ISSN 1063-7729, Astronomy Reports, 2020, Vol. 64, No. 7, pp. 556–562. © Pleiades Publishing, Ltd., 2020.
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Exoplanet Transients: Possible Variations in the Limb-Darkening Coefficients of Eclipsed Stars at Short Time Intervals

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Received February 27, 2020; revised March 30, 2020; accepted March 30, 2020

Abstract—The interpretation of high-precision transit light curves of binary systems with exoplanets Kepler-5b, Kepler-6b, and Kepler-7b is performed for three different epochs. It has been demonstrated that the values of the stellar limb-darkening coefficients differ significantly for each epoch, while the geometric parameters for each epoch agree well with each other within the error margin. It is shown that reliable determination of the limb-darkening coefficients requires methods that would "clear" the observed transit light curves from effects caused by surface inhomogeneity.

DOI: 10.1134/S1063772920080016

1. INTRODUCTION

Transit light curves provide important data not only on the geometrical parameters of a binary system (star radius, planet radius, orbit inclination), but also on the limb-darkening coefficients, which indirectly contain important information about the star's atmosphere. Thus, the adequacy of the classical limb darkening model to observational data is often questioned, in particular, due to various inhomogeneous structures on the star's surface.

As part of this problem, the authors obtained empirical values of the limb-darkening coefficients of the parent stars of well-studied binary systems with exoplanets Kepler-5b, Kepler-6b, and Kepler-7b for different epochs. The empirical values of the limb-darkening coefficients were obtained based on the transit light curves of the binary systems from [1–3]. The calculation of the limb-darkening coefficients is performed for the quadratic limb darkening law of the stellar disk.

The original transit light curves contain approximately 2100 brightness values obtained over 44 days. The authors divided these data into three equal sets ~15 days each. Each such observational set contained approximately 700 brightness values of the binary system. The interpretation results showed that the geometrical parameters of each obtained binary based on observational data of each set closely agrees with each other. At the same time, the values of the limb-darkening coefficients differ significantly.

2. INTERPRETATION METHOD

The method for interpreting the observed transit light curves of a binary system with an exoplanet is based on the algorithm for high-precision calculation of brightness during the planet transit across the star's disk; the algorithm is described in a series of studies [4–9].

A model of two spherical stars in a circular orbit in the absence of reflection and ellipsoidal effects was used. The relative radius of the Roche lobe is tens of times larger than the planet's radius [10]. Thus, the assumption that the planet is spherical is well founded. The same can be said of the optical star.

In the calculation of the light curve, the quadratic limb-darkening law was used as a function of the brightness distribution over the stellar disk:

$$I(\rho) = I_0\left[1 - x\left(1 - \sqrt{1 - \frac{\rho^2}{r_s^2}}\right) - y\left(1 - \sqrt{1 - \frac{\rho^2}{r_s^2}}\right)^2\right]. \quad (1)$$

Here, ρ is the polar distance from the stellar disk's center, I_0 is the brightness at the disk's center, and r_s is the stellar disk's radius. The brightness at the planet's center, as well as the brightness at any point on its disk, is assumed to be zero. The planet eclipses the star at orbital phase $\theta = \pi$. The unit of length in the models is the distance between the mass centers of the star and the planet a ($a = 1$); the orbit is considered circular. The "third light" in the model is absent. The planet's radius is denoted by r_p. The desired parameters of the model are the radii of the star and the planet r_s and r_p, the angle of orbital inclination i, the limb darkening

