS)	**Measurement of high-energy particles, 1-14 MeV(LPD), 2-240 MeV(HID), alpha particle detector, 4-6.5 MeV**
Plasma Analyzer (PACE)	**Charged particle energy and composition measurement, 5 eV/q – 28 keV/q**
Radio Science (RS)	**Detection of the tenuous lunar ionosphere using S and X-band carriers**
Plasma Imager (UPI)	**Observation of terrestrial plasmasphere from lunar orbit, XUV to VIS**
Public outreach	
High Definition TV Camera (HDTV)	**High definition imaging of "Earth's rise" and lunar surface**

Луна

Теория вращения Луны и ее внутренняя динамика

Приливные и неприливные вариации Луны

Приливные вариации селенопотенциала

Вариации сейсмической активности Луны и ее механизмы

W http://en.wikipedia.org/wiki/Internal_structure_of_the_Moon

Geology of the Moon

From Wikipedia, the free encyclopedia

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Exploring Shorty crater during the Apollo 17 mission to the Moon. This was the only Apollo mission to include a geologist. *NASA photo.*


ASTRONOMICAL and ASTROPHYSICAL TRANSACTIONS
CONTENTS
А. ЛЕОНОВ
В. ШАТАЛОВ
H. SCHMITT (APOLLO 17)
GORDON AND BREACH SCIENCE PUBLISHERS
VOLUME 18 ISSUE

W http://en.wikipedia.org/wiki/Internal_structure_of_the_Moon

Geology of the Moon

From Wikipedia, the free encyclopedia

Jump to: navigation, search

Impacts by meteorites and comets are the only abrupt geologic force acting on the Moon today, though the variation of Earth tides on the scale of the Lunar anomalistic month causes small variations in stresses. (Yu. V. Barkin, J. M. Ferrándiz and Juan F. Navarro, 'Terrestrial tidal variations in the selenopotential coefficients,' *Astronomical and Astrophysical Transactions*, Volume 24, Number 3 / June 2005, pp. 215 - 236.) [1] Some of the most important craters used in lunar stratigraphy formed in this recent epoch. For example, the Copernicus crater, which has a depth of 3.76 km and a radius of 93 km, is believed to have formed about 900 million years ago (though this is debatable). The *Apollo 17* mission landed in an area in which the material coming from the Tycho crater might have been sampled.

The Moon

1. The tidal periodic variations of selenopotential coefficients are bigger then corresponding variations of the geopotential coefficients (in 10 times) (Getino, Ferrandiz, 1993).

2. The main tidal variations are clearly are allocated from general list and are determined by formulas:

$$\delta J_2 = 1.534 \times 10^{-8} \cos(l_M),$$

$$\delta C_{22} = 0.7715 \times 10^{-8} \cos(l_M), \quad \delta S_{22} = 1.0389 \times 10^{-8} \sin(l_M),$$

$$\delta C_{21} = -2.221 \times 10^{-8} \cos(F),$$

$$\delta S_{21} = 0.1204 \times 10^{-8} \cos(l_M + F) - 0.1227 \times 10^{-8} \sin(l_M - F).$$

3. Due to terrestrial tides the period of the Moon rotation tests variation (with anomalistic period)

$$\delta T = T \times 0.6521 \times 10^{-8} \cos(l_M) = 15.3316 \text{ ms.}$$

The Earth

The Earth (Cheng, Taplay, 2004)

$$\delta J_2 = -(2.80 \pm 0.22) \times 10^{-10} \cos(V - 30 \pm 19)$$

Fig. 1.— Tidal variations δJ_2 in period $2005 - 2010$ (time measured in Julian days)

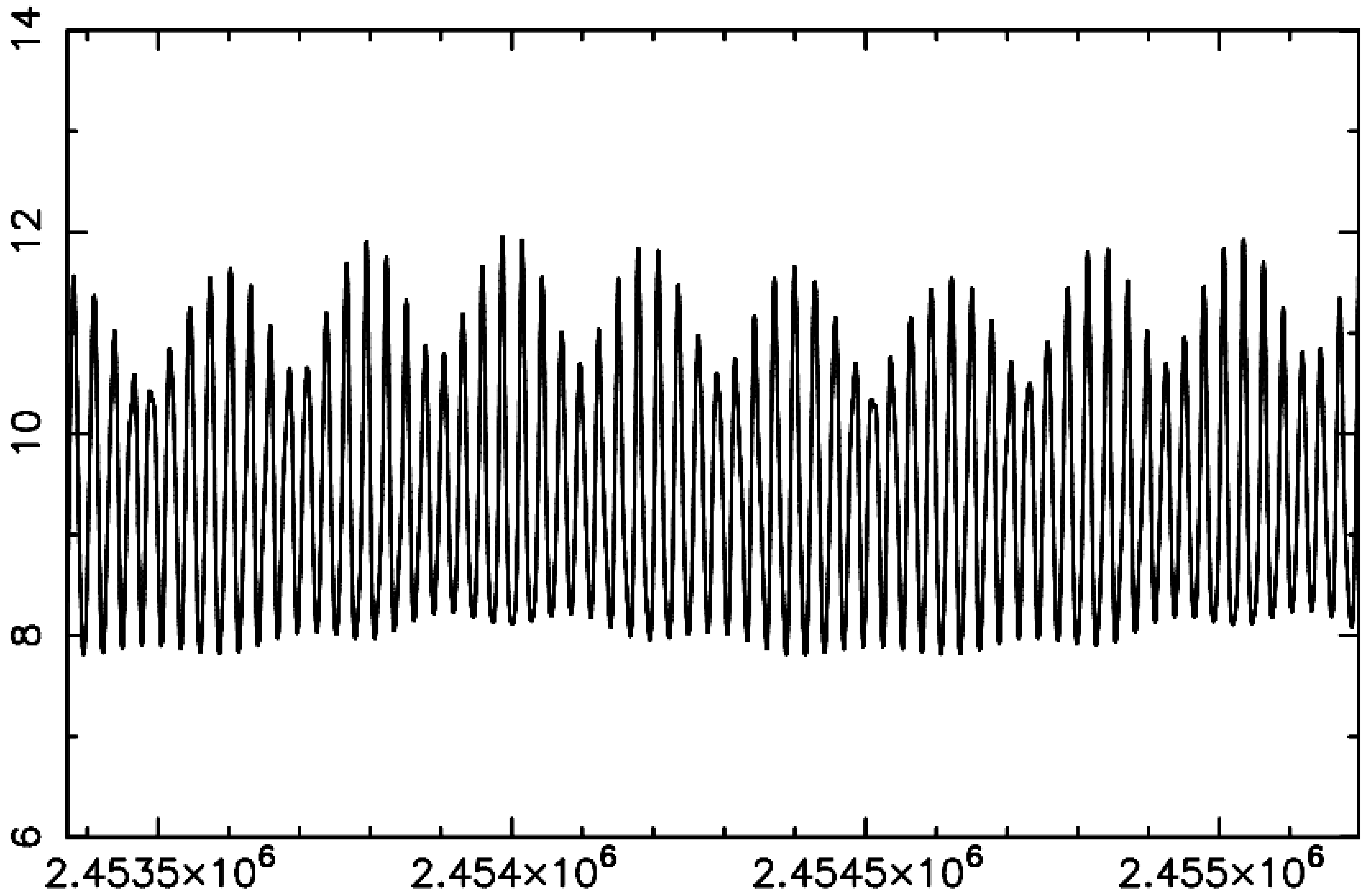


Fig. 2.— Tidal variations δC_{22} in period $2005 - 2010$ (time measured in Julian days)

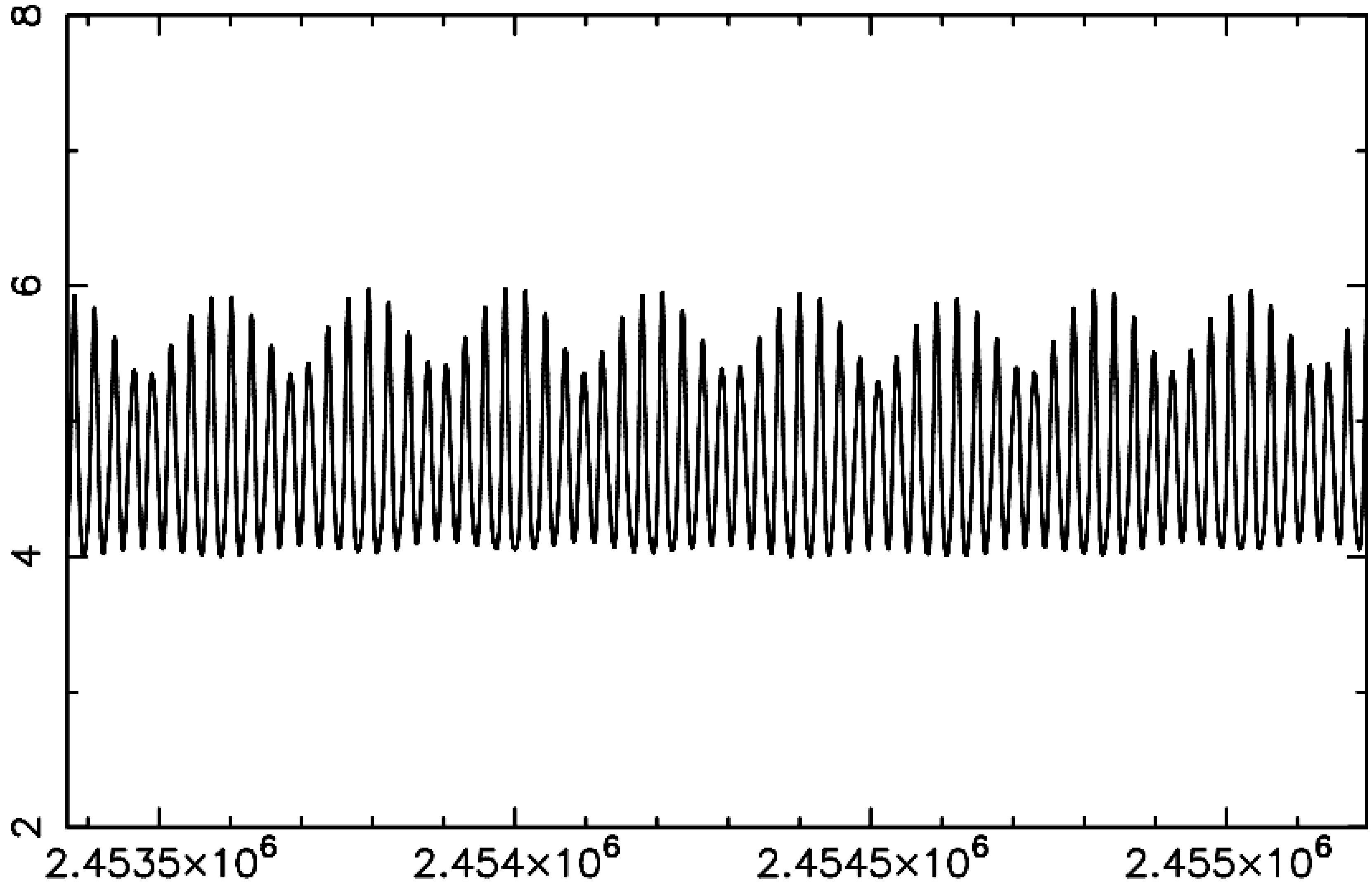
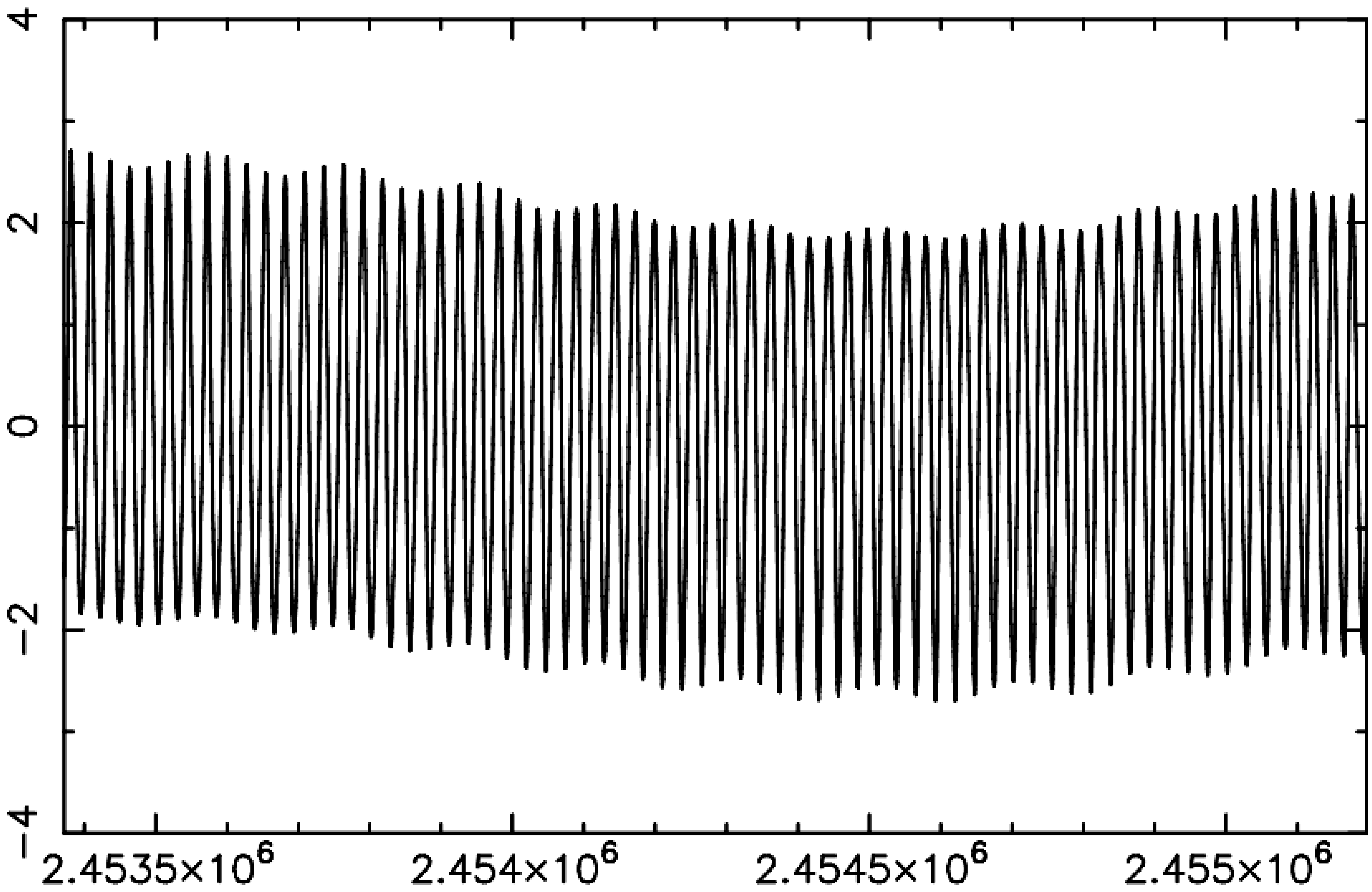