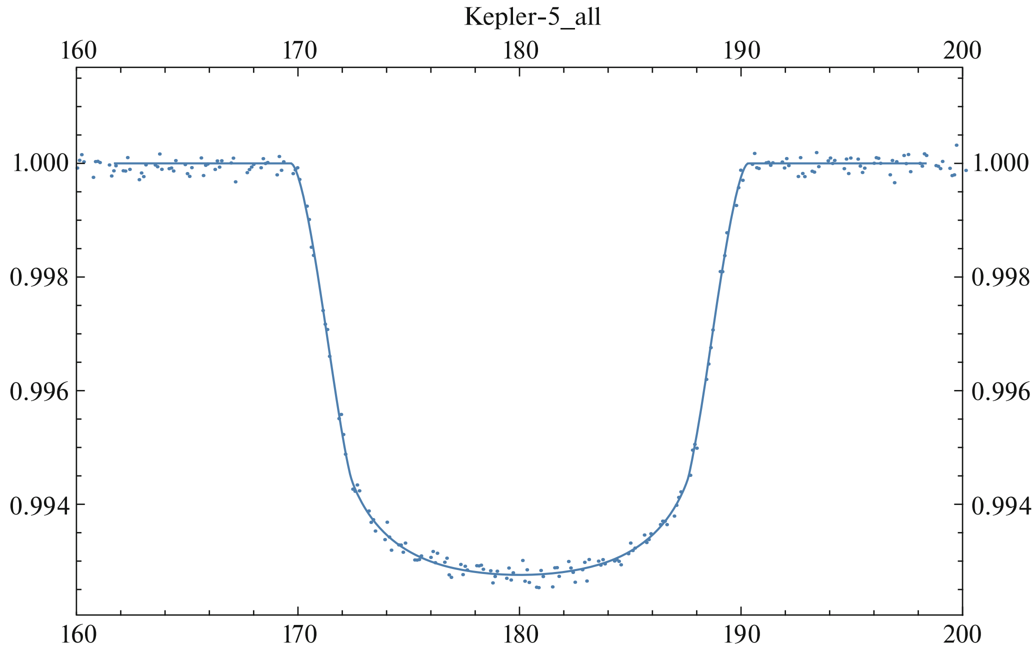

Fig. 1. The observed light curve of the binary system with exoplanet Kepler-5b obtained in the filter r from [1]. The solid line is the optimal theoretical curve obtained under the assumption of the quadratic law of the stellar disk's limb darkening.

coefficient x, as well as the limb-darkening coefficient y in the case of the quadratic limb-darkening law.

The system's total brightness is assumed to be known; in the adopted normalization, it is equal to unity. The observed brightness values are taken to be distributed normally. The standard deviations σ of the observed brightness values are also assumed to be known.

3. OBSERVATIONAL MATERIAL

In this study, the high-precision transit light curves of binary systems with exoplanets Kepler-5b, Kepler-6b, and Kepler-7b from [1–3] was studied. The light curves were obtained at the Kepler Space Observatory from May 1 to June 14, 2009.

The star systems Kepler-5b, Kepler-6b, and Kepler-7b are objects of ~13^m magnitude. The light curves are obtained in the photometric filter r of Gann's photometric system (ugriz). The central bandwidth λ_0 = 6550 Å; half bandwidth $\Delta\lambda$ = 900 Å. The transit light curves of each system include approximately 2100 individual brightness values, most of which lie in the extra-eclipse part of the light curve.

The accuracy of the transit light curves of the binary systems Kepler-5b, Kepler-6b, and Kepler-7b in intensities was $\sigma = 1.3759 \times 10^{-4}$, $\sigma = 1.2874 \times 10^{-4}$, and $\sigma = 1.0248 \times 10^{-4}$, respectively. The relative error (with respect to the eclipse depth) of the transit light curves is ~ 1%. The observed transit light curves of the Kepler-5b, Kepler-6b, and Kepler-7b systems are shown in Figs. 1–3, respectively.

4. ANALYTICAL RESULTS OF THE TRANSIT LIGHT CURVES

In astrophysical studies, there is a need to consider the limb-darkening of the stellar disk. The limb-darkening coefficients under the assumption of various darkening laws have been repeatedly calculated in numerous theoretical one-dimensional models of thin stellar atmospheres (e.g., ATLAS and PHOENIX). However, high-precision transit light curves make it possible to obtain empirical values of stellar limb-darkening coefficients directly from the observational material.