Fig. 3.— Tidal variations δC_{21} in period 2005 – 2010 (time measured in Julian days)



Основные коэффициенты селенопотенциала

$$J_2 = \left(203.428 \pm 0.09\right) \times 10^{-6}$$
$$C_{22} = \left(22.395 \pm 0.015\right) \times 10^{-6}$$

$$\delta J_2 = 1.534 \times 10^{-8} \cos(l_M)$$

$$J_2 = 20342.8 \times 10^{-8} + 1.534 \times 10^{-8} \cos(l_M) \pm 9 \times 10^{-8}$$

$$C_{22} = 2239.5 \times 10^{-8} + 0.77 \times 10^{-8} \cos(l_M) \pm 1.5 \times 10^{-8}$$

Celestial-mechanical nature of the Moon seismicity

36th Annual Lunar and Planetary Science Conference, 2005,

Texas, abstract no. 1076.

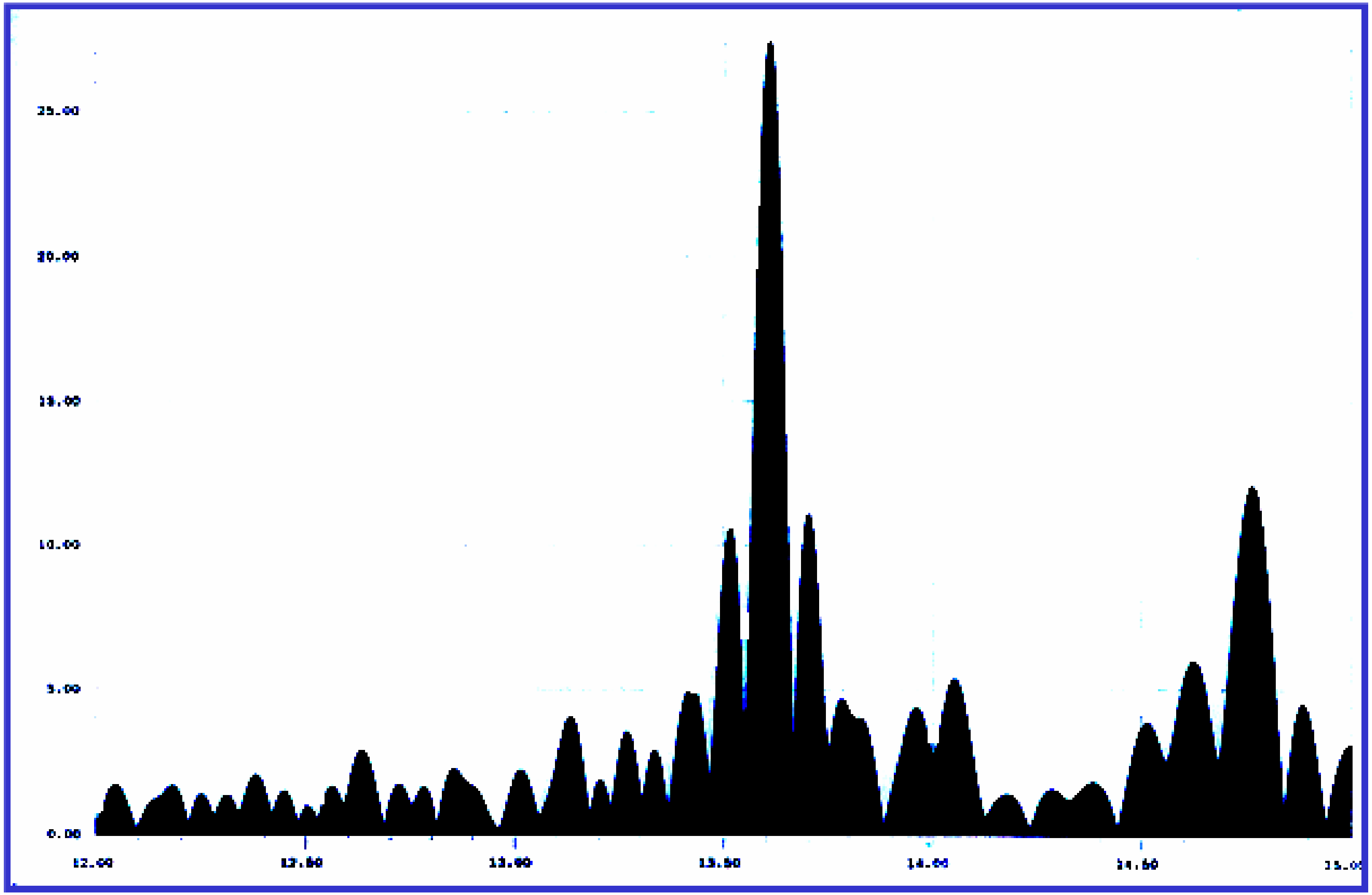
EARTH, MOON, MERCURY AND TITAN SEISMICITY: OBSERVED AND EXPECTED PHENOMENA

[1]Yu.V. Barkin, [2]J.M. Ferrandiz, [2]M. Garcia Ferrandez,
[1]Sternberg Astronomical Institute, Moscow, Russia,[2]Alicante University, Spain

Celestial-mechanical cyclicities of the Moon seismicity

27.4, 13.6, 9, 6.7, 206 days

Lammlein (1977)
Oleinic, Galkin, & Gamburtsev (2000),
Avsujk, Oleinic (2001), Oleinic (2002),
Nakamura Y. (2003) Base data on moonquakes.
Private communication.



25.00
20.00
15.00
10.00
5.00
0.00
12.00
12.50
13.00
13.50
14.00
14.50
15.00

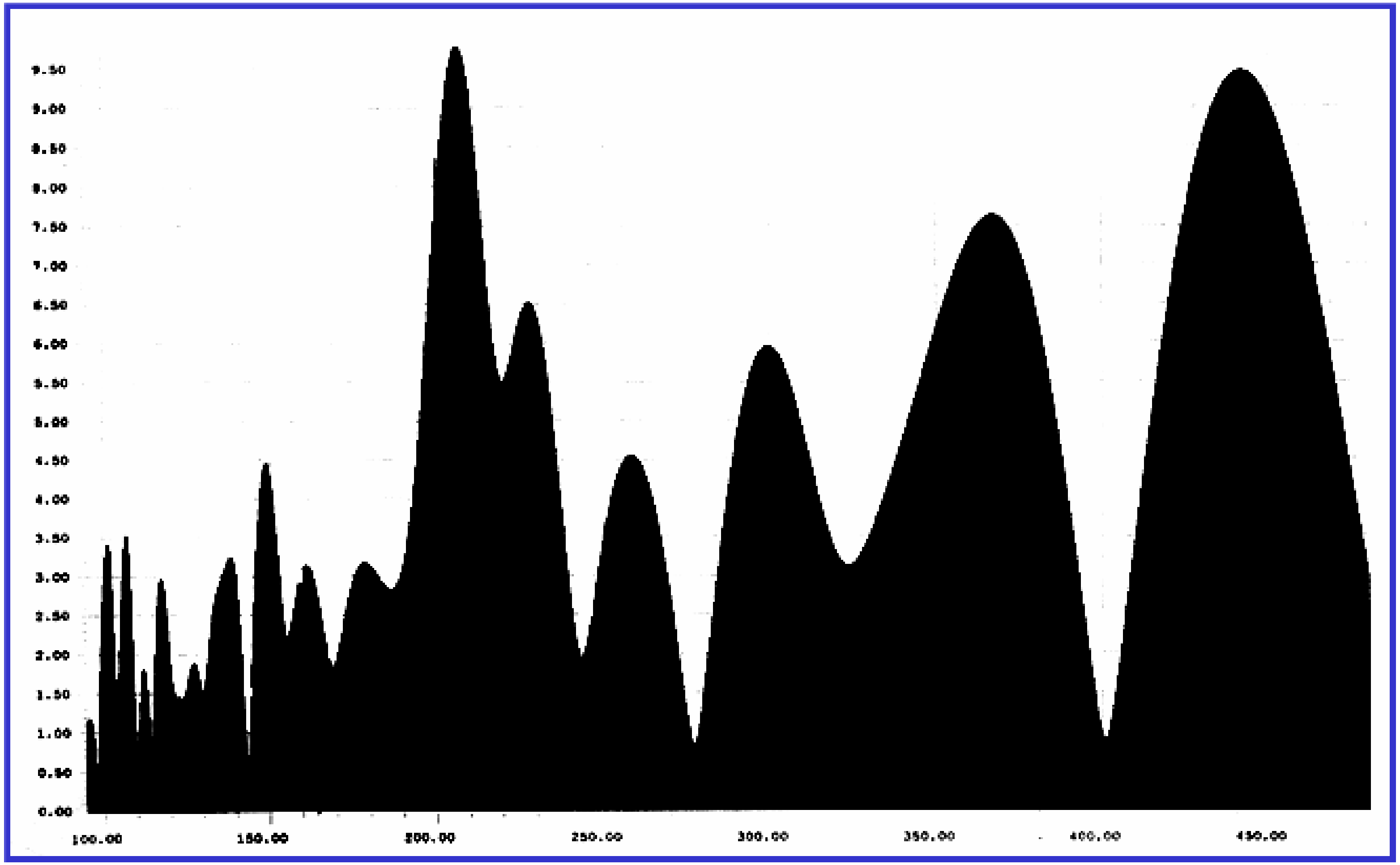
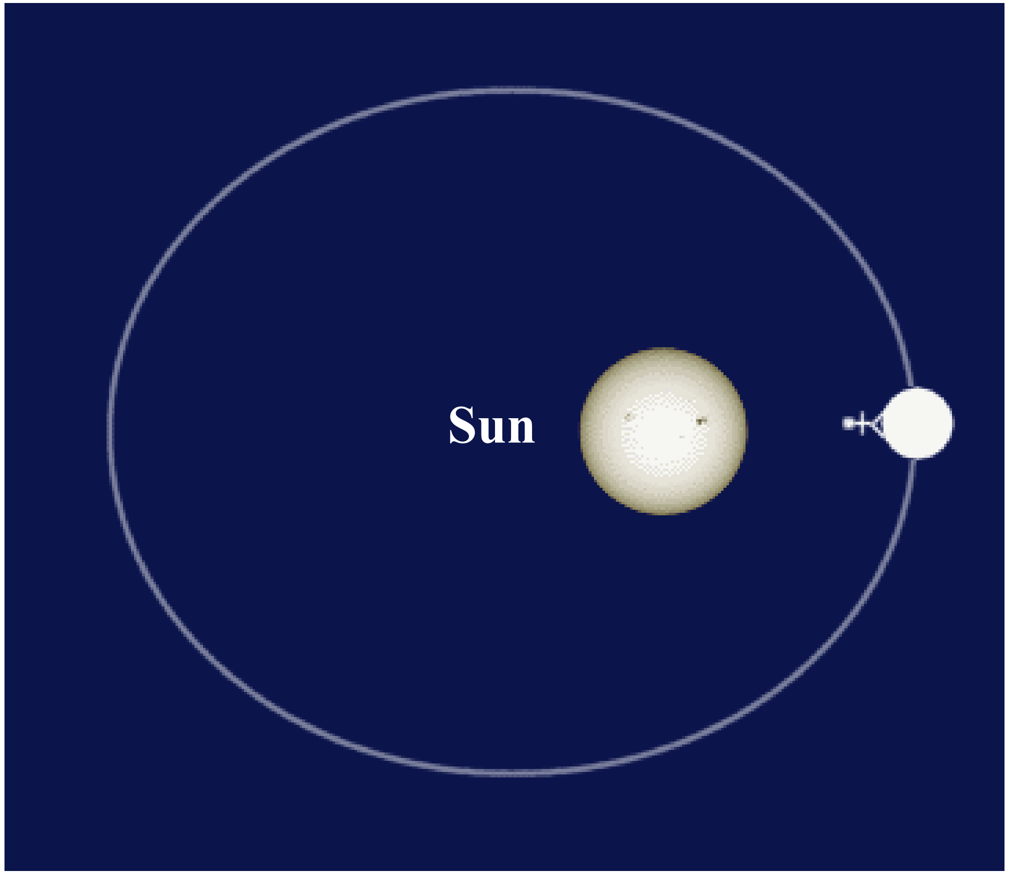

9.50
9.00
8.50
8.00
7.50
7.00
6.50
6.00
5.50
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
100.00
150.00
200.00
250.00
300.00
350.00
400.00
450.00

Table 1. Correlation of orbital periods with periods of variation of the Moon seismic activity. Predicted periods of seismic activity of Titan and Mercury.