The authors of this paper have set the task of finding out the reliability of limb-darkening coefficients based on the calculations of a high-precision transit light curve. The light curves of binary systems with exoplanets Kepler-5b, Kepler-6b, and Kepler-7b were used for this purpose. As noted above, each transit light curve included approximately 2100 individual brightness values obtained over a period of 44 days. The authors divided the 44-day light curve into three observational sets. The first set included individual brightness values obtained from the 1st day of observation to the 15th (t_1); the second set, from the 15th day to the 30th (t_2); and the third set, from the 30th day to the 44th (t_3). Each observational set included approximately 700 values of light curves. In this study, the

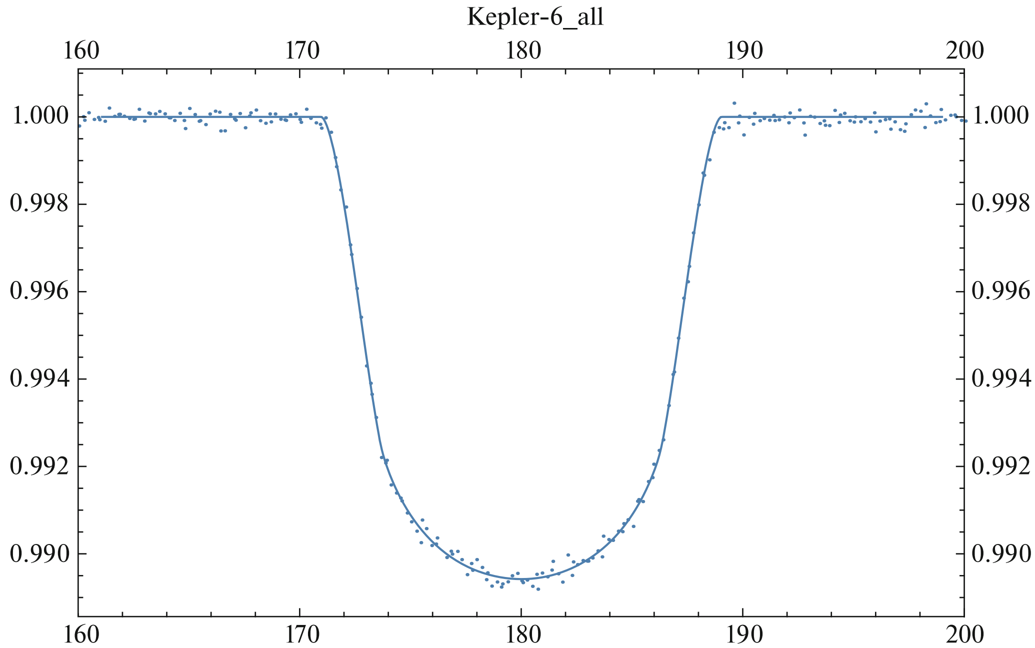

Fig. 2. The observed light curve of the binary system with exoplanet Kepler-6b obtained in the filter r from [2]. The solid line is the optimal theoretical curve obtained under the assumption of the quadratic law of the stellar disk's limb darkening.