Orbit periods (days)	Observed seismic periods	Predicted seismic periods	Conditional amplitudes	Titan seismicity	Mercury seismicity
$T_{drac}/2=$ 13.611	13.608	13.8	2371	7.9722	---
$T_{anom}=$ 27.54	27.570	27.4	1214	15.9455	87.969
$T_{syn}/2=$ 14.766	14.765	---	1042	7.9846	---
$T_{drac}/4=$ 6.805	6.804	6.76	815	3.9861	---
$T_{drac}=$ 27.22	27.18	27.4	680	15.9444	---
$T_{syn}=$ 29.53	29.56	---	495	15.9661	---
$T_{syn}/3=$ 9.844	9.840	---	373	5.3230	---
$T_{drac}/6=$ 4.537	4.535	---	4.535	2.6574	---
$T_{syn}/4=$ 7.383	7.380	7.24	348	3.9923	---
$T_{drac}/3=$ 9.074	9.071	9.02	314	5.3148	---
$T_{anom}/4=$ 6.884	6.871	6.76	289	3.9864	21.992
$T_{drac}/8=$ 3.403	3.395	---	285	1.9961	---
$T_{anom}/5=$ 5.508	5.500	---	261	3.1891	17.594
$T_{syn}/11=$ 2.685	2.685	---	253	1.4517	---
$T_{drac}/5=$ 5.444	5.455	---	249	3.1889	---
$T_{anom}/8=$ 3.442	3.470	---	249	1.9932	10.996
$T_{synod}/8=$ 3.692	3.735	---	236	1.9961	---
$T_{synod}/7=$ 4.219	4.205	---	234	2.2813	---

Mercury

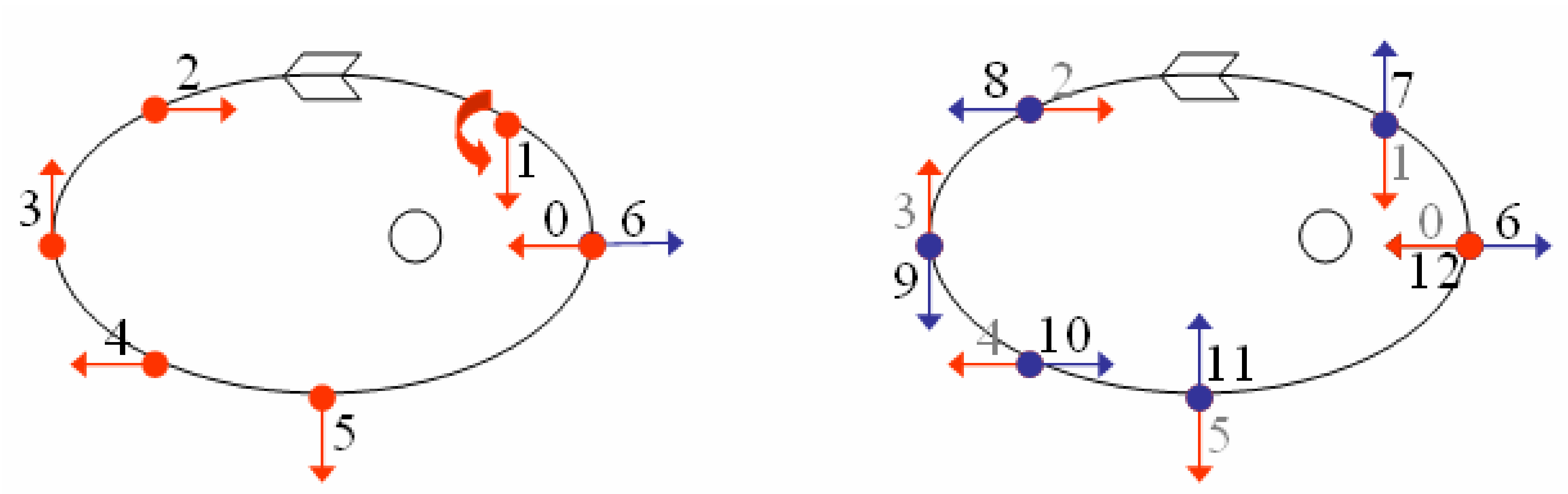
Mercury resonant motion 3:2



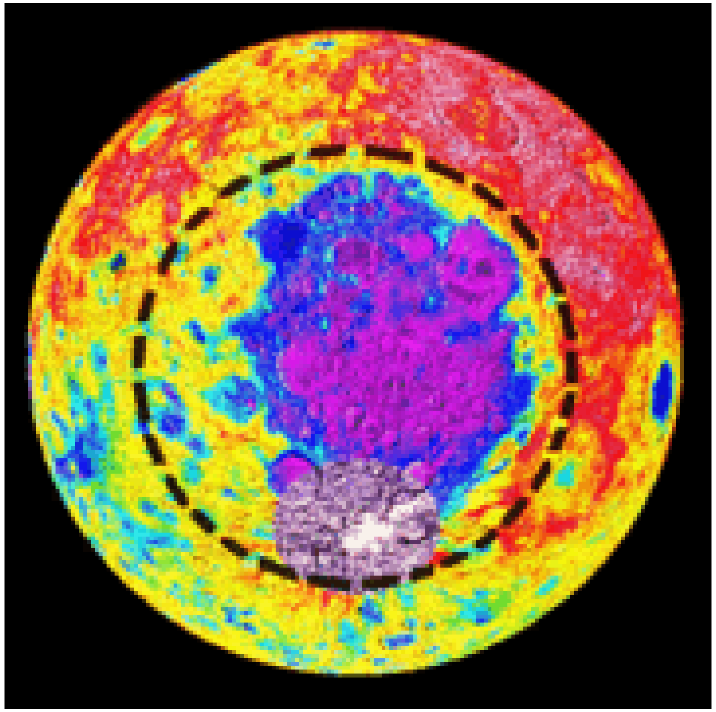
Mercury

Plane unperturbed Mercury rotation

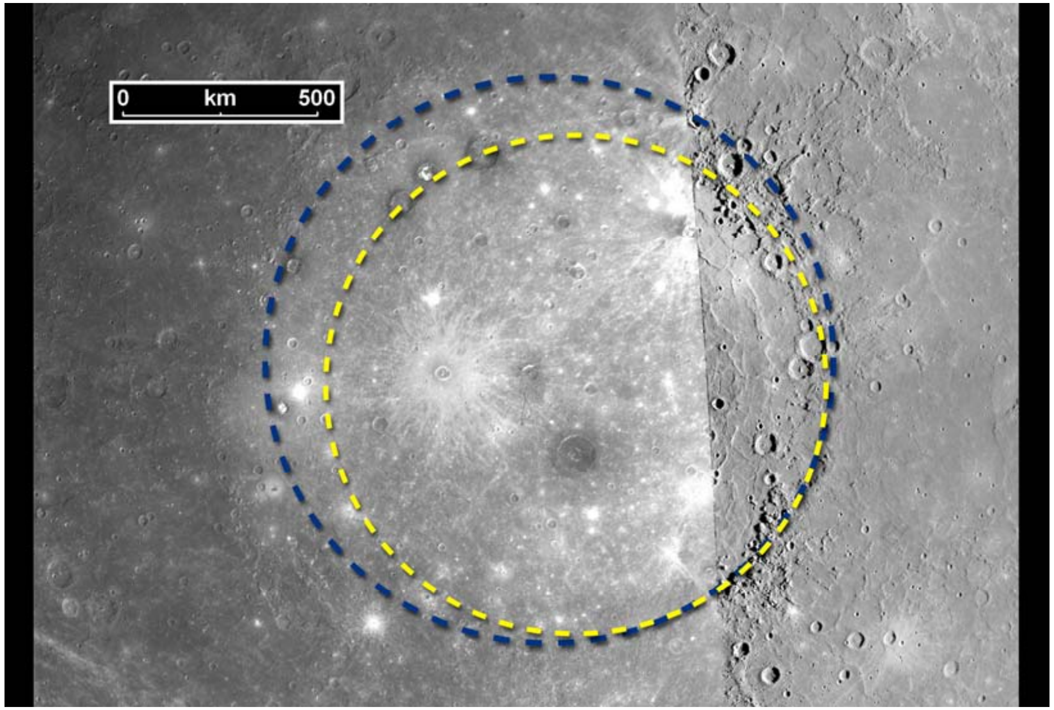
- **The libration is closely related to the planet's equatorial bulge**
- **Mercury's 3:2 spin-orbit resonance**
- **Gravitational torques tend to align this deformation in the Sun-Mercury direction**
- **Mercury is close to the Sun & large C22 ⇒ large libration**



Messenger and Bepi Colombo



The Moon



Mercury

Topographical map of the South Pole-Aitken basin based on [Clementine] data. Red represents high elevation, purple represents low elevation.

Expected results from Messenger

- They expect a global precision on X-band of 0.1 to 0.2 mm/s@60seconds; equivalent to 0.2 to 0.5mm/s@10sec.
- They will be there at high solar activity;
- They will reject data at elongation <5º;
- Problem of power consumption, so no regular tracking;
- NH: gravity up to 30-40 possible but unrealistic, depending on the coverage;
- Global field up to 8, meaning a S/N ratio =1 at degree 8; this doesn't mean they will obtain a precise field up to that degree; most probably precise up to 4-6;
- Line-of-sight measurement success depends on the coverage, which depends on the mission design;
- Degree 2 improvement expected: simulations have shown a precision of 0.5% on C_{20} and 0.2% on C_{22};
- Libration precision expected: 7%.

Mercury

	Mariner-10	
$J_2 = (6.0 \pm 2.0) \times 10^{-5}$	33%	
$C_{22} = (1.0 \pm 0.5) \times 10^{-5}$	50%	
	(Barkin et al., 2007)	MESSENGER
$J_2 = (4.23 \pm 0.06) \times 10^{-5}$	1.4%	0.5 %
$C_{22} = (0.85 \pm 0.05) \times 10^{-5}$	5.9%	0.2 %
$T_g = 12.2 \pm 0.3$ yr	2.5 %	---
$T_l = 426 \pm 25$ yr	5.9%	---
$T_h = 1462 \pm 69$ yr	4.7%	---
$T_{l_c} = 58.6252$ days		

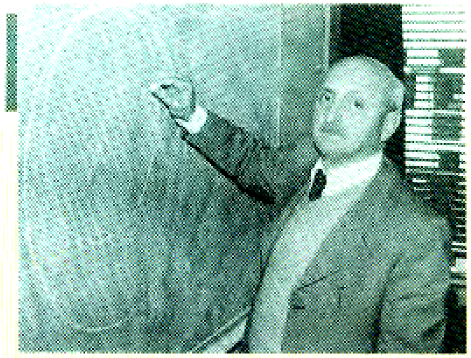
Barkin Yu.V., Ferrandiz J.M. (2007) Dynamic role of the liquid core of Mercury in its motion on Cassini's laws and in resonant librations. Abstracts of European Planetary Science Congress (Potsdam, Germany, 19 – 24 August 2007) Vol.2, EPSC 2007-A-00259.

BepiColombo Mercury ESA cornerstone mission

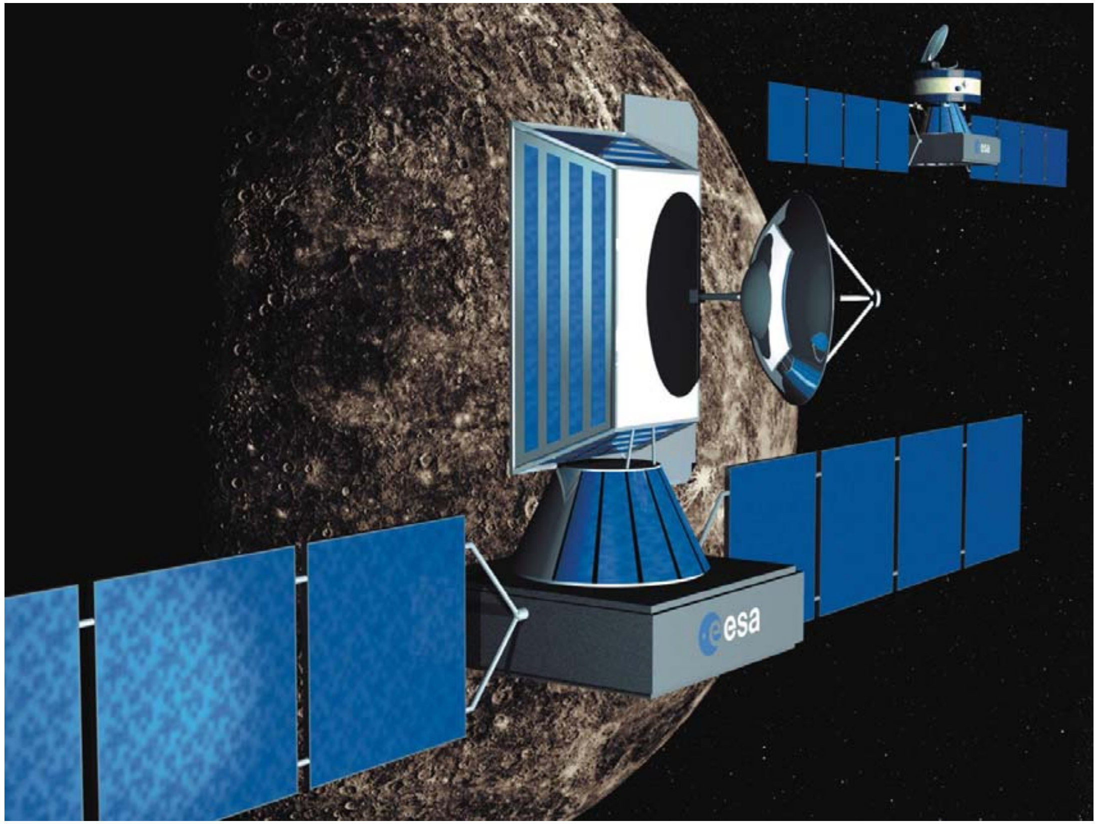
Detailed Scientific objectives:

- exploration of Mercury's unknown hemisphere;
- investigation of the geological evolution of the planet;
- understanding the origin of Mercury's high density;
- analysis of the planet's internal structure and search for the possible liquid outer core;
- investigation of the origin of Mercury's magnetic field;
- study of the planet's magnetic field interaction with the solar wind;
- characterisation of the surface composition;
- identification of the composition of the radar bright spots in the polar regions;
- determination of the global surface temperature;
- determination of the composition of Mercury's vestigial atmosphere (exosphere);
- determination of the source/sink processes of the exosphere;
- determination of the exosphere and magnetosphere structures;
- study of particle energisation mechanisms in Mercury's environment;
- fundamental physics: verification of Einstein's theory of gravity.

Bepi-Colombo dual spacecrafts



Giuseppe Colombo (1920-1984). The ESA Science Programme Committee (SPC) recognised the achievements of the late Giuseppe (Bepi) Colombo of the University of Padua by adopting his name for the Mercury Cornerstone mission. The Italian scientist was a mathematician and engineer of astonishing imagination who explained Mercury's peculiar habit of rotating three times around itself in every two revolutions around the Sun. He also advised NASA how to place Mariner-10 into an orbit that would enable it to perform three flybys of the planet Mercury in 1974-1975



Giuseppe Colombo (Bepi Colombo)

Giuseppe Colombo (1920-1984). The ESA Science Programme Committee (SPC) recognised the achievements of the late Giuseppe (Bepi) Colombo of the University of Padua by adopting his name for the Mercury Cornerstone mission. The Italian scientist was a mathematician and engineer of astonishing imagination who explained Mercury's peculiar habit of rotating three times around itself in every two revolutions around the Sun. He also advised NASA how to place Mariner-10 into an orbit that would enable it to perform three flybys of the planet Mercury in 1974-1975

On Cassini's laws

Sov. Astron. 22(1), Jan.-Feb. 1978

Yu. V. Barkin

P. K. Shternberg State Astronomical Institute

(Submitted July 7, 1976)

Astron. Zh. 55, 113–122 (January–February, 1978)

The generating periodic solutions of Poincaré for the problem of the rotational motion of a triaxial satellite about its center of mass in a circular uniformly precessing orbit are interpreted as generalized Cassini's laws. A qualitative analysis of the analytical conditions for the existence of Cassini motions is given. The results are used to refine the values of the parameters of the moon's physical libration.

[1] J. L. Lagrange, Mem. Acad. Berlin, in: Oevres de Lagrange, Vol. 5, J. H. Serret, ed., Gauthiers–Villars, Paris (1780).

[2] G. Colombo, Astron. J. 71, 891 (1966).

[3] S. J. Peal, Astron. J. 74, 483 (1969).

[4] V. V. Beletskii, "On Cassini's laws," Preprint No. 79, Inst. Prikl. Mat. Akad. Nauk SSSR (1971).

[5] M. L. Lidov and A. I. Neitshtadt, "The method of canonical transformations in problems on the rotation of celestial bodies and Cassini's laws," Preprint No. 9, Inst. Prikl. Mat. Akad. Nauk SSSR (1973).

[6] G. D. Cassini, Traite de Lorigine et du Progres de L'astronomie, Paris (1963).

[7] H. Andoyer, Mecanique Celeste, Vol. 1, Gauthier–Villars, Paris (1923).

[8] H. Konoshyta and Y. Kozai, Publ. Astron. Soc. Jpn. 25, 393 (1971).

[9] A. Poincare, Selected Works [Russian translation], Vol. 1, Nauka, Moscow (1971).

[10] Yu. V. Barkin, Astron. Zh. 53, 1110 (1976) [Sov. Astron. 20, 623 (1976)].

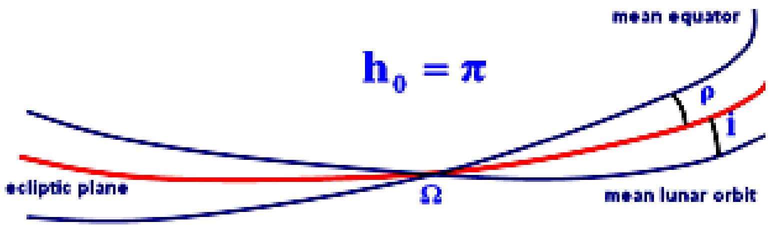
[11] H. Jeffreys, Mon. Not. R. Astron. Soc. 153, 73 (1971).

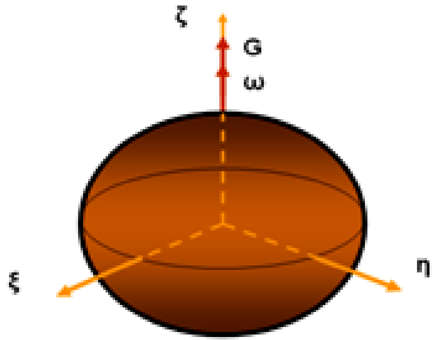
[12] K. A. Kulikov and V. B. Gurevich, Principles of Lunar Astrometry [in Russian], Nauka, Moscow (1972).

1. **Tides**
2. **Gravity analysis**
3. **Libration**
4. **Moment of inertia**
5. **Interior modeling**

Cassini´s laws from XVII century to our days

Cassini (1693)
Lagrange (1764)
Colombo (1965)
Goldreich, Peale (1966)
Peale (1969)
Beletskii (1971)
Lidov, Neishtadt (1973)
Ward (1976)
Barkin (1978)

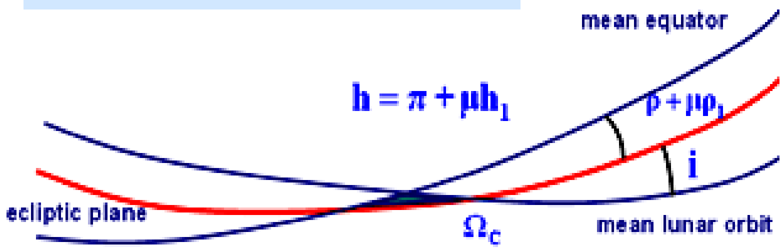


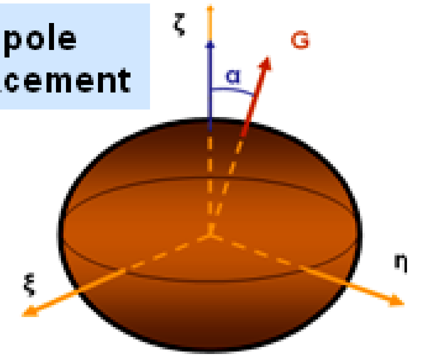


Splitting of Cassini's node

Barkin (1986-1990)

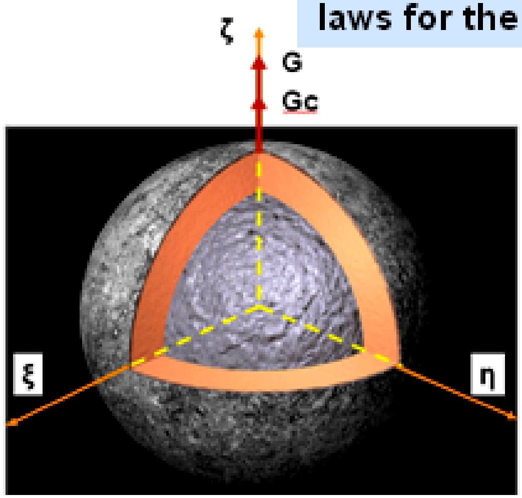
Mean pole displacement

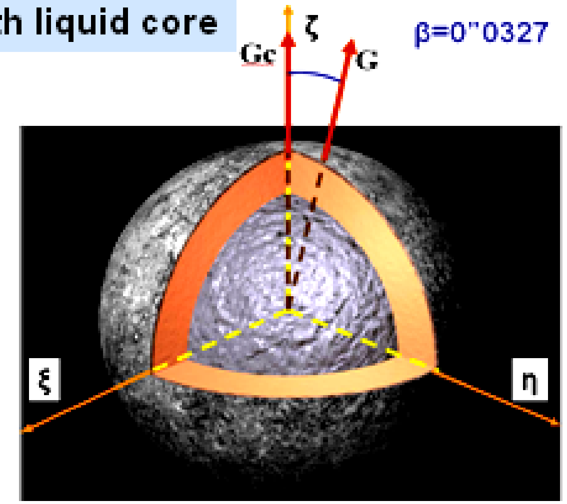




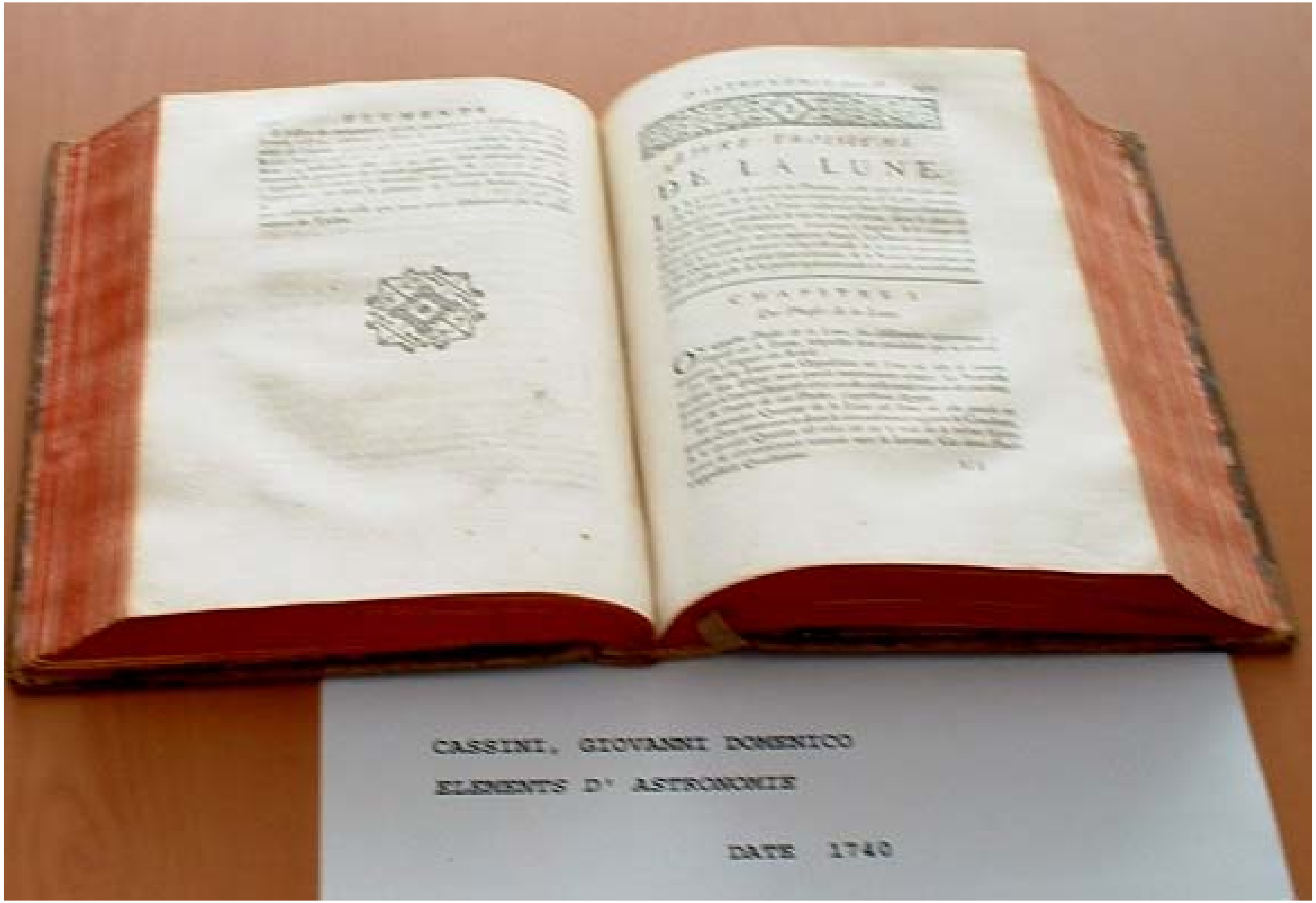
Barkin, Ferrandiz (2003)