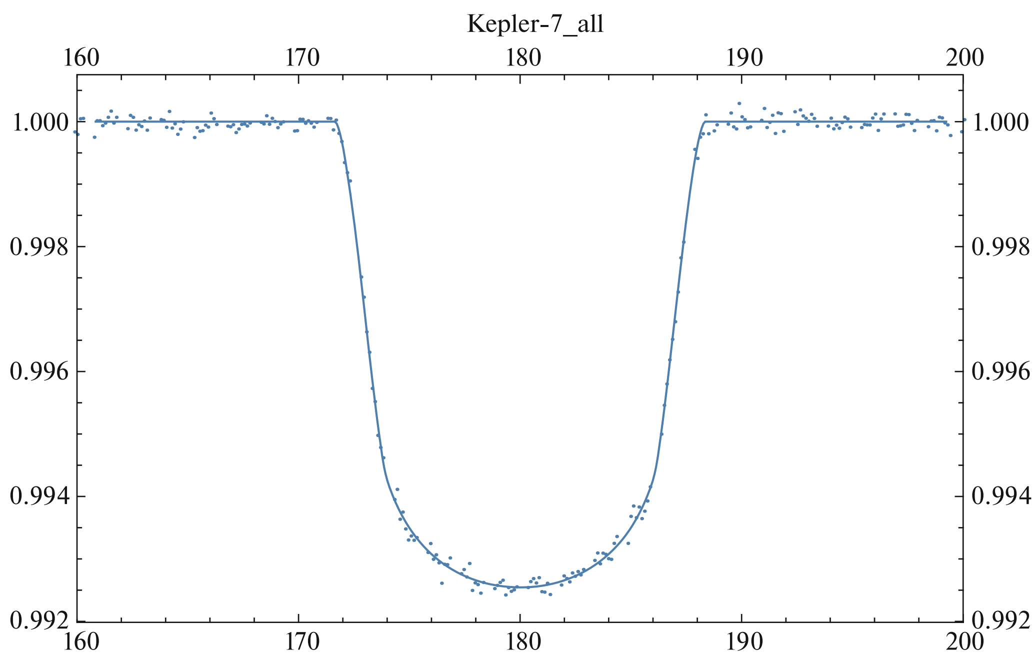

Fig. 3. The observed light curve of the binary system with exoplanet Kepler-7b obtained in the filter r from [3]. The solid line is the optimal theoretical curve obtained under the assumption of the quadratic law of the stellar disk's limb darkening.

parameters of the binary based on the transit light curves of these observational sets were determined. An analysis of the total transit light curve (t_4) that included ~2100 individual brightness values was performed as well.

The coefficients were calculated under the assumption of a quadratic limb-darkening law in accordance with (1). The discrepancy was minimized simultaneously in all parameters. The desired parameters were the radius of the star r_s, radius of the planet r_p, orbital

Table 1. Light curve analysis of the binary system with exoplanet Kepler-5b in the epochs t_1, t_2, t_3, t_4 and the theoretical values of the limb-darkening coefficients from [11]

Parameters	1^d–15^d (t_1)	15^d–30^d (t_2)	30^d–44^d (t_3)	1^d–44^d (t_4)	Theory
x	0.256 ± 0.26	-0.270 ± 0.42	-0.343 ± 0.33	-0.0632 ± 0.18	0.279
y	0.297 ± 0.36	1.041 ± 0.66	1.241 ± 0.57	0.744 ± 0.27	0.363
r_s	0.2069 ± 0.0046	0.2129 ± 0.0056	0.2029 ± 0.0063	0.2091 ± 0.0029	–
r_p	0.0172 ± 0.00056	0.0174 ± 0.00070	0.0163 ± 0.00074	0.0172 ± 0.00036	–
i, deg	82.03 ± 0.47	81.64 ± 0.62	82.65 ± 0.74	81.91 ± 0.33	–
χ^2	1.165	0.980	0.999	1.049	–

Table 2. Light curve analysis of the binary system with exoplanet Kepler-6b in the epochs t_1, t_2, t_3, t_4 and the theoretical values of the limb darkening coefficients from [11]

Parameters	1^d–15^d (t_1)	15^d–30^d (t_2)	30^d–44^d (t_3)	1^d–44^d (t_4)	Theory
x	0.319 ± 0.28	0.382 ± 0.22	0.748 ± 0.23	0.386 ± 0.24	0.366
y	0.440 ± 0.44	0.314 ± 0.35	-0.144 ± 0.33	0.374 ± 0.32	0.314
r_s	0.1801 ± 0.0025	0.1780 ± 0.0026	0.1949 ± 0.0027	0.1785 ± 0.0016	–
r_p	0.01795 ± 0.00040	0.01769 ± 0.00042	0.02023 ± 0.00044	0.0177 ± 0.00029	–
i, deg	82.98 ± 0.27	83.11 ± 0.0029	81.64 ± 0.30	83.15 ± 0.18	–
χ^2	1.051	1.015	1.47	1.073	–