Mercury generalized Cassini's laws for the planet with liquid core



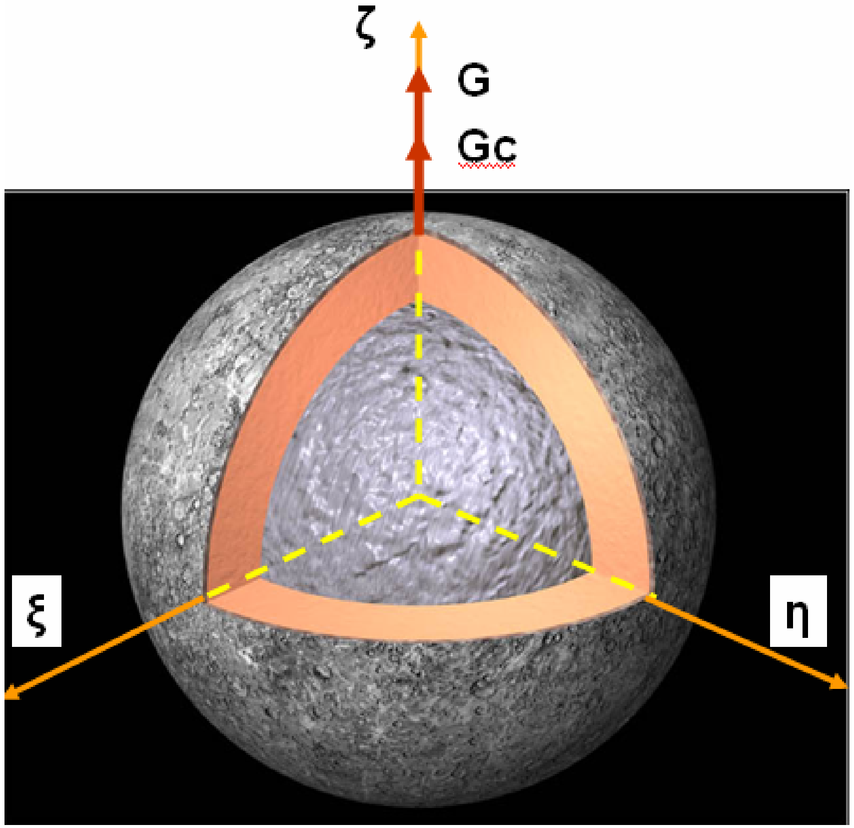


Cassini´s laws studies from XVII century

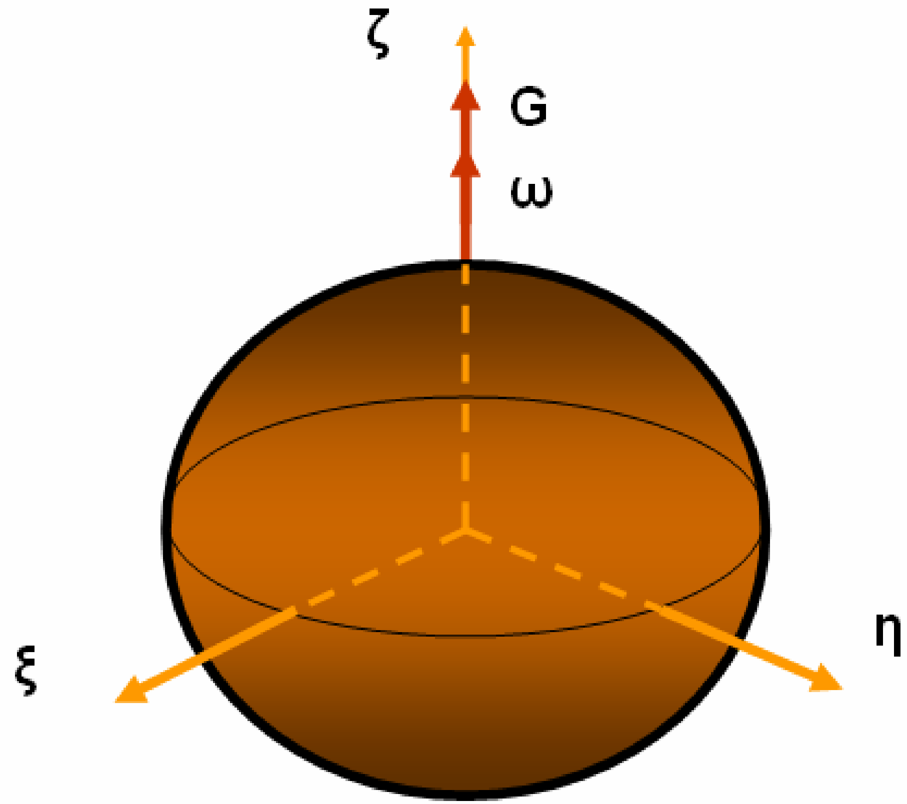


Трактат Кассини, 1740

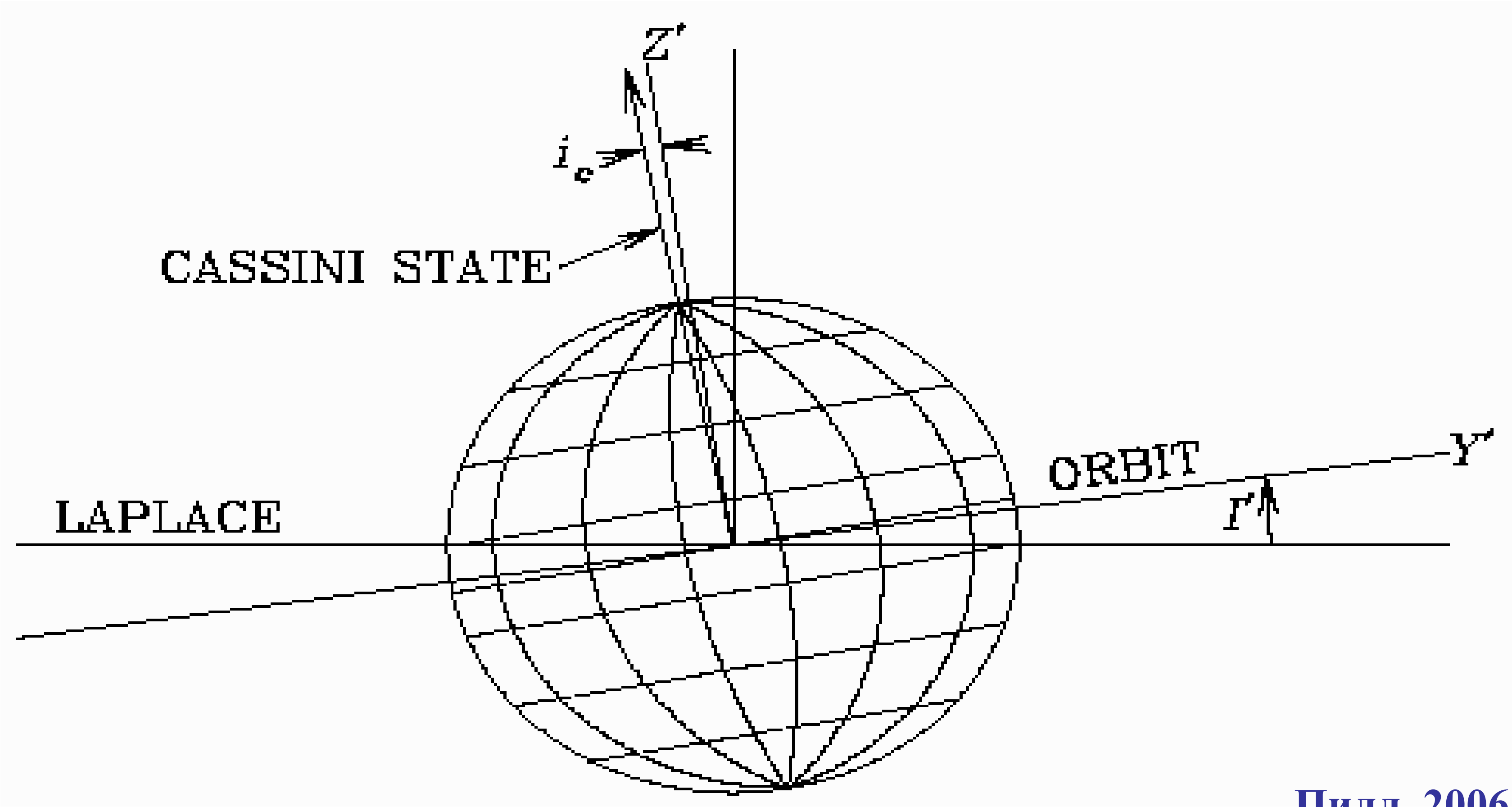
Angular momentums of Mercury and its core



Axial Cassini´s rotation



Движение Меркурия по Кассини-Коломбо



Пилл, 2006

Рис. Восходящий узел экватора Меркурия на плоскости орбиты и восходящий узел орбиты на плоскости Лапласа совпадают. Угол между нормалью к плоскости орбиты и осью вращения Меркурия составляет 2 минуты дуги. Плоскость орбиты прецессирует относительно нормали к плоскости Лапласа с периодом 283527 лет.

MEAN INCLINATION OF THE AXIS OF MERCURY ROTATION IN ORBITAL REFERENCE SYSTEM

Beletskii equation (1971)

$$\cos i + \varepsilon_1 \sin i \,\mathrm{ctg}\rho + \beta\cos\rho + \alpha = 0 \qquad \text{(B)}$$

$$\beta = -\frac{3}{8}\frac{n^2}{n_\Omega \omega}\left(\frac{A-C}{B}\right)\left[2(2\delta-1)X_0^{-3.0} + X_N^{-3.2}\varepsilon_2\right], \quad \alpha = -\frac{3}{8}\frac{n^2}{n_\Omega \omega}\left(\frac{A-C}{B}\right)X_N^{-3.2}\varepsilon_2$$

$\delta = \dfrac{A-B}{A-C}$ **(here** A, C **are equatorial moments of inertia).**

Peale equation (Peale, 1968, 1988, Wu et al., 1997)

$$\frac{mr_0^2}{C} = \frac{\mu}{n}\frac{\sin(i+\rho)}{\left[\sin\rho(1+\cos\rho)X_3^{-3.2}C_{22} - \cos\rho C_0^{-3.0}C_{22}\right]} \qquad \text{(P)}$$

$$C_0^{-3.0} = (1-e^2)^{-3/2}, \qquad X_3^{-3.2} = \frac{7}{2}e - \frac{123}{16}e^3 + \frac{4971}{2560}e^5 + \ldots$$

MEAN INCLINATION OF MERCURY AXIS OF ROTATION

Approximate analytical solution (Barkin, 1985)

$$\rho_0 = \frac{-\varepsilon_1 \sin i}{\cos i + \dfrac{n}{In_\Omega}\left[-C_{20}X_0^{-3,0}(e) + 2C_{22}X_3^{-3,2}(e)\varepsilon_2\right]}$$

$$\varepsilon_1 = \cos h_0 = \pm 1, \quad \varepsilon_2 = \cos 2(g_0 + h_0 - \omega_0) = \pm 1$$

Barkin Yu.V. (1985) $\rho = 1'24$; $\rho = 1'39$;

Rambaux N., Bois E. (2004) $\rho = 1'60$;
$\rho \in (1'33 - 2'65)$.

Rambaux N., Bois E. (2004) **Theory of the Mercury's spin-orbit motion and analysis of its main librations. Astronomy &Astrophysics.413, pp. 381-393.**

CASSINI'S LAWS FOR CORE-MANTLE SYSTEM

$$\theta = 0\,,\ \theta_c = \pi$$

$$p = q = 0\,, p_c = q_c = 0$$

$$n_\lambda = n_F = r = \Omega \quad (\lambda = n_F t + \lambda_0 = F),$$

$$\lambda_0 = F_0\,,$$

$$h_0 = \pi\,,$$

$$\bar{B}\cos 2\rho + \bar{E}\sin 2\rho + \bar{P}\cos\rho + \bar{I}\sin\rho = 0\,,$$

$$\bar{B} = \sum_{v_5}\left[A^{(1)}_{0.0.0.0.v_5} + \chi(A^{(1)}_{0.0.2.0.v_5} + A^{(1)}_{0.0.-2.0.v_5})\right](-1)^{v_5}\,,$$

$$\bar{E} = \frac{1}{4}\sum_{v_5}\left[-2A^{(0)}_{0.0.0.0.v_5} + A^{(2)}_{0.0.0.0.v_5} + \chi(A^{(2)}_{0.0.2.0.v_5} - 2A^{(0)}_{0.0.2.0.v_5} + A^{(2)}_{0.0.-2.0.v_5} - 2A^{(0)}_{0.0.-2.0.v_5})\right](-1)^{v_5}\,,$$

$$\bar{P} = \sum_{v_5}\chi\left[A^{(1)}_{0.0.2.0.v_5} - A^{(1)}_{0.0.-2.0.v_5}\right](-1)^{v_5}\,,$$

$$\bar{I} = -\frac{2n_\Omega\Lambda}{3(A+B-2C)n_0^2} + \frac{1}{2}\chi\sum_{v_5}\left[A^{(2)}_{0.0.2.0.v_5} - A^{(2)}_{0.0.-2.0.v_5}\right](-1)^{v_5}\,.$$

Astronomical and Astrophysical Transactions
Vol.23, December 2004, 533-554

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COMPARATIVE ROTATIONAL DYNAMICS OF THE MOON, MERCURY AND TITAN

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Astronomical and Astrophysical Transactions
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Dynamic structure and rotation of Mercury

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Science

AAAS

Large Longitude Libration of Mercury Reveals a Molten Core
J. L. Margot, *et al.*
Science **316**, 710 (2007);
DOI: 10.1126/science.1140514

Large Longitude Libration of Mercury Reveals a Molten Core

J. L. Margot,[1*] S. J. Peale,[2] R. F. Jurgens,[3] M. A. Slade,[3] I. V. Holin[4]

Observations of radar speckle patterns tied to the rotation of Mercury establish that the planet occupies a Cassini state with obliquity of 2.11 ± 0.1 arc minutes. The measurements show that the planet exhibits librations in longitude that are forced at the 88-day orbital period, as predicted by theory. The large amplitude of the oscillations, 35.8 ± 2 arc seconds, together with the Mariner 10 determination of the gravitational harmonic coefficient C_{22}, indicates that the mantle of Mercury is decoupled from a core that is at least partially molten.

Data of radar observations

ID	MJD	w (s)	τ (s)	σ	Spin rate
020513x00Goc	52407.889680	4.41	−12.36958	1.61×10^{-5}	0.999985
020522x00Goc	52416.871258	6.27	−12.69218	1.58×10^{-5}	0.999893
020602x00Goc	52427.845539	5.98	−11.84078	1.69×10^{-5}	0.999861
020612x00Goc	52437.816052	6.35	−11.21227	1.78×10^{-5}	0.999945
030113x00Goc	52652.760205	10.32	−10.93565	1.37×10^{-5}	1.000097
030123x00Goc	52662.725791	8.40	−11.29540	5.28×10^{-6}	1.000073
030531x00Goc	52790.846925	7.76	−8.30671	8.21×10^{-6}	0.999932
030601x00Goc	52791.844164	7.37	−8.36904	1.02×10^{-5}	0.999949
030918sA3Aoc	52900.630649	26.79	−14.67124	5.03×10^{-5}	1.000093
030919sA3Aoc	52901.628954	28.95	−14.70674	5.50×10^{-5}	1.000065
030920sA3Aoc	52902.627306	26.94	−14.71705	5.18×10^{-5}	1.000067
040331x00Goc	53095.968346	5.92	−7.41574	1.20×10^{-5}	1.000098
041212x00Goc	53351.866334	7.86	−7.74059	1.55×10^{-5}	1.000070
041218x00Goc	53357.848521	7.75	−7.38323	1.05×10^{-5}	1.000067
041219x00Goc	53358.845401	8.13	−7.39330	9.67×10^{-6}	1.000075
050313x00Goc	53443.004320	7.04	−4.78491	3.19×10^{-5}	1.000035
050314x00Goc	53444.001094	6.87	−4.98779	9.83×10^{-6}	1.000056
050316x00Goc	53445.994761	6.72	−5.31606	1.45×10^{-5}	1.000047
050318x00Goc	53447.988621	6.24	−5.53562	1.52×10^{-5}	1.000056
060629x00Goc	53915.735467	8.03	−11.27627	8.35×10^{-6}	0.999866
060712x00Goc	53928.676641	7.94	−10.71493	6.43×10^{-6}	0.999882

$$\phi = \frac{3}{2}\frac{(B-A)}{C} f(e)\sin(nt) \qquad (1)$$

Meteoritics & Planetary Science 37, 1269–1283 (2002)
Available online at http://www.uark.edu/meteor

A procedure for determining the nature of Mercury's core

STANTON J. PEALE[1]*, ROGER J. PHILLIPS[2], SEAN C. SOLOMON[3], DAVID E. SMITH[4] AND MARIA T. ZUBER[5]

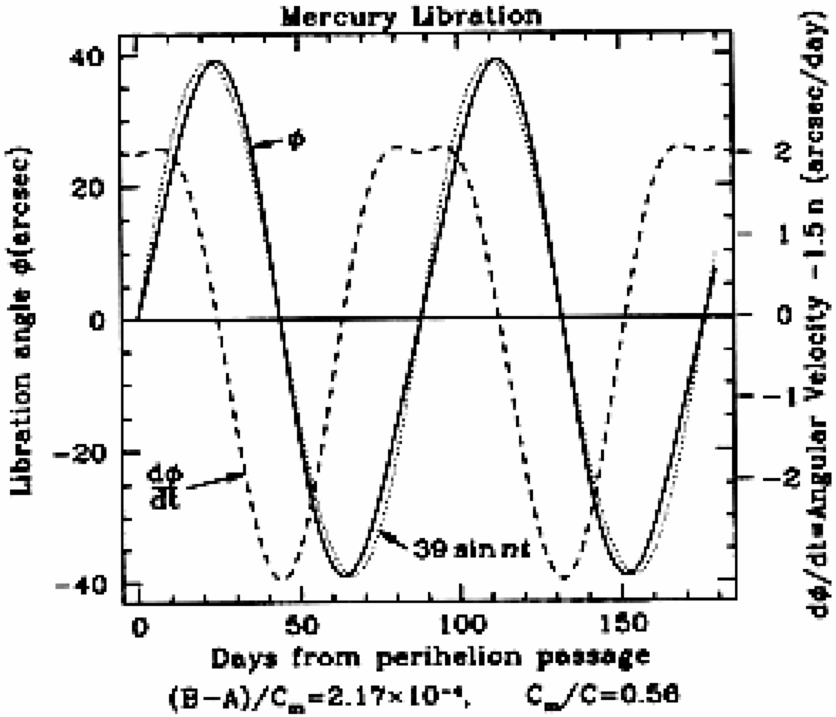
We estimate a maximum relative velocity between the core fluid and mantle by assuming the core to be uniformly rotating during the libration of the mantle about this rotation rate. From Eqs. (1) and (3), we have

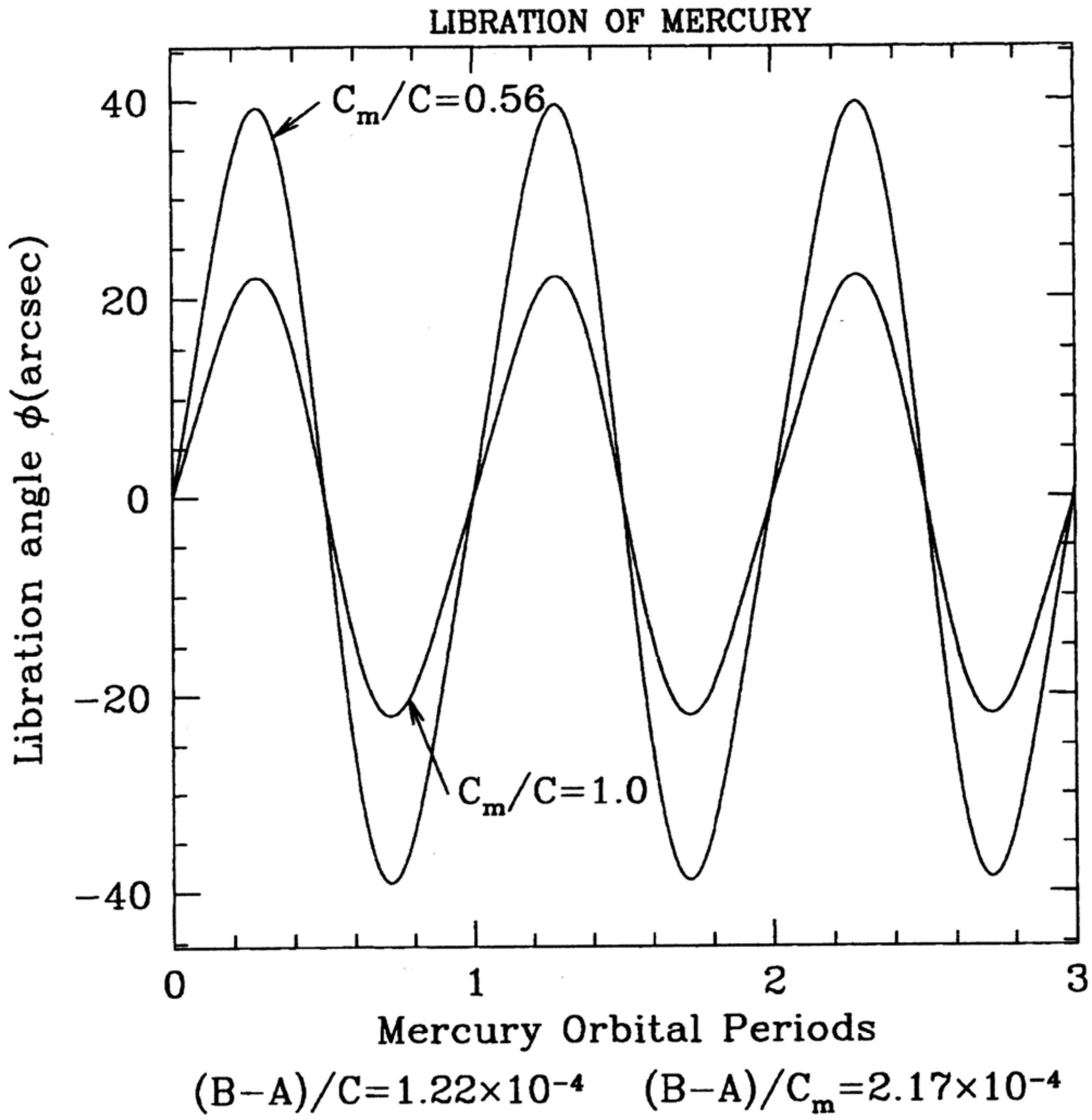
$$\phi_0 = 3.42 C_{22} \frac{MR^2}{C} \frac{C}{C_m} \tag{13}$$

where $C_{22} = (1.0 \pm 0.5) \times 10^{-5}$ (Anderson *et al.*, 1987).

Properties of Mercury's interior shown in Table 2 follow from Schubert *et al.* (1988) with the density distribution of Siegfried and Solomon (1974) cited therein. As before, the subscripts "c", "m", and "ic" designate core, mantle, and inner core, respectively, and those symbols without subscripts refer to the entire planet. For the model of Table 2, $C_m/C = 0.56$ and $C/MR^2 = 0.33$ such that $19 \le \phi_0 \le 57$ arcsec. If we choose $\phi_0 = 40$ arcsec,

$$\phi = 40(\text{arcsec}) \sin nt, \qquad \frac{d\phi}{dt} = 1.61 \times 10^{-10} \cos nt \ (\text{rad/s}) \tag{14}$$




LIBRATION OF MERCURY
$C_m/C=0.56$
$C_m/C=1.0$
Libration angle ϕ(arcsec)
40
20
0
−20
−40
0
1
2
3
Mercury Orbital Periods
$(B-A)/C=1.22\times10^{-4}$ $(B-A)/C_m=2.17\times10^{-4}$

Периодическая либрация Меркурия с периодом 88 суток.
“Кулачки” угловой скорости вращения Меркурия (Баркин, 1979).