Table 3. Light curve analysis of the binary system with exoplanet Kepler-7b in the epochs t_1, t_2, t_3, t_4 and the theoretical values of the limb darkening coefficients from [11]

Parameters	1^d–15^d (t_1)	15^d–30^d (t_2)	30^d–44^d (t_3)	1^d–44^d (t_4)	Theory
x	0.511 ± 0.21	0.0428 ± 0.75	0.104 ± 0.30	0.226 ± 0.15	0.316
y	0.0384 ± 0.28	0.657 ± 1.0	0.658 ± 0.50	0.435 ± 0.22	0.344
r_s	0.1710 ± 0.0028	0.1734 ± 0.0046	0.1701 ± 0.0038	0.1711 ± 0.0019	–
r_p	0.01453 ± 0.00037	0.01448 ± 0.00058	0.01407 ± 0.00054	0.01433 ± 0.00025	–
i, deg	83.24 ± 0.29	83.14 ± 0.45	83.46 ± 0.44	83.3 ± 0.19	–
χ^2	0.919	1.0693	1.20	1.054	–

inclination i, linear coefficient x, and quadratic coefficient y of stellar limb darkening in accordance with (1).

The analytical results of individual light curve sets and the total light curve of the binary systems Kepler-5b, Kepler-6b, and Kepler-7b are presented in Tables 1–3. The errors of the sought parameters were found using the Monte Carlo method.

The last column ("Theory") of Tables 1–3 shows the theoretical values of the limb-darkening coefficients from [11] obtained under the assumption of a quadratic limb-darkening law. The theoretical values of the limb-darkening coefficients are given for the r filter (ugriz photometric system).

The part of the light curve near the minimum is most sensitive to the values of the limb-darkening coefficients. Figures 4–6 show parts of optimal theoretical light curves for the observational data from epochs t_1, t_2, t_3, and t_4. It can be seen that the value scatter of the optimal light curves for different epochs near the minimum is hundredths and thousandths of a percent.

5. DISCUSSION

The above tables, which contain the limb-darkening coefficients and the geometric parameters, show that the values of the geometric parameters for different observational sets closely agree with each other. At the same time, the values of the limb-darkening coefficients differ significantly. It should also be noted that the obtained limb-darkening coefficients based on the

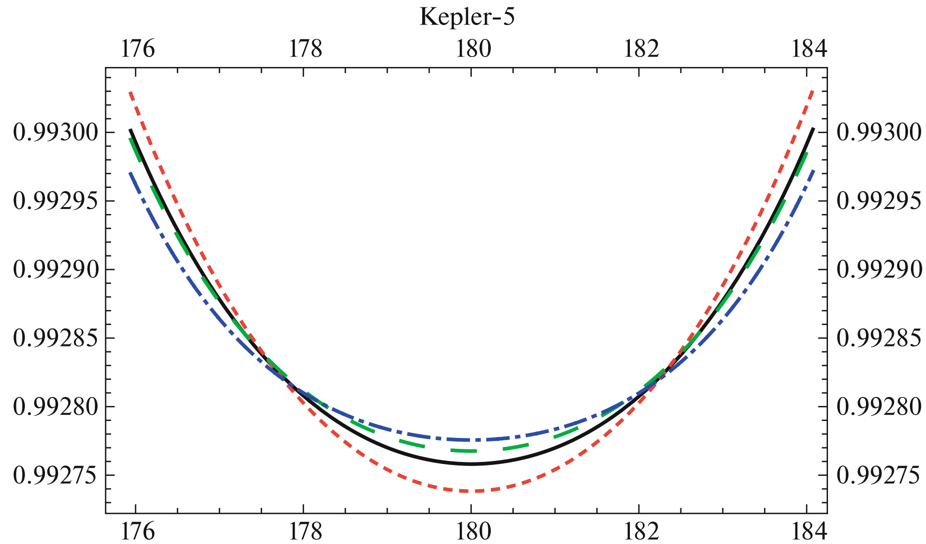

Fig. 4. Fragments of optimal light curves of the Kepler-5b binary system obtained from the observational data in the epochs t_1 (small dashed (red) line), t_2 (dashed (green) line), t_3 (dash-dotted (blue) line), and t_4 (solid (black) line).