Опуская промежуточные выкладки, приведем окончательные формулы для изучаемого периодического решения:

$$
\begin{aligned}
a &= a_0 + \nu\{a_0^{(1)} + a_1^{(1)}\cos\tau + a_2^{(1)}\cos 2\tau + a_3^{(1)}\cos 3\tau + \ldots\} + \ldots,\\
e &= e_0 + \nu\{e_0^{(1)} + e_1^{(1)}\cos\tau + e_2^{(1)}\cos 2\tau + e_3^{(1)}\cos 3\tau + \ldots\} + \ldots,\\
&\boxed{H = H_0 + \nu\{H_1^{(1)}\cos\tau + H_2^{(1)}\cos 2\tau + H_3^{(1)}\cos 3\tau + \ldots\} + \ldots},\\
l &= n^{(0)}t + \nu\{l_1^{(1)}\sin\tau + l_2^{(1)}\sin 2\tau + l_3^{(1)}\sin 3\tau + \ldots\} + \ldots,\\
g &= \nu\{g_1^{(1)}\sin\tau + g_2^{(1)}\sin 2\tau + g_3^{(1)}\sin 3\tau + \ldots\} + \ldots,\\
&\boxed{h = n_1^{(0)}t + \nu\{h_1^{(1)}\sin\tau + h_2^{(1)}\sin 2\tau + h_3^{(1)}\sin 3\tau + \ldots\} + \ldots},
\end{aligned}
\tag{22}
$$

где $\tau = n^{(0)}t$, а коэффициенты $a_i^{(1)}, e_i^{(1)}, \ldots, h_i^{(1)}$ $(i = 1, 2, 3)$ определяются последовательностью формул

“Cams” of angular velocity of Mercury

Динамические параметры, характеризующие поступательно-вращательное движение Луны и Меркурия

Параметры движения	Луна	Меркурий
m_2	$0{,}0123001 \cdot m_{\oplus}$	$1{,}63398 \cdot 10^{-7} \cdot m_{\odot}$
a_0	$384{,}400 \cdot 10^{6}$ м	$578{,}867 \cdot 10^{8}$ м
e_0	$0{,}055$	$0{,}206$
δ	$1{,}5$	$1{,}18$
ν	$4{,}0357 \cdot 10^{-4}$	$1{,}695 \cdot 10^{-4}$
ν'	$0{,}330 \cdot 10^{-8}$	$0{,}124 \cdot 10^{-12}$
ν''	$0{,}399 \cdot 10^{-3}$	$1{,}695 \cdot 10^{-4}$

Аналогичные формулы для параметров Меркурия, принятых в таблице, имеют вид:

$$\nu a_1 = 0{,}0085 \cos\tau + 0{,}0044 \cos 2\tau + 0{,}0010 \cos 3\tau,$$

$$\nu e_1 = 0{,}51 \cdot 10^{-12} \cos\tau + 0{,}14 \cdot 10^{-12} \cos 2\tau + 0{,}034 \cdot 10^{-12} \cos 3\tau,$$

$$\boxed{\nu \left(\frac{H_1}{B}\right) = n_1^{(0)} \{0{,}489 \cdot 10^{-4} \cos\tau} - 0{,}119 \cdot 10^{-4} \cos 2\tau\},$$

$$\nu l_1 = 0{,}4240'' \cdot 10^{-6} \sin\tau + 0{,}1264'' \cdot 10^{-6} \sin 2\tau + 0{,}0416'' \cdot 10^{-6} \sin 3\tau,$$

$$\boxed{\nu h_1 = 10{,}098'' \cdot \sin\tau} - 2{,}414'' \cdot \sin 2\tau, \quad \tau = n^{(0)} t.$$

Astronomical and Astrophysical Transactions, 2001, Vol. 19, pp. 769–780
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ON INTEGRABLE CASES OF THE POINCARE PROBLEM

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(Received June 25, 1999)

The equations of motion for the classical Poincaré problem of the rotational motion of a rigid body with an ellipsoidal cavity containing liquid in canonical Andoyer variables have been obtain. Three integrable cases of this problem were established and their full systems of integrals and general solutions were constructed.

KEY WORDS Poincaré problem, integrable cases, Andoyer variables

1 INTRODUCTION

Oscillations of a rotating liquid core in the cavity of a rigid envelope have been studied by many authors, starting from the classical works of Poincaré (1910). The basis of these studies usually contains the equations of motion in quasi-coordinates, but in recent years a new approach to the classical problem has been suggested, based on the equations of motion in Andoyer variables (Sevilla and Romero, 1987; Getino and Ferrandiz, 1997; etc.). Andoyer variables and angle-action variables (for the Euler–Poinsot problem) were effectively used for studies of integrability of the native Kirgoff problem (Barkin and Borisov, 1989).

The papers of Sevilla and Romero (1987), Getino and Ferrandiz (1997) and others were directed to the problems of the Earth's rotation. The principal effects of the liquid core in the Earth's rotation were studied, and amplitudes of lunar and solar perturbations, corrected by the liquidcore influence, were constructed (Getino and Ferrandiz, 1997). The purpose of the above-mentioned papers was to give an analytical description of the Earth's rotation effects and the authors naturally used some simplifications of the equations of motion.

In this work we consider an exact treatment of the classical Poincaré problem in Andoyer variables to study integrable cases of this problem.

The canonical equations of the Poincaré problem in Andoyer variables were obtained. Three cases of the integrability of these equations have been identified

Equations of plane Mercury motion with the liquid core

$$\frac{d(g_s)}{dt} = \frac{\partial \mathrm{K}}{\partial (G_s)}, \quad \frac{d(G_s)}{dt} = -\frac{\partial \mathrm{K}}{\partial (g_s)} \quad . \tag{14}$$

On the basis of formulae (5) - (10) for the Hamiltonian K we obtain the following expression:

$$\mathrm{K} = \frac{1}{2}\left[G^2 \frac{C_c}{\Delta} + G_c^2 \frac{C}{\Delta}\right] + GG_c \frac{D_c}{\Delta} - U(g,t), \tag{15}$$

where the force function is defined by trigonometric series (8), $\Delta = C_c C - D_c^2$. The right hand sides of equations (14), (15) and (8) are the periodic functions of time. It means that the Poincare theory of periodic solutions can be applied to these equations. $C_c = \frac{1}{5} m_c (a^2 + b^2),\ D_c = \frac{2}{5} m_c ab$

2.1.2 Force function of the problem. This function in considered problem is identified with second harmonic of the gravitational potential of Mercury and the Sun. In accordance with Barkin (1979b) for this function we have following trigonometric development:

$$U_2 = n^2 \frac{m^*}{m^* + m} \frac{C}{I} \sum_{\sigma=0}^{\infty} \left\{ \frac{1}{2} J_2 X_\sigma^{-3,0} \cos \sigma M + 3C_{22} \left[X_\sigma^{-3,2} \cos(\sigma M - 2g) + X_{-\sigma}^{-3,2} \cos(\sigma M + 2g) \right] \right\}, \tag{8}$$

Mercury resonant librations

$$\varphi = \frac{3}{2}\frac{B-A}{C_m} f(e)\sin(M)$$

Peale, 1976; 2003
Dehant et al., 2004

Barkin (1976, 1979)

$$\delta g = -6C_{22}\frac{C}{I[C - C_c(D_c^2/C_c^2)]}\sum_{\substack{\sigma=1\\ \sigma\neq 3}}^{\infty}\left(\frac{X_{\sigma}^{-3.2}}{(\sigma - N)^2}\sin[(\sigma - N)M] - \frac{X_{-\sigma}^{-3.2}}{(\sigma + N)^2}\sin[(\sigma + N)M]\right),$$

$$\delta\omega = -6C_{22}n\frac{1}{I}\frac{C}{[C - C_c(D_c^2/C_c^2)]}\sum_{\substack{\sigma=1\\ \sigma\neq 3}}^{\infty}\left(\frac{X_{\sigma}^{-3.2}}{\sigma - N}\cos[(\sigma - N)M] - \frac{X_{-\sigma}^{-3.2}}{\sigma + N}\cos[(\sigma + N)M]\right),$$

$$\frac{C_c}{\Delta} = \frac{1}{C - C_c(D_c^2/C_c^2)} \approx \frac{1}{C - C_c} = \frac{1}{C_m}$$

Analytical formulae of Mercury librations

$$\phi = \frac{3}{2}\frac{(B-A)}{C} f(e)\sin(nt)$$

(Peale, 1988)

$$\begin{aligned}\delta g = \frac{C_{22}}{I}\frac{C}{C - C_c(D_c^2/C_c^2)}\Bigg[& 6\left(1 - 11e^2 + \frac{959}{48}e^4 - \frac{3641}{288}e^6 + \frac{11359}{2880}e^8\right)\sin M \\ & - \frac{3}{4}e\left(1 + \frac{421}{12}e^2 - \frac{32,515}{384}e^4 + \frac{2,186,863}{32,256}e^6 - \frac{428,399,713}{15,482,880}e^8\right)\sin(2M) \\ & - \frac{1}{24}e^4\left(533 - \frac{13,827}{10}e^2 + \frac{728,889}{560}e^4\right)\sin(3M) \\ & + \frac{1}{128}e^3\left(1 - \frac{57,073}{20}e^2 + \frac{7,678,157}{960}e^4 - \frac{298,080,597}{34,560}e^6\right)\sin(4M) + \cdots\Bigg].\end{aligned}$$

(Barkin, 1979)

Phenomenon of Mercury non-perturbation

(ii) In the time intervals of 7.4 days before passing the pericentre of the orbit and 7.4 days after it, the angular velocity of Mercury with a high accuracy maintains a constant pericentre value. This interval of time of 14.8 days can be called 'the period of Mercury's non-perturbation'.

Barkin (1979, 2003)
Barkin, Ferrandiz, 2005

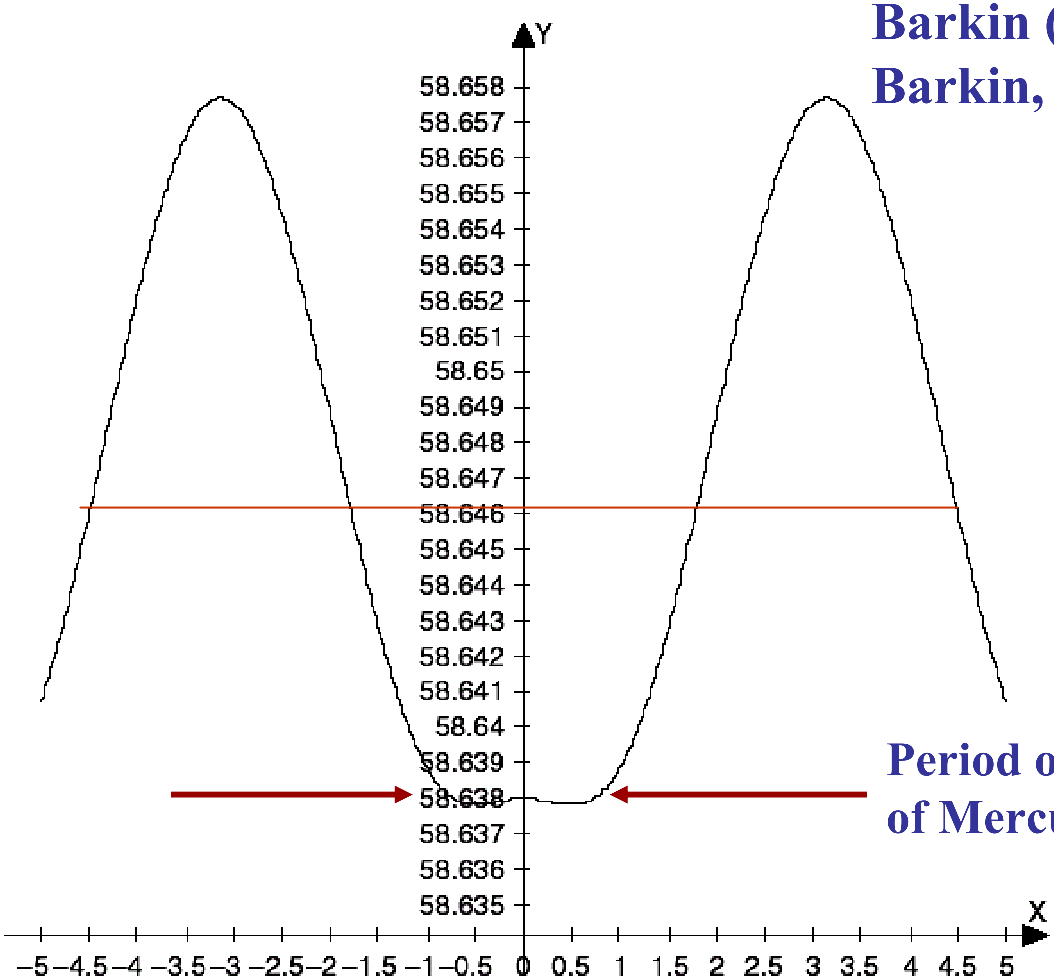


Figure 2. Seasonal variation in Mercury's period: $Y = T$ (in days); $X = M$ (in radians).