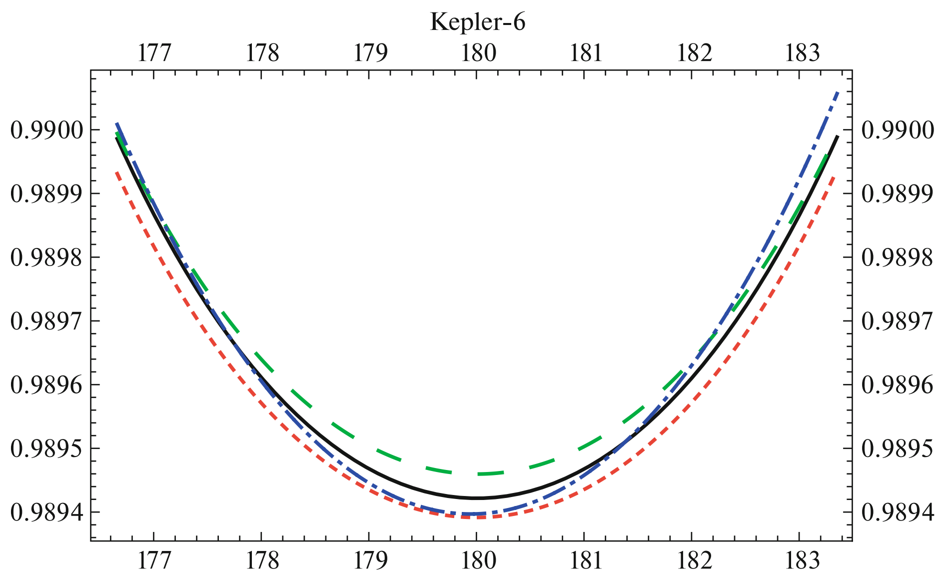

Fig. 5. Same as in Fig. 4, for the Kepler-6b binary system.

light curves of different epochs also differ significantly from the theoretical values of [11] in the vast majority of cases.

The limb-darkening coefficients are very sensitive to inhomogeneities of the transit light curve. These inhomogeneities can be associated with various physical processes on the star's surface, as well as processes in a binary system.

First of all, inhomogeneities in the light curve can be caused by spots on the star's surface. For example, in [12], modulations of star activity and their seasonality were revealed based on a four-year analysis of the light curve of a K-class star Kepler-210. It was shown that the development of activity modulations follows a certain pattern, which resembles variations from sunspots during the solar magnetic cycle. Based on the assumption of differential rotation, the authors of [12] estimated the spot lifetime of the Kepler-210 star at 60–90 days. The mapping of the surfaces of late-type stars with a surface temperature of 3000–6500 K [13] on the basis of transit light curves reliably confirm the presence of spots on the surface of these stars.