Earth-based libration observing strategy

- **Illuminate Mercury with monochromatic radio signal from Goldstone radar (l=3.5 cm) during ~10 minutes round-trip light time.**
- **Record echoes at Goldstone and at the Green Bank Telescopes for ~10 minutes.**
- **Perform cross-correlations between amplitude fluctuations recorded at both telescopes.**

Wisdom, Margot

Large Longitude Libration of Mercury Reveals a Molten Core

J. L. Margot,[1]* S. J. Peale,[2] R. F. Jurgens,[3] M. A. Slade,[3] I. V. Holin[4]

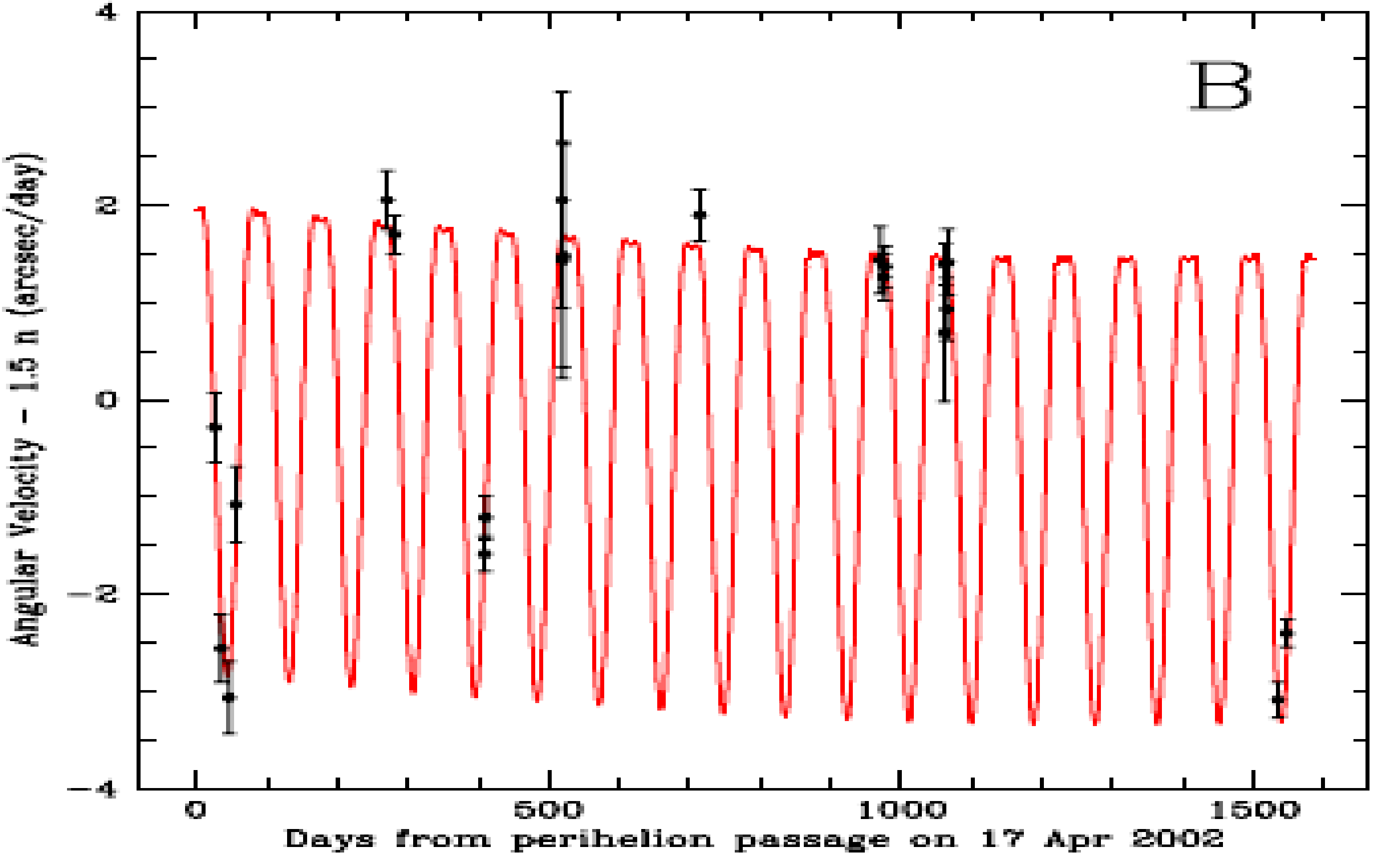
Observations of radar speckle patterns tied to the rotation of Mercury establish that the planet occupies a Cassini state with obliquity of 2.11 ± 0.1 arc minutes. The measurements show that the planet exhibits librations in longitude that are forced at the 88-day orbital period, as predicted by theory. The large amplitude of the oscillations, 35.8 ± 2 arc seconds, together with the Mariner 10 determination of the gravitational harmonic coefficient C_{22}, indicates that the mantle of Mercury is decoupled from a core that is at least partially molten.

(2’27   Баркин, 1978-2007)

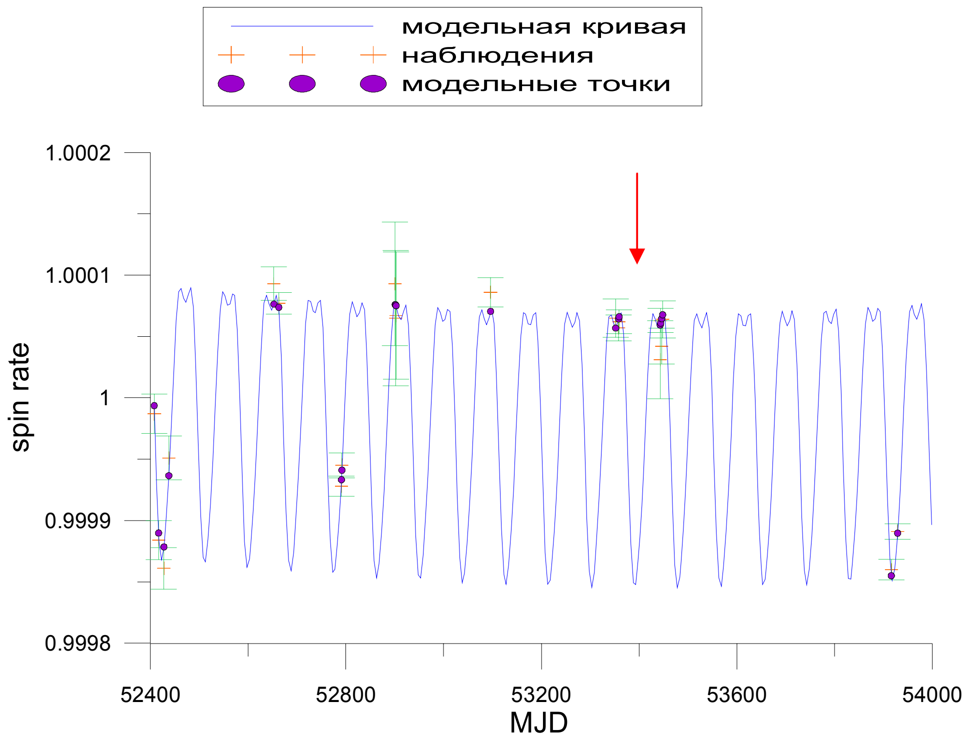
Stan Peale and his wife, Namur, 2005

Barkin – Ward, Cannes, 2004

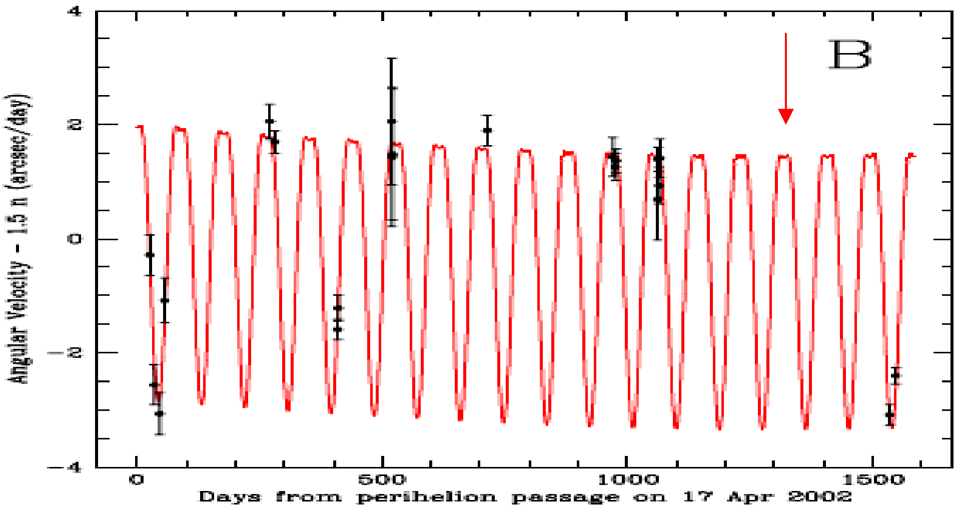
Modeled and observed variations of the angular velocity of Mercury



(Margot, Peale, Jurgens, Slade, Holin, 2007)



Barkin, Zotov, 2007



Margot, Peale et al., 2007

Амплитуда резонансной либрации в долготе

$$\mathbf{A_g = 6'9 \pm 2'0}$$

Вариации угла вращения Меркурия

$$\boldsymbol{\delta g = (34''9 \pm 2''9)\sin(M) + (4''5 \pm 1''1)\sin(2M + 13^0 4) - (1''3 \pm 0''6)\cos(3M - 43^0 8)}$$

$$\mathbf{+(413''6 \pm 117.1)\cos(L - 9^0 1)}$$

.

$$M = M^* - 254^0 1 \qquad L = L^* - 254^0 1$$

Solar tides and variations of gravitational potential of Mercury

Tidal variations of coefficients of gravitational potential of Mercury

2.3 *Tidal variations in Mercury's gravitational field*

Above we have discussed the constant components of the gravitational parameters J_2 and C_{22} caused by tidal and rotational deformations of satellites. Here we also present the general formulae for periodic tidal variations in the coefficients of the second harmonic of Mercury's gravitational potential ($N = 3$):

Love number $k_2 = 0.37$

(Dehant et al., 2004)

$$\delta J_2 = 3\frac{D_t}{mR^2}\sum_{\sigma=1}^{\infty} X_\sigma^{-3.0}(e)\cos(\sigma M),$$

$$\delta C_{22} = \frac{3D_t}{2mR^2}\sum_{\sigma=1}^{\infty}[X_\sigma^{-3.2}(e)\cos(\sigma M - 2g) + X_{-\sigma}^{-3.2}(e)\cos(\sigma M + 2g)], \tag{6}$$

$$\delta S_{22} = \frac{3D_t}{2mR^2}\sum_{\sigma=1}^{\infty}[X_\sigma^{-3.2}(e)\sin(\sigma M - 2g) + X_{-\sigma}^{-3.2}(e)\sin(\sigma M + 2g)].$$

Barkin, Yu.V. 2004. **Comparative rotational dynamics of the Moon, Mercury and Titan.** Astr.& Astroph. Transact., v. 23, Issue 5, pp.481-492.

Tidal variations of coefficients of second harmonic of gravitational potential of Mercury (Barkin, 2004)

$$
\begin{aligned}
(\delta J_2)_{\text{periodic}} = 10^{-8}[&2.6991\cos M + 0.8203\cos(2M) + 0.2450\cos(3M) \\
&+ 0.0723\cos(4M) + 0.0211\cos(5M) + 0.0061\cos(6M) \\
&+ 0.0018\cos(7M) + 0.0005\cos(8M) + 0.0002\cos(9M)], \\
(\delta C_{22})_{\text{periodic}} = 10^{-8}[&5.0907\cos M + 0.1489\cos(2M) + 0.2220\cos(3M) \\
&+ 0.0816\cos(4M) + 0.0283\cos(5M)], \\
(\delta S_{22})_{\text{periodic}} = 10^{-8}[&-2.3734\ \sin M + 1.0012\sin(2M) + 0.2220\sin(3M) \\
&+ 0.0816\sin(4M) + 0.0283\sin(5M)].
\end{aligned}
$$

The Earth (Cheng, Taplay, 2004)

$$\delta J_2 = -(2.80 \pm 0.22) \times 10^{-10} \cos(V - 30 \pm 19)$$

EPSC Abstracts,
Vol. 2, EPSC2007-A-00259, 2007
European Planetary Science Congress 2007

Dynamic role of the liquid core of Mercury in its motion on Cassini's laws and in resonant librations

Yu. Barkin (1), J.M. Ferrandiz (2)

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Mercury

		Mariner-10
$J_2 = (6.0 \pm 2.0) \times 10^{-5}$		33%
$C_{22} = (1.0 \pm 0.5) \times 10^{-5}$		50%
	(Barkin et al., 2007)	MESSENGER
$J_2 = (4.23 \pm 0.06) \times 10^{-5}$	1.4%	0.5 %
$C_{22} = (0.85 \pm 0.05) \times 10^{-5}$	5.9%	0.2 %
$\delta C_{22} = 0.005 \times 10^{-5} \cos M$	0.6 %	
$T_g = 12.2 \pm 0.3$ yr	2.5 %	---
$T_l = 426 \pm 25$ yr	5.9%	---
$T_h = 1462 \pm 69$ yr	4.7%	---
$T_{l_c} = 58.6252$ days		

Barkin Yu.V., Ferrandiz J.M. (2007) Dynamic role of the liquid core of Mercury in its motion on Cassini's laws and in resonant librations. Abstracts of European Planetary Science Congress (Potsdam, Germany, 19 – 24 August 2007) Vol.2, EPSC 2007-A-00259.

Выводы

Приливные деформации Луны являются значительными. Основные вариации коэффициентов селенопотенциала будут доступны спутниковым наблюдениям в современных японских миссиях.

Годовая вариация коэффициента C22 гравитационного потенциала Меркурия в принципе доступна наблюдениям по программе MESSENGER в ближайшие годы.

Важнейшей задачей небесной механики и астрометрии является развитие и построение аналитических теорий вращения Луны и Меркурия, рассматриваемых как системы оболочек (упругая мантия, жидкое ядро, твердое ядро).