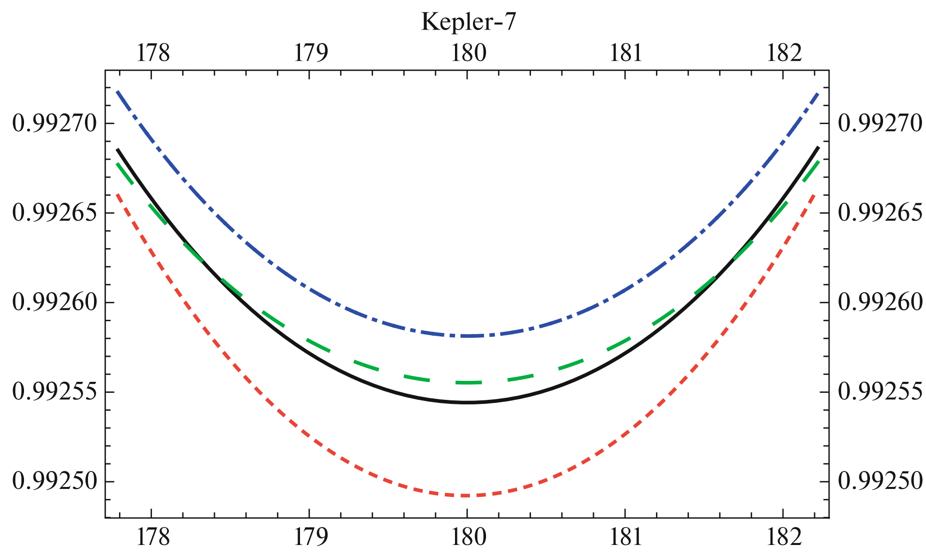

Fig. 6. Same as in Fig. 4, for the Kepler-7b binary system.

We can reasonably assume that the radii and rotation periods of the stars Kepler-5, Kepler-6, and Kepler-7 are close to the radius and rotation period of the Sun. A few or dozens of medium-sized spots (~10 000–30 000 km) on the Sun give variations in the Sun's brightness of several hundredths and thousandths of a percent.

A similar behavior of light curves at time intervals of the half period of the Sun's rotation around its axis is also observed for the stars Kepler-5, Kepler-6, and Kepler-7. As noted above, the optimal light curves for epochs t_1, t_2, t_3 differ from each other (see Figs. 4–6) by thousandths and hundredths of a percent (or 10^{-5}–10^{-4} in relative intensity values). Thus, the observed variations in the light curves near the minimum at time intervals of ~15 days indirectly imply the presence of spots on the surface of the stars Kepler-5, Kepler-6, and Kepler-7.

It should be noted that inhomogeneity of the transit light curve can also be induced by a planet with a shape that differs from a sphere (despite the fact that the planet does not fill the Roche lobe). Three-dimensional gas-dynamic modeling shows that the atmosphere of a hot Jupiter actively escapes due to its proximity to the star [14]. The picture of a hot Jupiter's outflow is complicated manifold by the shock wave formed in the collision of the stellar wind and the planet's atmosphere, as well as by the magnetic field interaction of the star and the planet; as a result, the approximation of the planet by a sphere may not be sufficiently correct [15].

It is possible that the discrepancy between the theoretical and observed limb-darkening coefficients found in [6, 16] is caused by physical processes beyond the scope of the model used for the interpretation of the binary system. In order to make responsible conclusions about the values of the observed limb-darkening coefficients, the transit light curves should be maximally "cleared" of the possible influence of inhomogeneities of the stellar surface. This will require observations during the periods of a star's minimal activity.

6. CONCLUSIONS

A significant difference in the limb-darkening coefficients indicates the presence of active physical processes on the star's surface. Determining the exact values of the limb-darkening coefficients based on the transit curve is difficult and requires special methods that compensate and consider the influence of the inhomogeneous surface of the star on the transit curve (e.g., [17]).

Otherwise, the limb-darkening coefficients should be determined on an observational data set covering several rotation periods of a star. An observational data set containing brightness values for several rotation periods will statistically average and thereby minimize the influence of spots when determining the limb-darkening coefficients of a star.

It is important to note that transit light curves do not only provide data about the geometrical parameters of a binary system (stellar radius, planetary radius, orbital inclination), yet they are also a unique source of information regarding limb-darkening of various spectral-class stars. From this point of view, a binary system with an exoplanet is a unique opportunity to acknowledge the stellar atmosphere physics and the formation of the limb-darkening effect of a stellar disk.

ACKNOWLEDGMENTS

The authors thank Academician A.M. Cherepashchuk for helpful advice and discussions on this study.

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Translated by M. Chubarova